Soil Mechanics for Unsaturated Soils

Delwyn G. Fredlund
University of Saskatchewan
Saskatoon, Sask., Canada

and

Harianto Rahardjo
Nanyang Technological University
Singapore

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# Soil Mechanics for Unsaturated Soils

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Notes:

- Different approaches could be taken towards synthesizing the material on the behavior of unsaturated soil mechanics. The approach deemed most desirable was to maintain a relatively close parallel to classical saturated soil mechanics.
- The emphasis was not focused solely on “problematic soils”, but rather on the areas of seepage, shear strength and volume change of all soils with negative pore-water pressures.
- The prepared notes closely follow the material in the textbook. Additional notes have been prepared on new and important subjects that have emerged during the years subsequent to 1993.
- The textbook appears to have filled an important role in centralizing our understanding of the behavior of unsaturated soils from the standpoint of two independent stress state variables.

Additional notes:
Format For Each Constitutive Behavior

- **Constitutive Relationship** (e.g., seepage, shear strength, volume change)
  - *Theory* associated with the Constitutive Relationship
  - *Measurement* of the associated unsaturated soil properties
  - *Estimation* of the unsaturated soil property functions *through use of SWCC*
  - *Application* of the constitutive relationship to practical engineering problems
Important Objective of This Course

• To teach Geotechnical Engineers to think the way the **Unsaturated Soil** behaves

• The **Physics** must be correct

• Many behavioral aspects related to **Unsaturated Soils** are the opposite to saturated soil behavior (e.g., hydraulic conductivity of sands capillary barriers)
Categorization of Soil Mechanics

Emphasis of Soil Mechanics textbooks

- Combination of Engineering Mechanics and the Properties of Soils

SOIL MECHANICS

SATURATED SOIL MECHANICS

NATURAL SILTS & CLAYS

SANDS AND GRAVELS*

u_w GENERALLY \geq 0

Two-Phase Behavior

(\sigma - u_w) or
(\sigma - u_a)

* may be saturated or dry

UNSATURATED SOIL MECHANICS

NATURAL, DESICCATED SILTS AND CLAYS
TRANSPORTED SOILS
RESIDUAL SOILS

COMPACTED SILTS & CLAYS

u_w GENERALLY \leq 0

More than Two-Phases

(\sigma - u_a) and
(u_a - u_w)

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CATEGORIES OF UNSATURATED SOIL MECHANICS PROBLEMS

Description of STRESS STATE VARIABLES $(\sigma - u_a)$ and $(u_a - u_w)$

SEEPAGE
- Flux Boundary Conditions
- Saturated/Unsaturated Modelling
- Geo-Environmental
- Contaminant Transport

SHEAR STRENGTH
- Slope Stability
- Bearing Capacity
- Lateral Earth Pressures

VOLUME CHANGE
- Swelling
- Shrinkage
- Collapsing
- Deformation

Based on Constitutive Behaviour

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CLIMATIC CHARACTERISTICS

- Approximately 33% of the earth’s surface is Arid or Semi-arid
- Climate gives rise to a continuously changing FLUX Boundary Condition at the ground surface

- An UPWARD Flux is produced as water is removed through:
  i.) Evaporation of water from the soil surface,
  ii.) Evapo-transpiration from vegetative cover
- The result is a Drying, Cracking and Desaturation of the soil

ACTUAL EVAPORATIVE FLUX depends on the pore-water stress state and is difficult to predict
POTENTIAL EVAPORATIVE FLUX is from a water surface and depends primarily on temperature (e.g., Thornthwaite Moisture Index)

- A DOWNWARD Flux is produced by rain and other forms of precipitation. The result is an attempt to saturate the soil

- The DIFFERENCE between the UPWARD and the DOWNWARD Flux largely dictates the location of the water table, and therefore the location of the negative pore-water pressure zone

Moisture flux is an aspect originally omitted from Soil Mechanics

Coupled mass-transport & thermal analysis

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CAPILLARY FLOW PHENOMENA

Syphon Effect on an Earth Dam with a Core-wall

Water Table

Intercept Ditch for a Highway on a Side-hill Location
EXAMPLES OF PROBLEMS REQUIRING AN UNDERSTANDING OF UNSATURATED SOIL MECHANICS

1.) THE DESIGN, CONSTRUCTION AND OPERATION OF MAN-MADE STRUCTURES SUCH AS A DAM
   a.) During construction of the fill
   b.) During the filling of the reservoir and Rapid Drawdown
   c.) After Steady State seepage conditions are established, but environmental changes occur

2.) NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES
   - Natural slopes often fail after being subjected to heavy precipitation for a long time
   - The slip surface may be relatively shallow, passing through the unsaturated zone

3.) MOUNDING BELOW WASTE RETENTION PONDS
   - Wastes are often stored on the ground surface where the water table is deep
   - Contaminant may move to the groundwater even if the soil remains unsaturated

4.) STABILITY OF VERTICAL OR NEAR VERTICAL EXCAVATIONS
   - Excavation back-slopes often fail some time after excavation
   - Negative pore-water pressures are dissipated due to infiltration

Compacted Soils

Infiltration

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5.) LATERAL EARTH PRESSURES
   - Dry, clayey soils are often used as backfill
   - Can exert high lateral pressures as they become wet

6.) BEARING CAPACITY FOR SHALLOW FOUNDATIONS
   - Foundation design is generally based on unconfined compression tests on soils from above the groundwater table
   - Analysis assumes that negative pore-water pressures are maintained with time

7.) GROUND MOVEMENTS INVOLVING EXPANSIVE SOILS
   - Light structures suffer distress as a result of environmental changes or man-made effects

8.) COLLAPSING SOILS
   - There may be volume change or a loss of shear strength resulting from pore-water pressure changes

9.) FLOW THROUGH RESIDUAL SOILS
   - Water infiltration into Residual soils often results in instability of slopes

Problematic Soils

Bernatzik, 1948

UNSATURATED SOIL TECHNOLOGY
ENVIRONMENTAL EFFECTS DURING THE OPERATION OF THE RESERVOIR

The effect of rainfall on steady state flow through a dam

Water phase

Excessive rainfall (surface flux)

Phreatic surface

Water level

Rock fill

SOME RELEVANT QUESTIONS MIGHT BE ASKED AS STEADY STATE CONDITIONS ARE ESTABLISHED

?
SOME RELEVANT QUESTIONS THAT MIGHT BE ASKED ARE REFERRED TO THE STABILITY OF THE EXCAVATION SLOPES
LATERAL EARTH PRESSURES

Lateral earth pressures against a retaining wall as water infiltrates the compacted backfill.

Effect of a membrane?

SOME RELEVANT QUESTIONS MIGHT BE ASKED PERTAINING TO LATERAL EARTH PRESSURES

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LATERAL EARTH PRESSURES AGAINST WALLS

Example of lateral earth pressures generated subsequent to backfilling with dry soils

Precipitation and lawn watering

Compacted clay backfill

House basement wall

Natural clay

Drain with sand backfill

Lateral earth pressure against a house basement wall
IS THERE A NEED FOR UNSATURATED SOIL MECHANICS?

YES!

The Geotechnical Engineer has the greatest potential to assist the public in circumventing problems associated with Unsaturated Soils.

There is need for an APPROPRIATE TECHNOLOGY which is:
  a.) Practical
  b.) Not too costly to employ
  c.) Has a sound theoretical basis, and
  d.) Runs parallel in concept to Saturated Soil Mechanics

Concern for the environment and advances in computing power greatly assisted in the promotion of Unsaturated Soil Mechanics.
AN UNSATURATED SOIL ELEMENT WITH A CONTINUOUS AIR PHASE

PHASES OF AN UNSATURATED SOIL

An Unsaturated Soil is postulated to have 4 phases
1.) Solids
2.) Air
3.) Water
4.) Contractile Skin (Air-Water Interface)
TERMINOLOGY AND DEFINITIONS

Will use a MACROSCOPIC, PHENOMENOLOGICAL approach to Unsaturated Soil behavior.

Additional terms required from Continuum Mechanics

1.) STATE - Non material variable required for the characterization of a system
2.) STRESS STATE VARIABLE - Variables required for the characterization of the stress state
3.) DEFORMATION STATE VARIABLES - Variables required for the characterization of the deformation conditions or deviations from an initial state
4.) CONSTITUTIVE RELATIONS - Single-valued equations expressing the relationship between state variables

Fung (1969) - are single-valued expressions which relate one state variable to one or more other state variables. They always incorporate the material properties.

Examples are:
- Stress versus Strain Relations (E, μ)
- Ideal Gas Law, relates pressure to density and temperature. Gas constant is the property
- Shear Strength equation (c', φ', φb)
- Pore Pressure Parameter equation (S, m_v, β_w)
Categorization of Soil Mechanics Based on Stress State Variables

Visualized Soil Mechanics

Unsaturated Soil Mechanics

Negative pore-water pressures
Net normal stress \( (\sigma - u_a) \)
Matric suction \( (u_a - u_w) \)

\( (\sigma - u_w) \)
Effective stress

Positive pore-water pressures

Saturated Soil Mechanics

Visualization of the World of Saturated-Unsaturated Soil Mechanics
Stress State at a Point in an Unsaturated Soil and in a Saturated Soil
Summary of the description of the Stress State for an Unsaturated Soil Element with the Preferred Stress State Variables

The stress state variables for an unsaturated soil using the combination of $(\sigma - u_a)$ and $(u_a - u_w)$
Stress State Variables (Unsaturated Soils)

Net Total Stress Tensor

\[
\begin{bmatrix}
\sigma_x-u_a & \tau_{yx} & \tau_{zx} \\
\tau_{xy} & \sigma_y-u_a & \tau_{zy} \\
\tau_{xz} & \tau_{yz} & \sigma_z-u_a \\
(u_a-u_w) & 0 & 0 \\
0 & (u_a-u_w) & 0 \\
0 & 0 & (u_a-u_w)
\end{bmatrix}
\]

• Stress Tensors form the basis for a Science because we live in a 3-D Cartesian coordinate world
Variations in Stress State Description

\[ \sigma' = (\sigma - u_a) + \chi (u_a - u_w) \]

\[ \sigma' = \text{effective stress} \]

\[ \chi = \text{parameter related to saturation} \]

\[ \sigma_{ij}^* = \sigma_{ij} - [S u_w + (1 - S) u_a] \delta_{ij} \]

\[ \sigma_{ij} = \text{total stress tensor}, \]

\[ \delta_{ij} = \text{Kroneker delta or substitution tensor}, \]

\[ \sigma_{ij}^* = \text{Bishop’s soil skeleton stress} \ (\text{Jommi} \ 2000) \]

Above proposed equations are constitutive relations
Separation of the Zones above the Water Table

Legend
- Solids
- Water
- Air
- Dry soil
- Two fluid phases
- Capillary fringe

Unsaturated Soil

Saturated Soil

Occluded air bubbles
Constitutive Relations for Classic Problems in Unsaturated Soils Mechanics

- **Empirical, semi-empirical and theoretical relationships are proposed and verified**
  - Volume change *(Stress versus Strain)*
  - Shear strength *(Stress versus Stress)*
  - Flow *(Velocity versus Stress)*

- **Demanding laboratory experiments**
  - Careful experiments required for uniqueness studies
  - May alter test procedures for economic reasons
Constitutive Relations for Saturated/Unsaturated Soil Mechanics

Shear strength

\[ \tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \]

Unsaturated Soil

Seepage

\[ v = k_w(-u_w) \frac{dh}{dy} \]

Volume change

\[ de = a_1 d(\sigma - u_a) + a_2 d(u_a - u_w) \]

Saturated Soil

\[ v = k_w \frac{dh}{dy} \]

where: \[ h = \frac{u_w}{\gamma_w} + Y \]


Selected Reading Materials

Review of Procedure Typical to Saturated Soil Mechanics

1.) Define geometry of the problem (Surface & Stratigraphy)

2.) Determine pore-water pressures

3.) Determine the soil parameters ($c'$ & $\phi'$)

4.) Use analysis to incorporate total stresses

Pore-water pressures must be assessed because behavior is controlled by $(\sigma - u_w)$
Additional Features to Accommodate Rainfall Induced Landslides

• Slope must be visualized as a transient analysis on a saturated-unsaturated soil profile
• Unsaturated soil has water storage capabilities
• Unsaturated soil has highly varying coefficient of permeability and infiltration conditions
• Shear strength of the unsaturated soil must be taken into account
• Actual (or real-time) flux moisture conditions (i.e., rainfall) must be taken into account
• Calculation of factor of safety must account for unsaturated soil behavior
Analysis Can Be Viewed as a Combination of the Following Elements of Physics

- **Saturated-unsaturated seepage analysis** (Permeability and Storage)
- **Stress analysis** for the shear and normal forces (Method of Slices or Stress Analysis)
- **Shear strength** evaluation of the unsaturated soil (angle $\phi^b$)
- Evaluation of **surface moisture flux** conditions (Percentage of $k_{sat}$)
- Calculation of **factor of safety**, $F_s$
NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES

An example of the effect of excavations on a natural slope subjected to environmental changes

RELEVANT QUESTIONS?
Initial conditions for the seepage analysis

Average recharge flux on surface AF = 2080 mm/year

Only 10% flux is specified on protected surface EE and ED

Groundwater table

Elevation (m)

Distance (m)
2080 mm/year = 6.6 \times 10^{-8} \text{ m/s}

Quite an extreme condition

Initial steady state conditions

Initial groundwater condition and pore-water pressure head contours
Initial steady state conditions along Section X - X
Seepage and slope stability results under high intensity rainfall conditions

Flux = $1.3 \times 10^{-5}$ m/s

After 120 minutes

Elapsed time = 120 min.
After 480 minutes
When the rain stops

Flux = 0.0
After 1080 minutes

Elapsed time = 1080 min.
Heavy rainfall creates a wetting front.
Factors of safety with respect to elapsed time from the beginning of rainfall

- Loss of strength
- Regain of strength
- Rain stops
- 50% of $\phi'$