

Soil Mechanics for Unsaturated Soils

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SOIL MECHANICS FOR UNSATURATED SOILS

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Soil Mechanics for Unsaturated Soils

TABLE OF CONTENTS

Chapter

→	1	Introduction to Unsaturated Soil Mechanics	
	2	Phase Properties and Relations	
→	3	Stress State Variables	
	4	Measurement of Soil Suction	
→	5	Flow Laws	
→	6	Measurement of Permeability	Estimations of ---
→	7	Steady State Flow	
	8	Pore Pressure Parameters	
→	9	Shear Strength Theory	
→	10	Measurement of Shear Strength Parameters	Estimations of ---
→	11	Plastic and Limit Equilibrium	
	12	Volume Change Theory	
	13	Measurement of Volume Change Indices	Estimations of ---
	14	Volume Change Predictions	
	15	One-dimensional Consolidation and Swelling	
→	16	Two- and Three-dimensional Unsteady-state Flow and Non-isothermal Analyses	



Notes at the bottom of each PowerPoint Slide

- **Notes:**
 - *Different approaches could be taken towards synthesizing the material on the behavior of unsaturated soil mechanics. The approach deemed most desirable was to maintain a relatively close parallel to classical saturated soil mechanics.*
 - *The emphasis was not focused solely on “problematic soils”, but rather on the areas of seepage, shear strength and volume change of all soils with negative pore-water pressures.*
 - *The prepared notes closely follow the material in the textbook. Additional notes have been prepared on new and important subjects that have emerged during the years subsequent to 1993.*
 - *The textbook appears to have filled an important role in centralizing our understanding of the behavior of unsaturated soils from the standpoint of two independent stress state variables.*
 - **Additional notes:**



Format For Each Constitutive Behavior

- ***Constitutive Relationship*** (e.g., seepage, shear strength, volume change)
 - ***Theory*** associated with the Constitutive Relationship
 - ***Measurement*** of the associated unsaturated soil properties
 - ***Estimation*** of the unsaturated soil property functions (***through use of SWCC***)
 - ***Application*** of the constitutive relationship to practical engineering problems



Important Objective of This Course

- ***To teach Geotechnical Engineers to think the way the **Unsaturated Soil behaves*****
- ***The **Physics** must be correct***
- ***Many behavioral aspects related to **Unsaturated Soils** are the opposite to saturated soil behavior (e.g., hydraulic conductivity of sands capillary barriers)***



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Categorization of Soil Mechanics

Emphasis of Soil Mechanics textbooks

- Combination of Engineering Mechanics and the Properties of Soils

SOIL MECHANICS

SATURATED SOIL MECHANICS

UNSATURATED SOIL MECHANICS

NATURAL SILTS & CLAYS

SANDS AND GRAVELS*

NATURAL, DESICCATED SILTS AND CLAYS
TRANSPORTED SOILS
RESIDUAL SOILS

COMPACTED SILTS & CLAYS

u_w GENERALLY ≥ 0

u_w GENERALLY < 0

*may be saturated or dry

Two-Phase Behavior

$$\begin{aligned} &(\sigma - u_w) \text{ or} \\ &(\sigma - u_a) \end{aligned}$$

More than Two-Phases

$$\begin{aligned} &(\sigma - u_a) \text{ and} \\ &(u_a - u_w) \end{aligned}$$



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CATEGORIES OF UNSATURATED SOIL MECHANICS PROBLEMS

Description of STRESS STATE VARIABLES
($\sigma - u_a$) and ($u_a - u_w$)

SEEPAGE

Flux Boundary
Conditions

Saturated/
Unsaturated
Modelling

Geo-Environmental

Contaminant
Transport

SHEAR
STRENGTH

Slope Stability

Bearing Capacity

Lateral Earth
Pressures

VOLUME
CHANGE

Swelling

Shrinkage

Collapsing

Deformation

Based on Constitutive Behaviour



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CLIMATIC CHARACTERISTICS

*Moisture flux
is an aspect
originally
omitted from
Soil Mechanics*

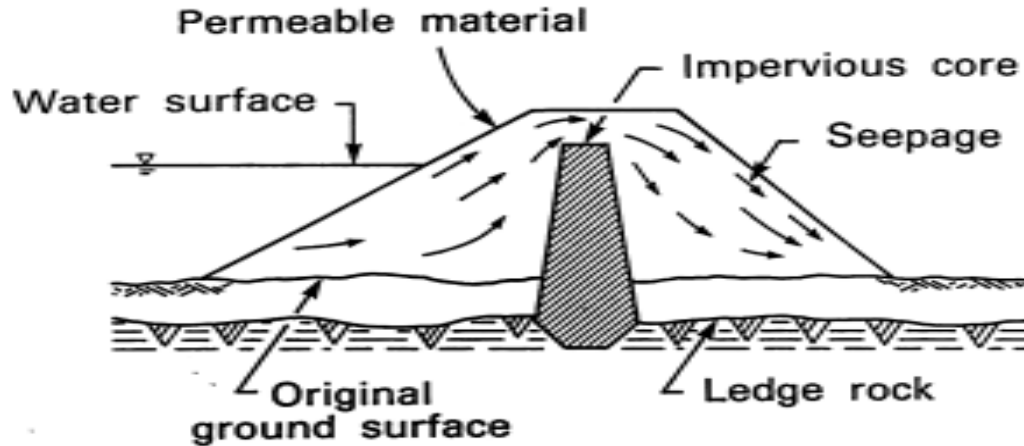
*Coupled mass-
transport & thermal
analysis*

- Approximately 33% of the earth's surface is Arid or Semi-arid
- Climate gives rise to a continuously changing FLUX Boundary Condition at the ground surface
- An UPWARD Flux is produced as water is removed through;
 - i.) Evaporation of water from the soil surface,
 - ii.) Evapo-transpiration from vegetative cover
- The result is a Drying, Cracking and Desaturation of the soil
- ACTUAL EVAPORATIVE FLUX depends on the pore-water stress state and is difficult to predict
- POTENTIAL EVAPORATIVE FLUX is from a water surface and depends primarily on temperature (e.g., Thornthwaite Moisture Index)
- A DOWNWARD Flux is produced by rain and other forms of precipitation. The result is an attempt to saturate the soil
- The DIFFERENCE between the UPWARD and the DOWNWARD Flux largely dictates the location of the water table, and therefore the location of the negative pore-water pressure zone



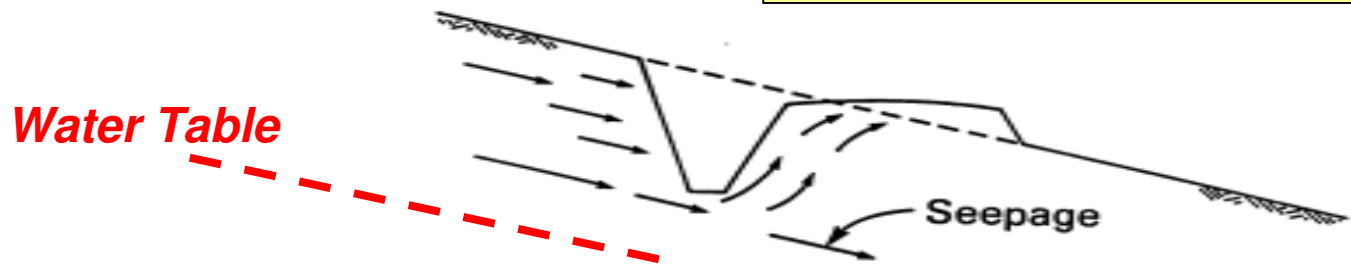
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ILLUSTRATIONS OF UNSATURATED FLOW PHENOMENA IN THE FIELD (from Hogentogler and Barber, 1941)



Syphon Effect on an Earth Dam with a Core-wall

CAPILLARY FLOW PHENOMENA



Intercept Ditch for a Highway on a Side-hill Location

EXAMPLES OF PROBLEMS REQUIRING AN UNDERSTANDING OF UNSATURATED SOIL MECHANICS

**Compacted
Soils**

- 1.) THE DESIGN, CONSTRUCTION AND OPERATION OF MAN-MADE STRUCTURES SUCH AS A DAM
 - a.) During construction of the fill
 - b.) During the filling of the reservoir **and Rapid Drawdown**
 - c.) After Steady State seepage conditions are established, but environmental changes occur

Infiltration

- 2.) NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES
 - Natural slopes often fail after being subjected to heavy precipitation for a long time
 - The slip surface may be relatively shallow, passing through the unsaturated zone
- 3.) MOUNDING BELOW WASTE RETENTION PONDS
 - Wastes are often stored on the ground surface where the water table is deep
 - Contaminant may move to the groundwater even if the soil remains unsaturated
- 4.) STABILITY OF VERTICAL OR NEAR VERTICAL EXCAVATIONS
 - Excavation back-slopes often fail some time after excavation
 - Negative pore-water pressures are dissipated due to infiltration



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5.) LATERAL EARTH PRESSURES

- Dry, clayey soils are often used as backfill
- Can exert high lateral pressures as they become wet

6.) BEARING CAPACITY FOR SHALLOW FOUNDATIONS

- Foundation design is generally based on unconfined compression tests on soils from above the ground-water table
- Analysis assumes that negative pore-water pressures are maintained with time

7.) GROUND MOVEMENTS INVOLVING EXPANSIVE SOILS

- Light structures suffer distress as a result of environmental changes or man-made effects

8.) COLLAPSING SOILS

- There may be volume change or a loss of shear strength resulting from pore-water pressure changes

**Problematic
Soils**

9.) FLOW THROUGH RESIDUAL SOILS

- Water infiltration into Residual soils often results in instability of slopes

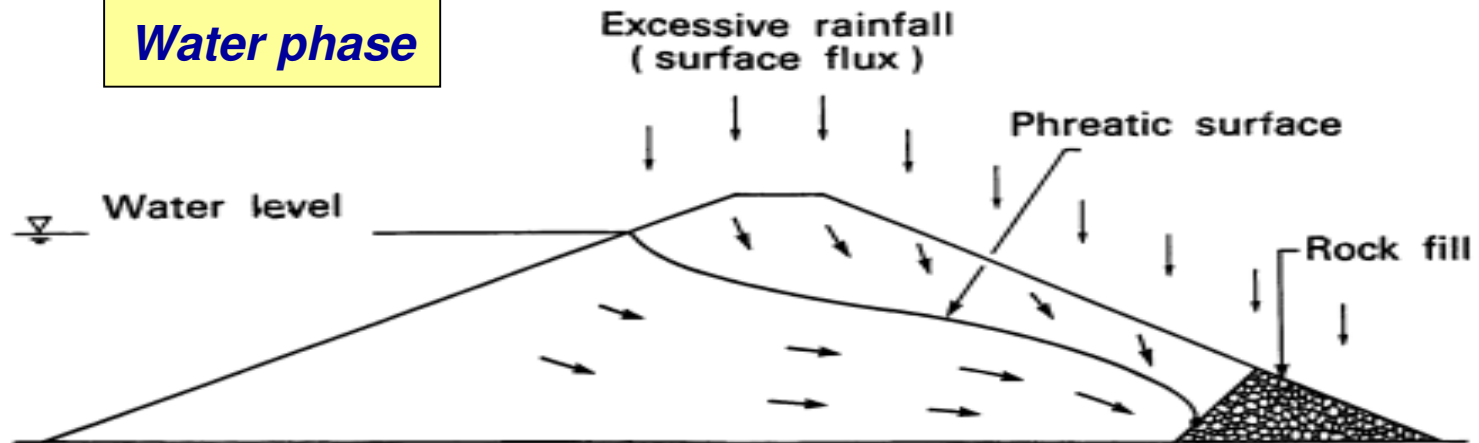


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ENVIRONMENTAL EFFECTS DURING THE OPERATION OF THE RESERVOIR

The effect of rainfall on steady state flow through a dam

Water phase



SOME RELEVANT QUESTIONS MIGHT BE ASKED AS STEADY STATE CONDITIONS ARE ESTABLISHED

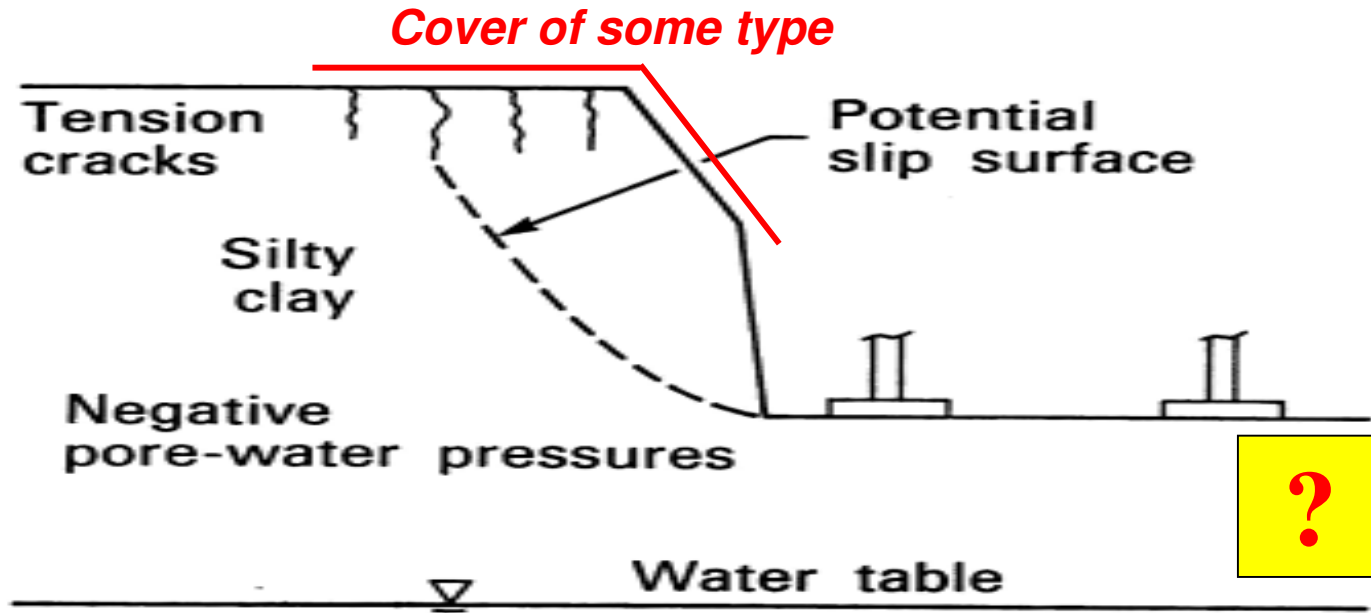
?



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STABILITY OF VERTICAL OR NEAR VERTICAL EXCAVATIONS

An example showing potential instability of a near vertical excavation used during the construction of a foundation



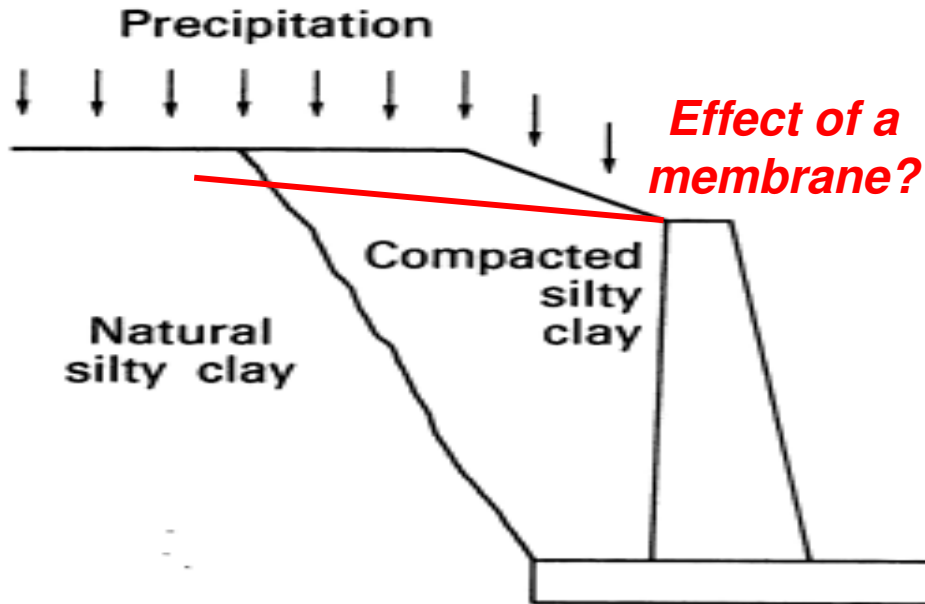
SOME RELEVANT QUESTIONS THAT MIGHT BE ASKED ARE REFERRED TO THE STABILITY OF THE EXCAVATION SLOPES



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LATERAL EARTH PRESSURES

Lateral earth pressures against a retaining wall as water infiltrates the compacted backfill



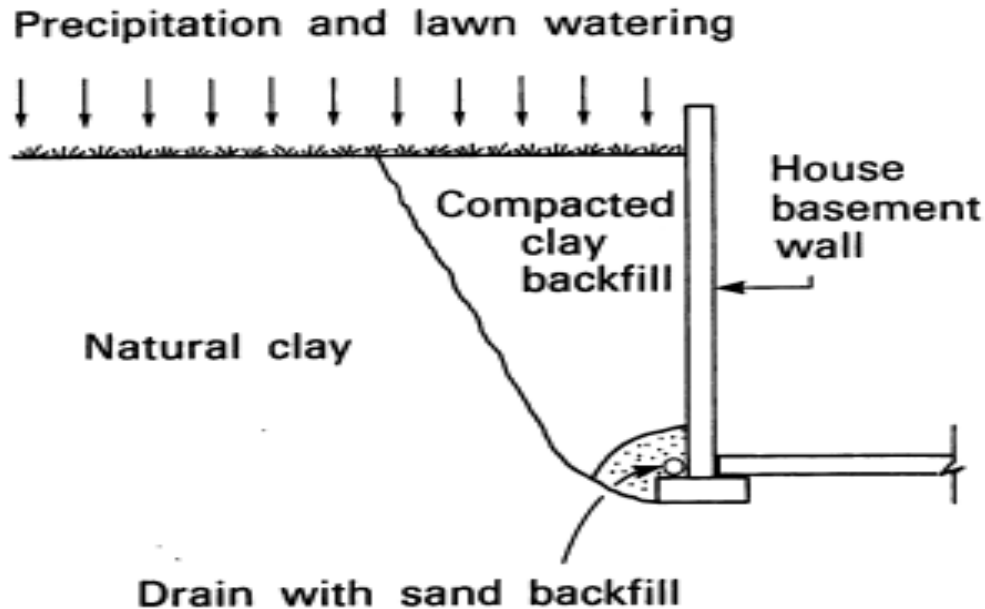
SOME RELEVANT QUESTIONS MIGHT BE ASKED
PERTAINING TO LATERAL EARTH PRESSURES



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LATERAL EARTH PRESSURES AGAINST WALLS

Example of lateral earth pressures generated subsequent to backfilling with dry soils



Lateral earth pressure against a house basement wall



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IS THERE A NEED FOR UNSATURATED SOIL MECHANICS?

YES!

The Geotechnical Engineer has the greatest potential to assist the public in circumventing problems associated with Unsaturated Soils

There is need for an APPROPRIATE TECHNOLOGY which is:

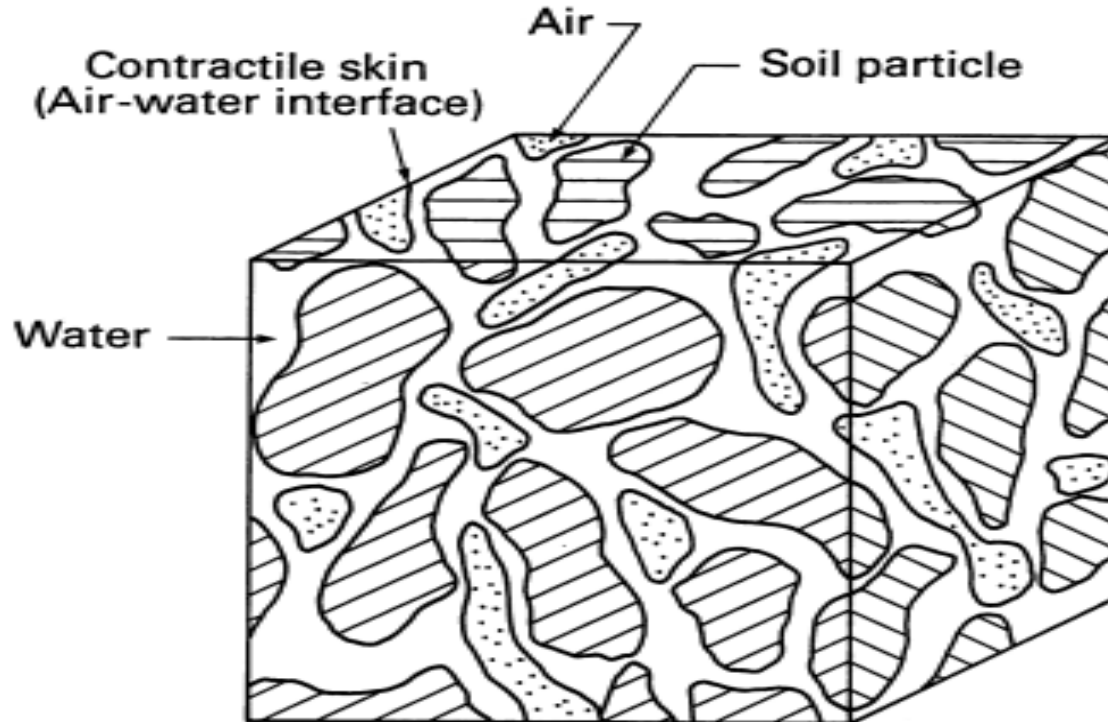
- a.) Practical
- b.) Not too costly to employ
- c.) Has a sound theoretical basis, and
- d.) Runs parallel in concept to Saturated Soil Mechanics

Concern for the environment and advances in computing power greatly assisted in the promotion of **Unsaturated Soil Mechanics**



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AN UNSATURATED SOIL ELEMENT WITH A CONTINUOUS AIR PHASE



PHASES OF AN UNSATURATED SOIL

An Unsaturated Soil is postulated to have 4 phases

- 1.) Solids
- 2.) Air
- 3.) Water
- 4.) Contractile Skin (Air-Water Interface)



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TERMINOLOGY AND DEFINITIONS

Will use a MACROSCOPIC, PHENOMENOLOGICAL approach to Unsaturated Soil behavior

Additional terms required from Continuum Mechanics

- 1.) STATE - Non material variable required for the characterization of a system
- 2.) STRESS STATE VARIABLE - Variables required for the characterization of the stress state
- 3.) DEFORMATION STATE VARIABLES - Variables required for the characterization of the deformation conditions or deviations from an initial state
- 4.) CONSTITUTIVE RELATIONS - Single-valued equations expressing the relationship between state variables

Fung (1969) - are single-valued expressions which relate one state variable to one or more other state variables. They always incorporate the material properties

Examples are:

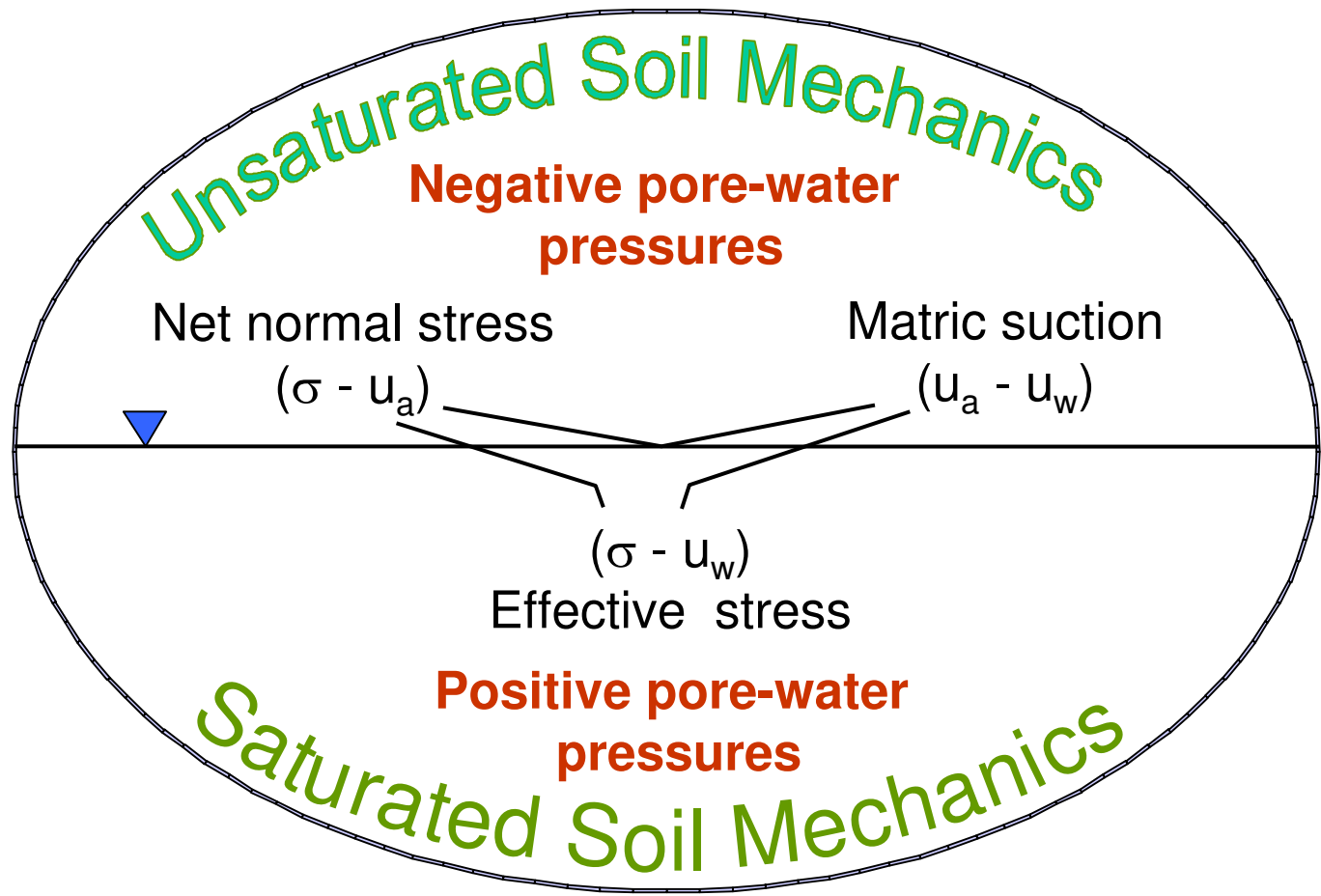
- Stress versus Strain Relations (E , μ)
- Ideal Gas Law, relates pressure to density and temperature. Gas constant is the property
- Shear Strength equation (c' , ϕ' , ϕ^b)
- Pore Pressure Parameter equation (S , m_v , β_w)



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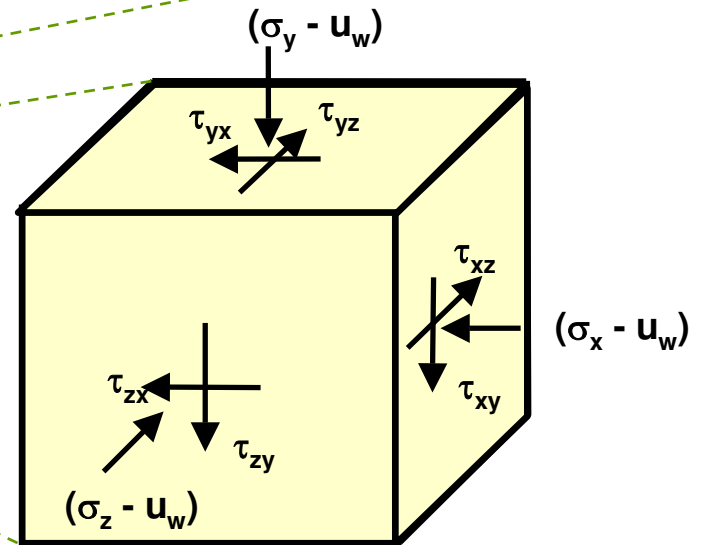
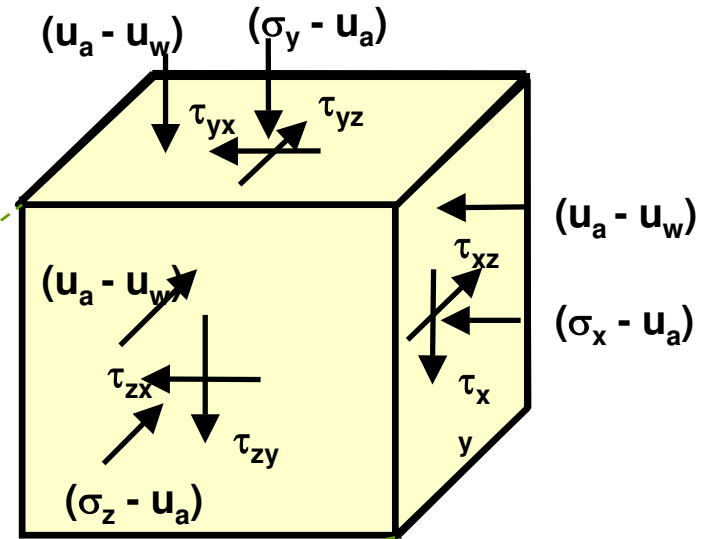
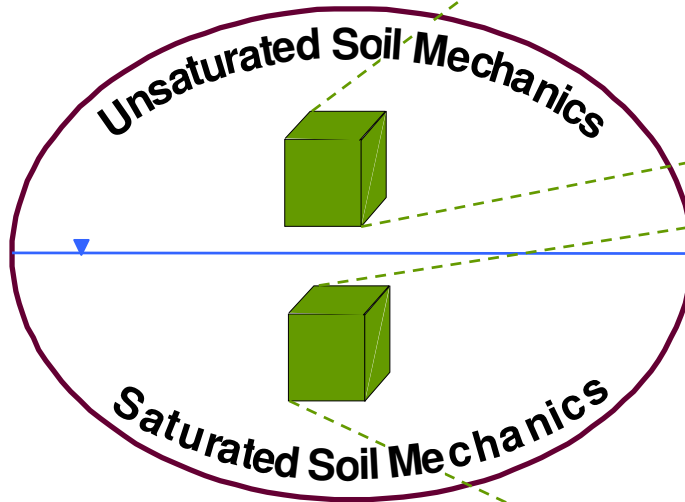
Categorization of Soil Mechanics Based on Stress State Variables

Visualization
of the World
of Saturated-
Unsaturated
Soil
Mechanics

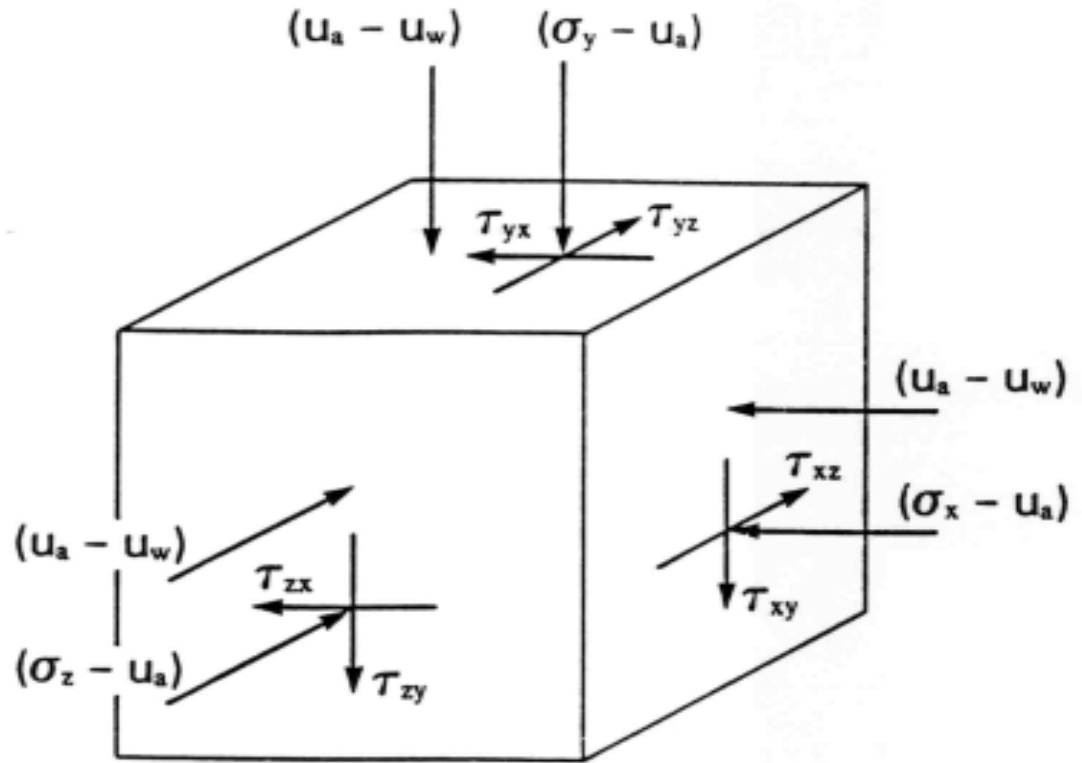
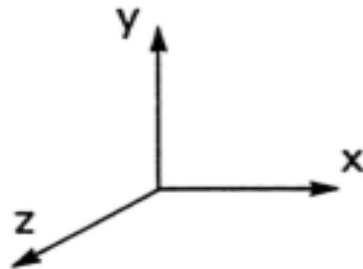


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**Stress State
at a Point in an
Unsaturated Soil
and in a
Saturated Soil**



Summary of the description of the Stress State for an Unsaturated Soil Element with the Preferred Stress State Variables



The stress state variables for an unsaturated soil using the combination of $(\sigma - u_a)$ and $(u_a - u_w)$



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Stress State Variables (Unsaturated Soils)

Net Total Stress Tensor

- **Stress Tensors form the basis for a Science because we live in a 3-D Cartesian coordinate world**

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \\ (u_a - u_w) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (u_a - u_w) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (u_a - u_w) \end{bmatrix}$$

Matric Suction Stress Tensor



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Variations in Stress State Description

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w)$$

σ' = ***effective stress***

χ = ***parameter related to saturation***

$$\sigma^{*}_{ij} = \sigma_{ij} - [S u_w + (1 - S) u_a] \delta_{ij}$$

σ_{ij} = ***total stress tensor,***

δ_{ij} = ***Kroneker delta or substitution tensor,***

σ^{*}_{ij} = ***Bishop's soil skeleton stress (Jommi 2000)***

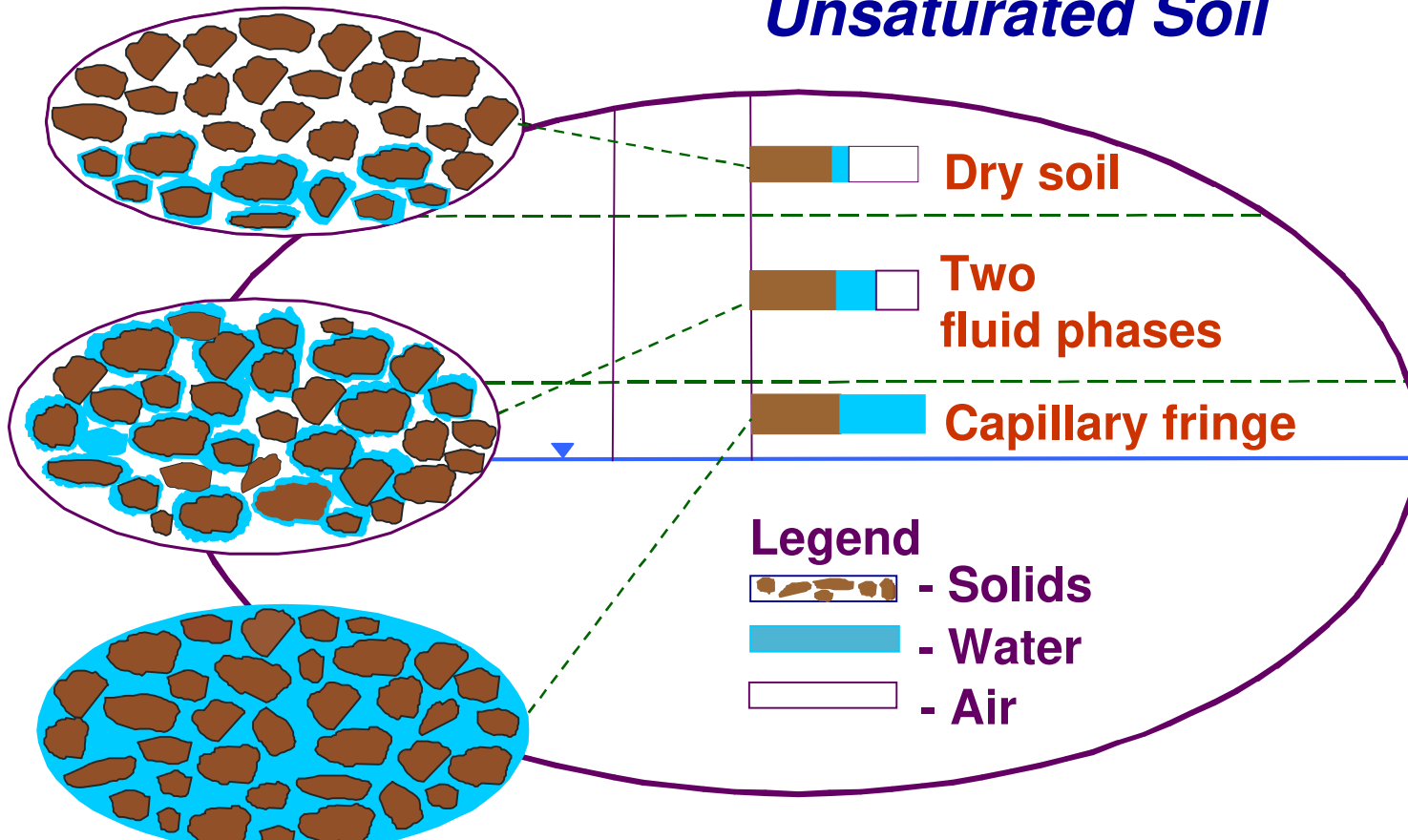
Above proposed equations are constitutive relations



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Separation of the Zones above the Water Table

Unsaturated Soil



Occluded air bubbles

Saturated Soil



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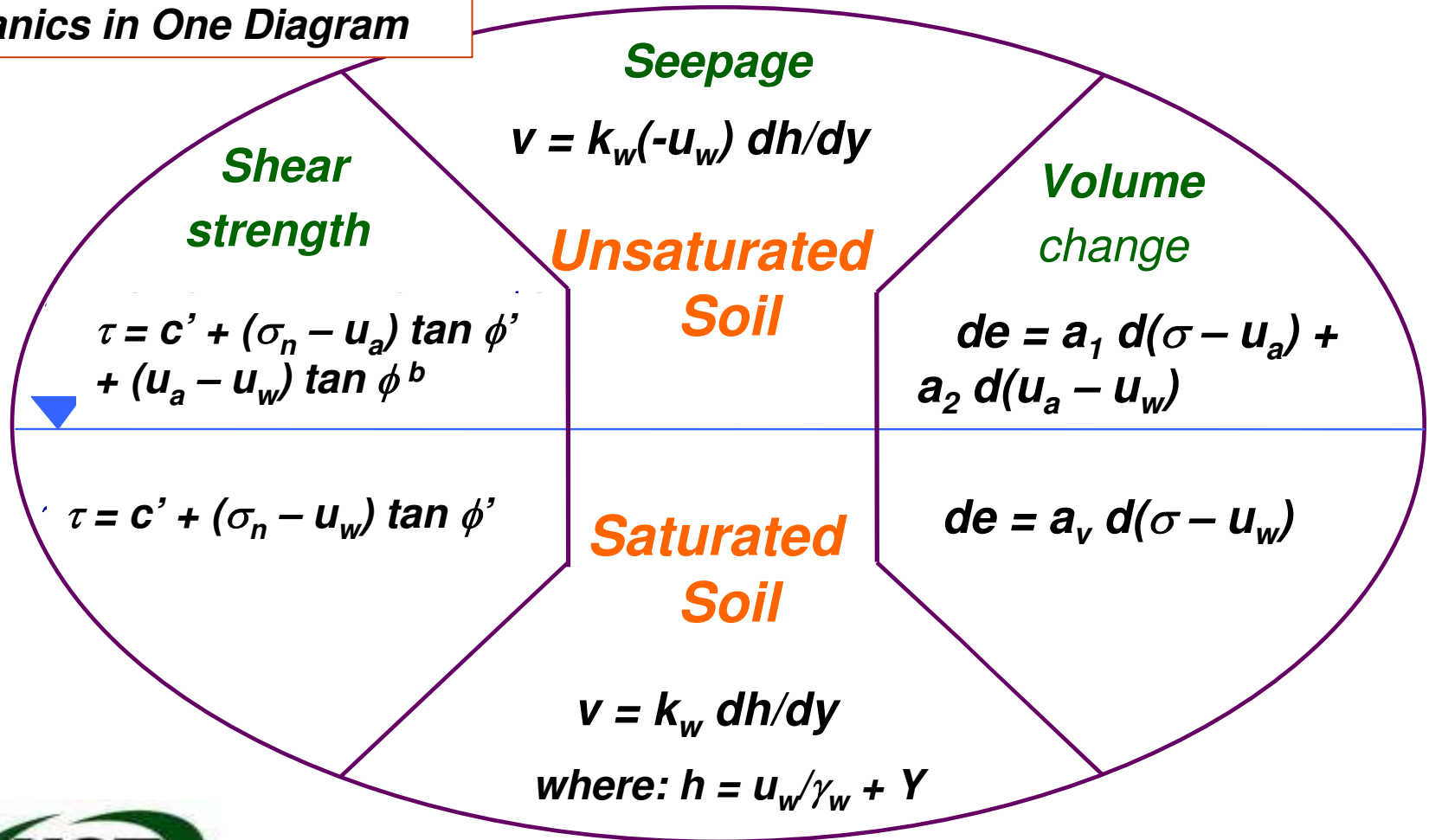
Constitutive Relations for Classic Problems in Unsaturated Soils Mechanics

- ❑ ***Empirical, semi-empirical and theoretical relationships are proposed and verified***
 - ***Volume change (Stress versus Strain)***
 - ***Shear strength (Stress versus Stress)***
 - ***Flow (Velocity versus Stress)***
- ❑ ***Demanding laboratory experiments***
 - ***Careful experiments required for uniqueness studies***
 - ***May alter test procedures for economic reasons***



Constitutive Relations for Saturated/Unsaturated Soil Mechanics

Fundamentals of Soil Mechanics in One Diagram



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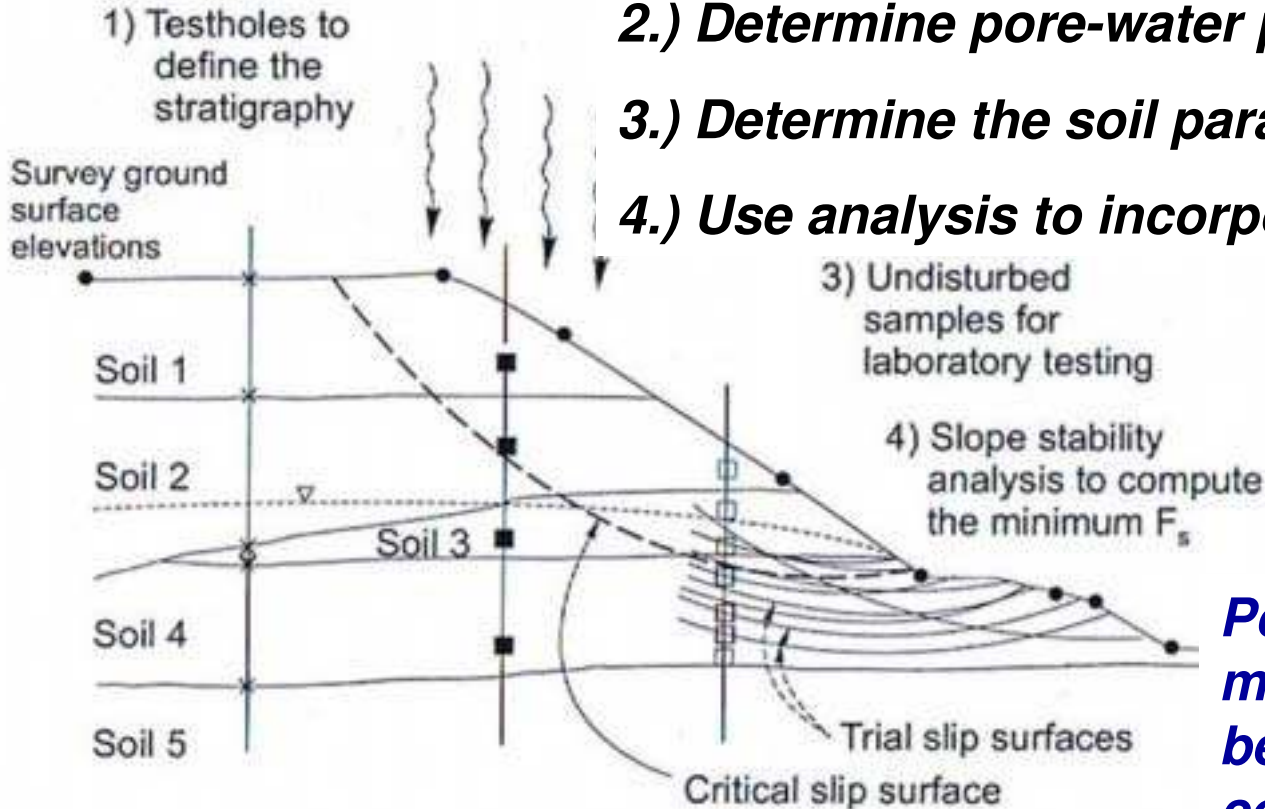
Selected Reading Materials

- ***Fredlund, D.G. (2000). “The 1999 R.M. Hardy Lecture: The Implementation of Unsaturated Soil Mechanics into Geotechnical Engineering”, Canadian Geotechnical Journal, 37 (5), 963-986***
 - ***Fredlund, D.G. (2002). “Use of the Soil-Water Characteristic Curve in the Implementation of Unsaturated Soil Mechanics”, Keynote Address, March 10-13, UNSAT 2002, Recife, Brazil, Vol. 3.***
 - ***Ha, T.V. Pham, and Fredlund, D. G. (2002). “The Application of Dynamic Programming to Slope Stability Analysis”, Canadian Geotechnical Journal, pp. 830-847.***
-
- ***Gitirana, Gilson, and Fredlund, D. G., (2003). “From Experimental Evidence Towards the Assessment of Weather-Related Railway Embankment Hazards” Proc. Of the Conf. on “From Experimental Evidence Towards Numerical Modelling of Unsaturated Soils”, Sept. 18-19.***



Review of Procedure Typical to **Saturated Soil** Mechanics

- 1.) Define geometry of the problem (Surface & Stratigraphy)
- 2.) Determine pore-water pressures
- 3.) Determine the soil parameters (c' & ϕ')
- 4.) Use analysis to incorporate total stresses



Pore-water pressures must be assessed because behavior is controlled by $(\sigma - u_w)$



Additional Features to Accommodate Rainfall Induced Landslides

- ***Slope must be visualized as a **transient** analysis on a **saturated-unsaturated soil profile*****
- ***Unsaturated soil has **water storage capabilities*****
- ***Unsaturated soil has highly varying **coefficient of permeability and infiltration conditions*****
- *****Shear strength of the unsaturated soil must be taken into account*****
- ***Actual (or real-time) **flux moisture conditions** (i.e., rainfall) must be taken into account***
- *****Calculation of factor of safety** must account for unsaturated soil behavior***



Analysis Can Be Viewed as a *Combination* of the Following Elements of Physics

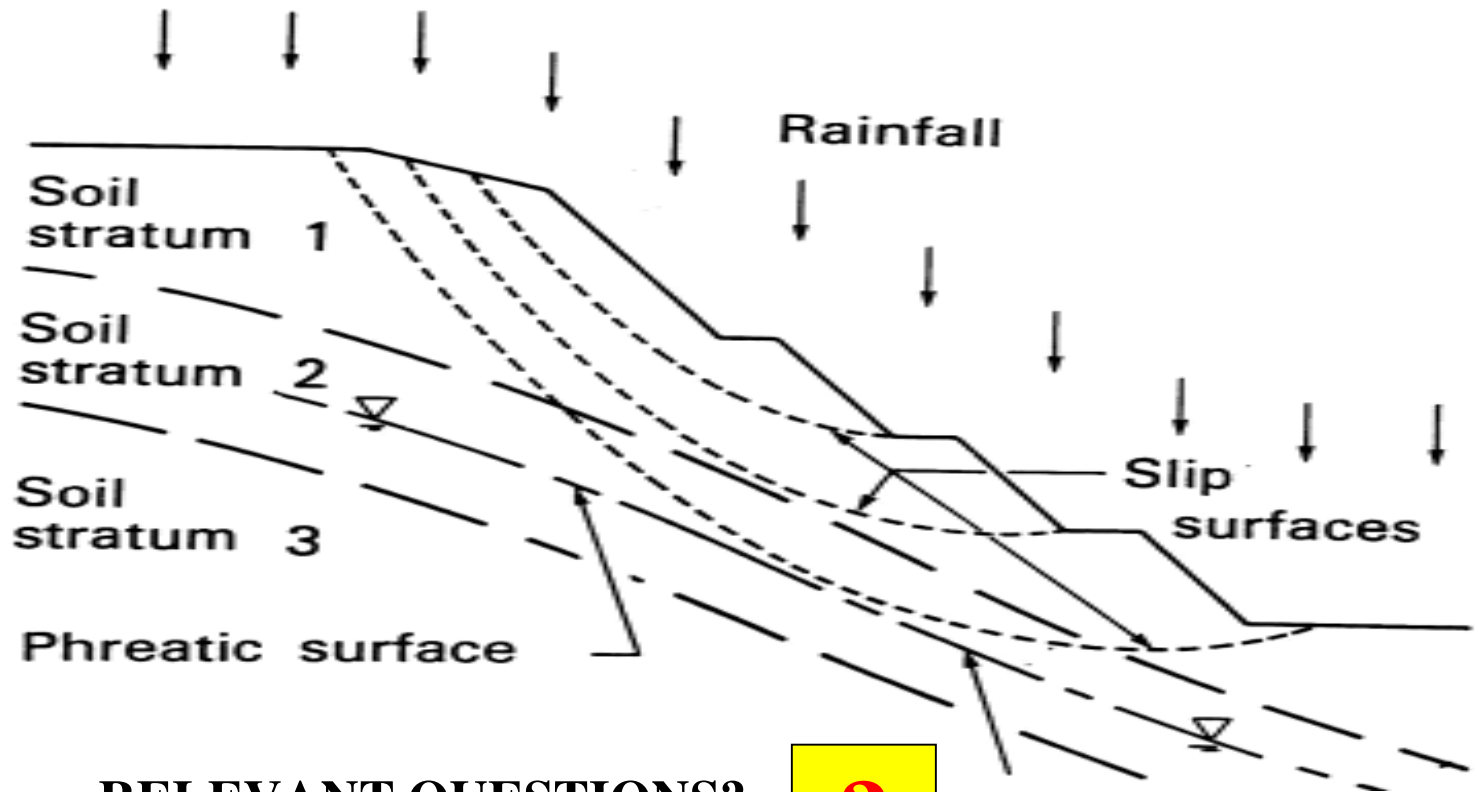
- **Saturated-unsaturated seepage analysis** (Permeability and Storage)

- **Stress analysis** for the shear and normal forces (Method of Slices or Stress Analysis)
- **Shear strength** evaluation of the unsaturated soil (angle ϕ^b)
- Evaluation of **surface moisture flux** conditions (Percentage of k_{sat})
- Calculation of **factor of safety, F_s**



NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES

An example of the effect of excavations on a natural slope subjected to environmental changes

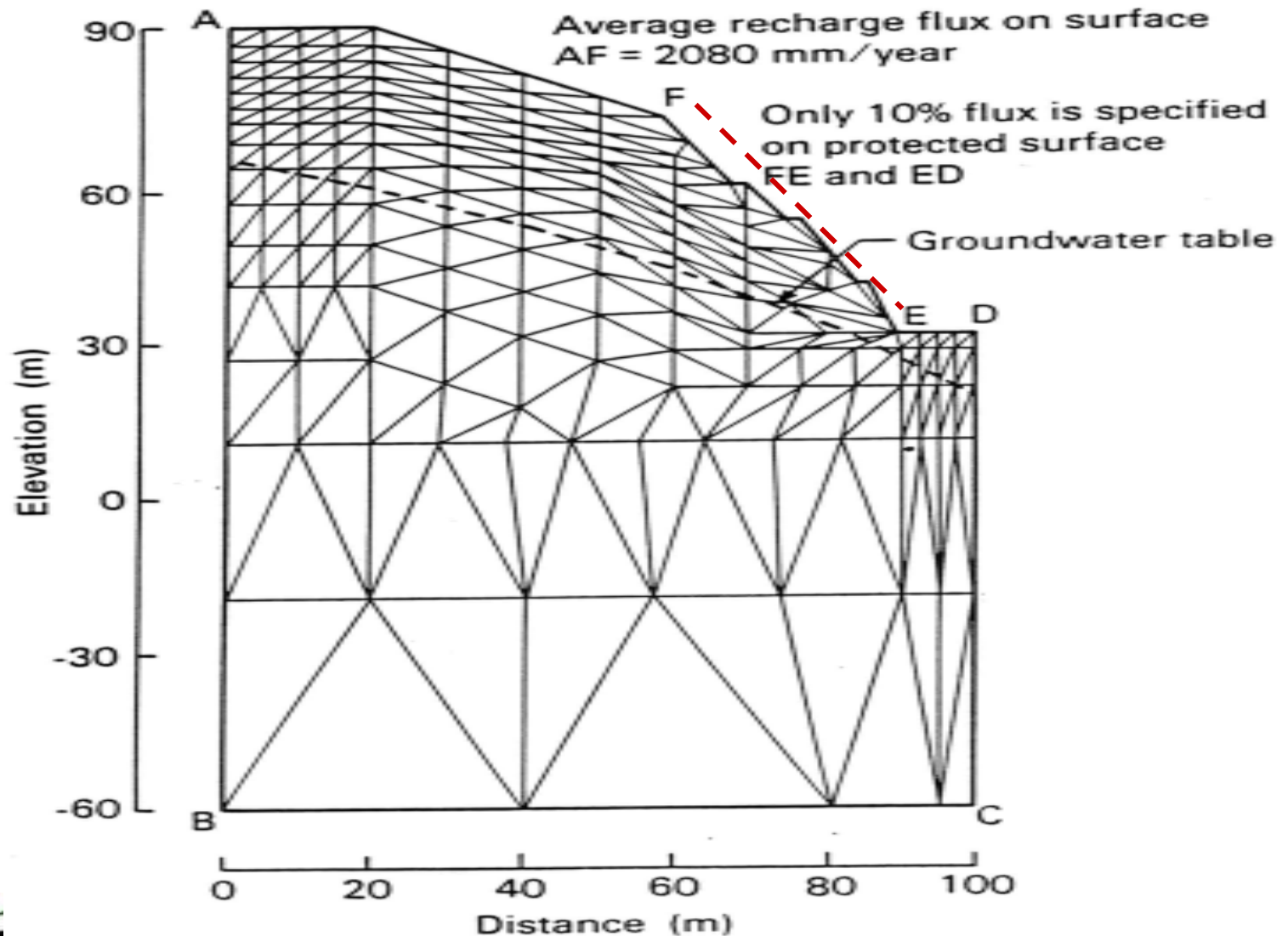


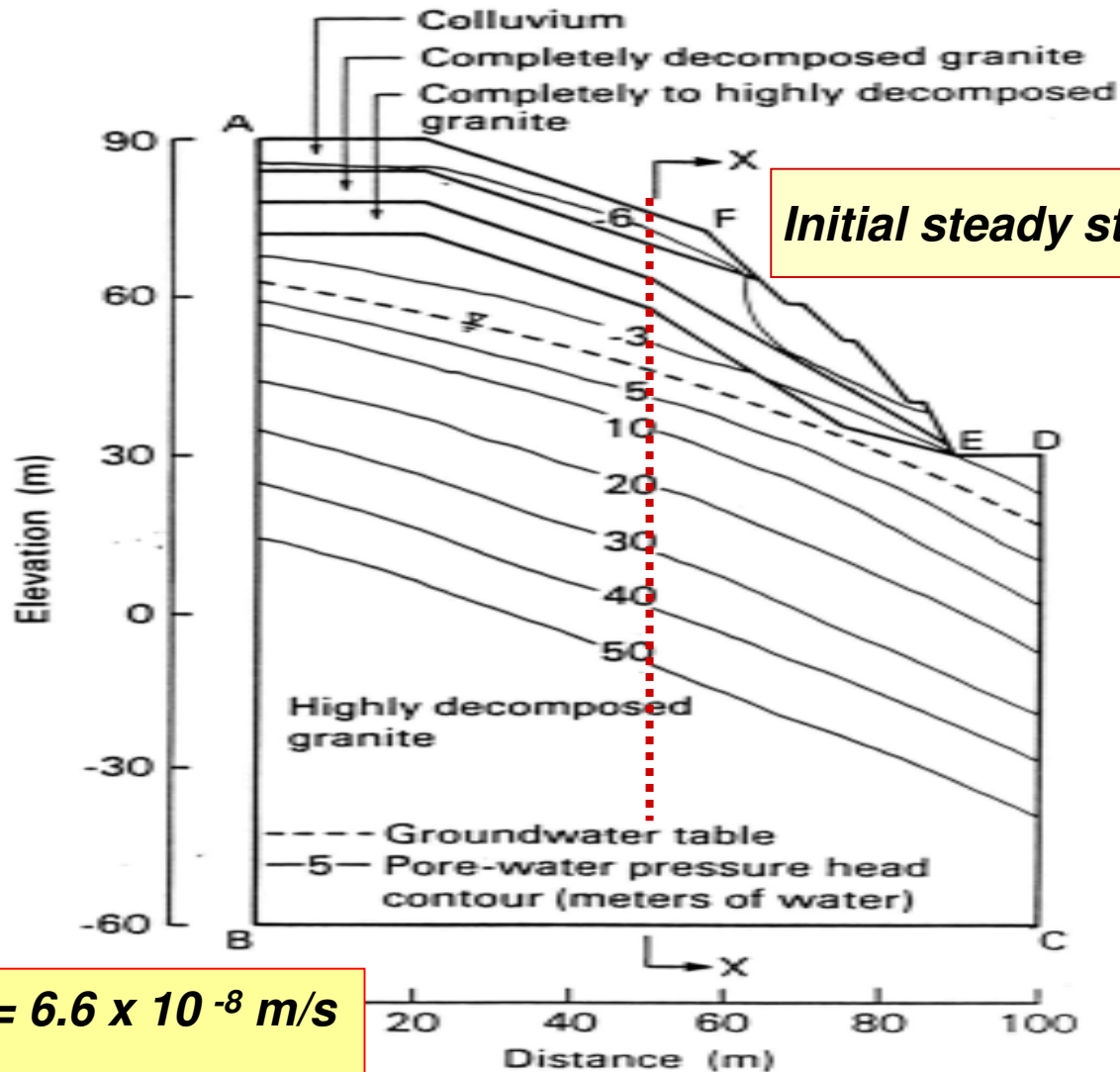
RELEVANT QUESTIONS?



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Initial conditions for the seepage analysis





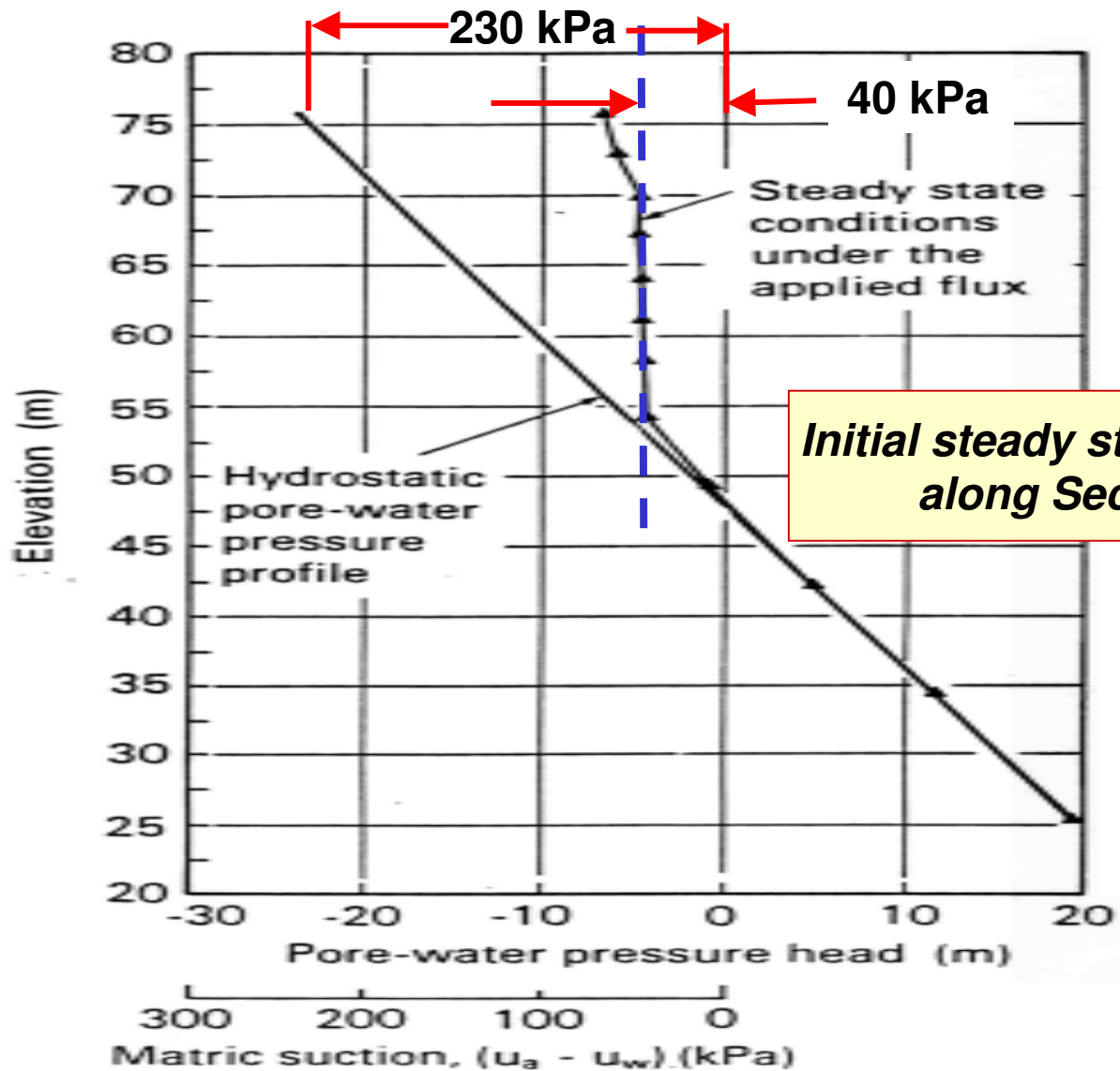
$2080 \text{ mm/year} = 6.6 \times 10^{-8} \text{ m/s}$

Quite an extreme condition



Initial groundwater condition and pore-water pressure head contours

logy



Matric suction profiles for section X-X under steady state flux conditions

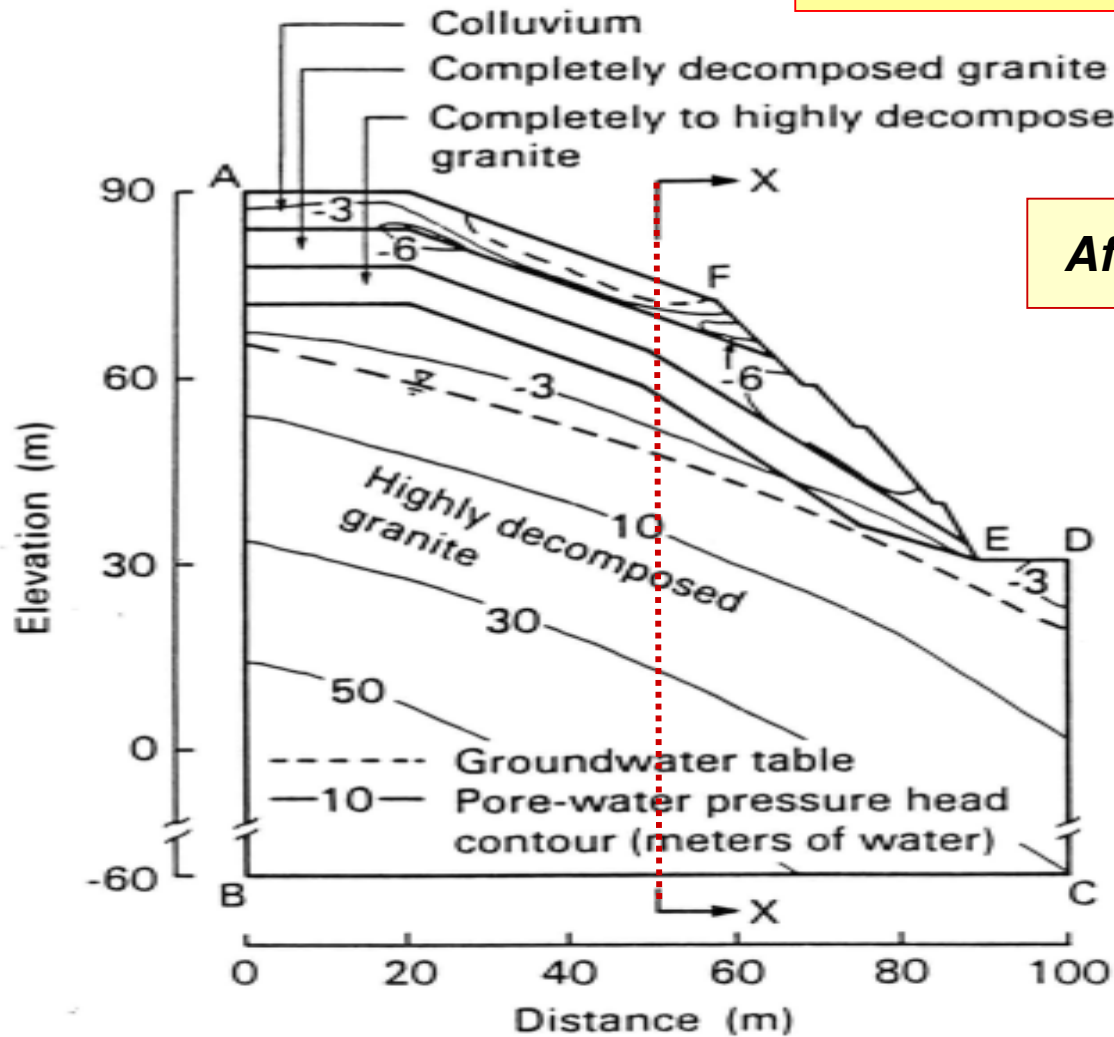


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Seepage and slope stability results under high intensity rainfall conditions

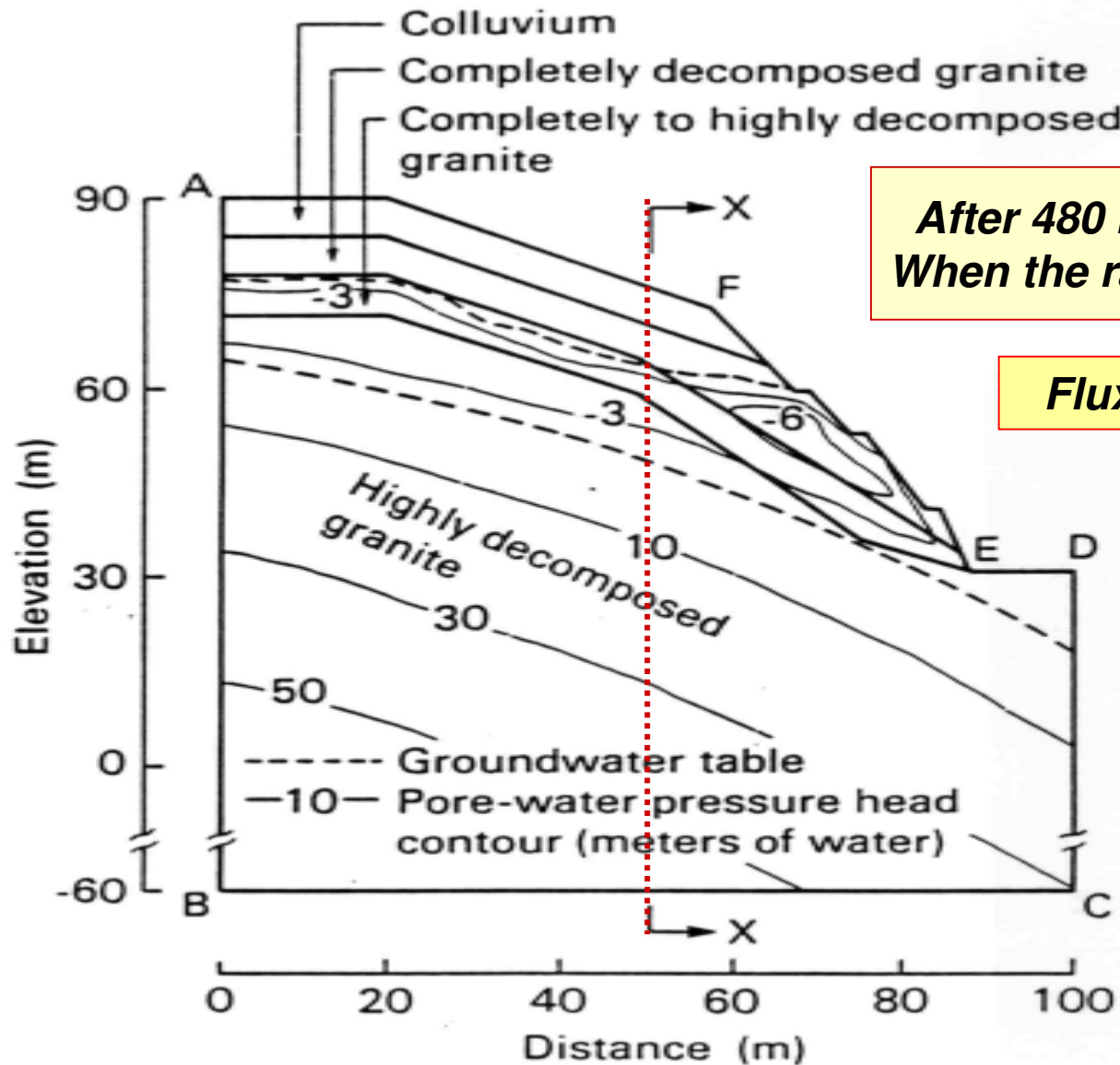
$Flux = 1.3 \times 10^{-5} \text{ m/s}$

After 120 minutes



Elapsed time = 120 min.

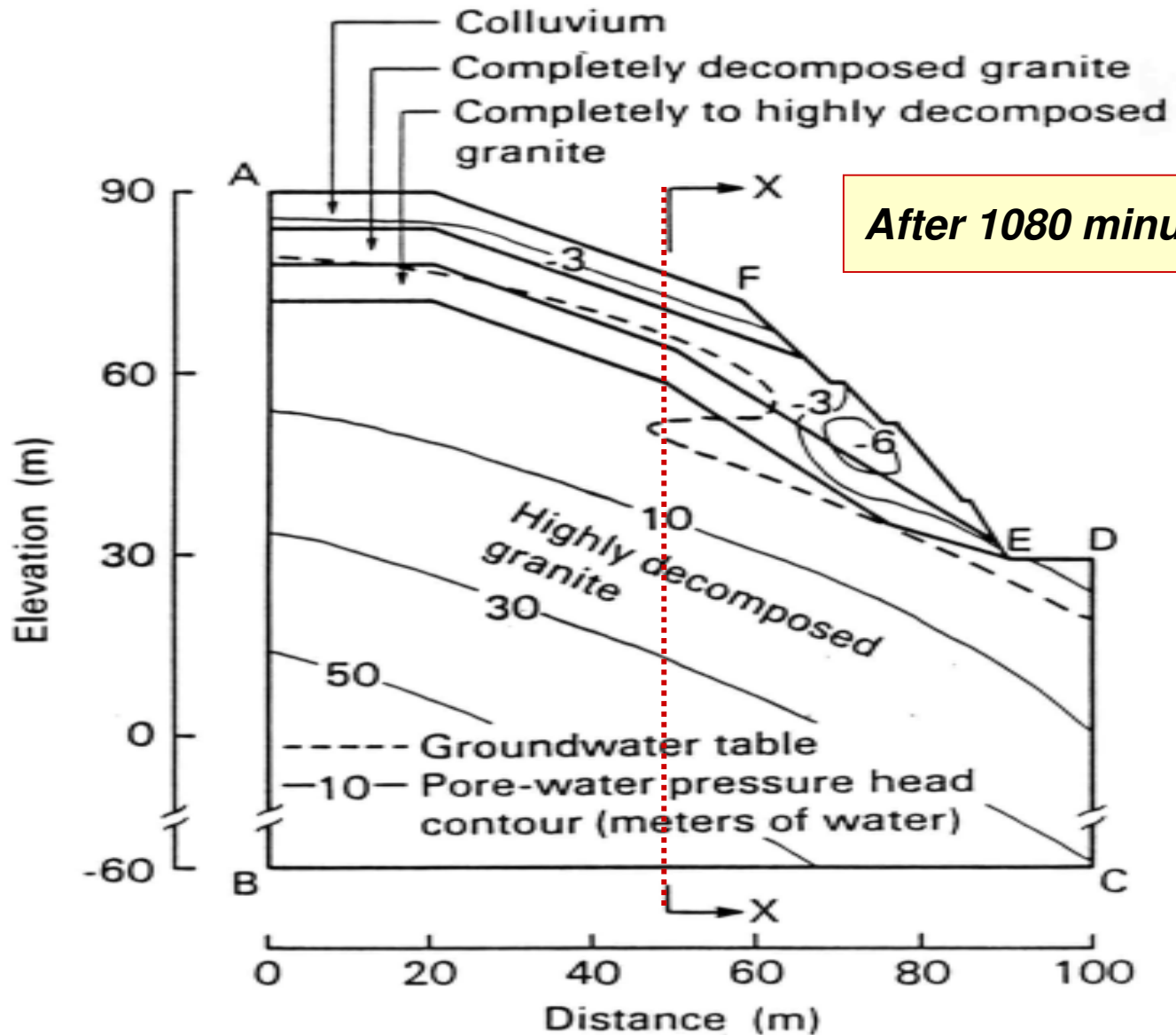




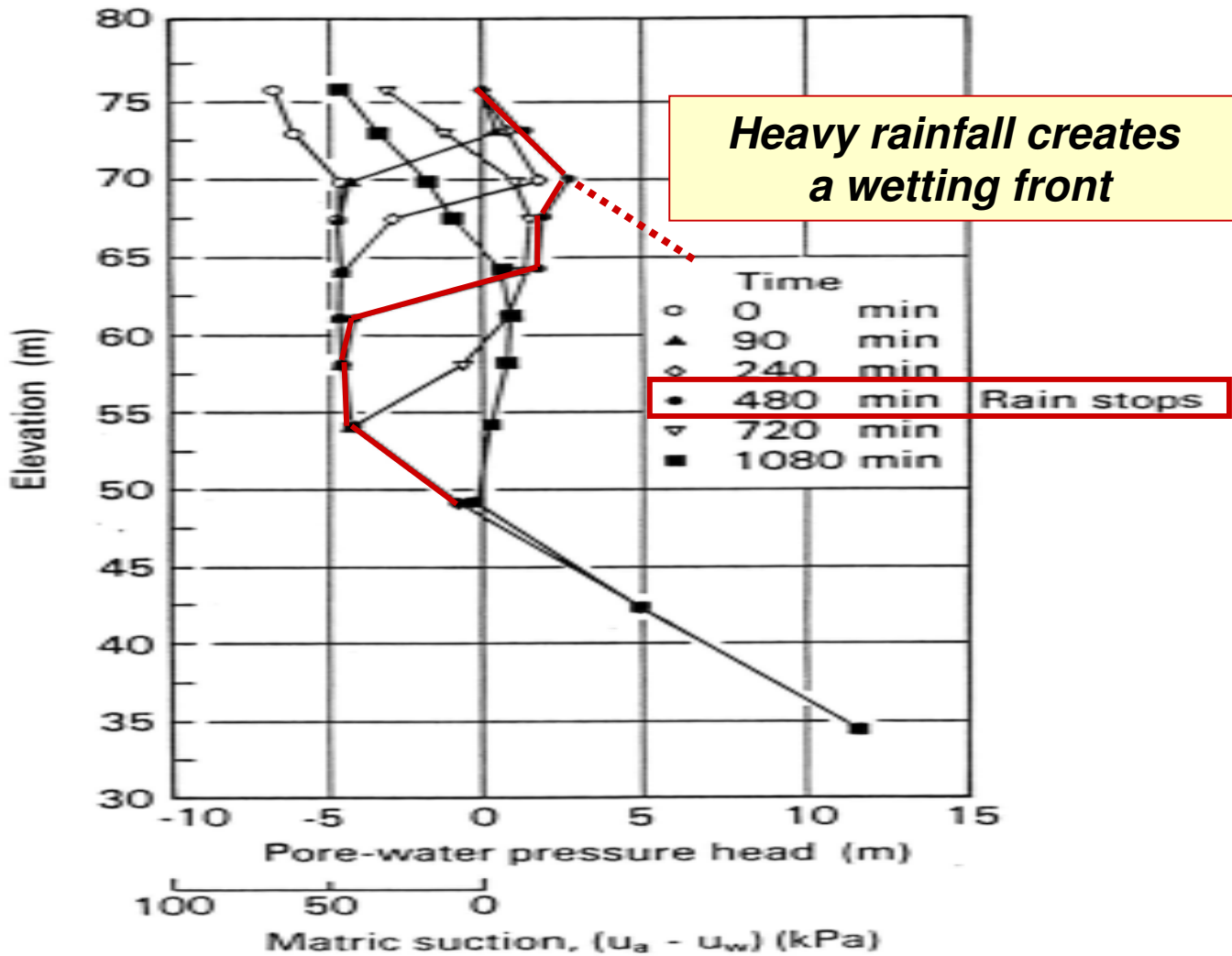
Elapsed time = 480 min.



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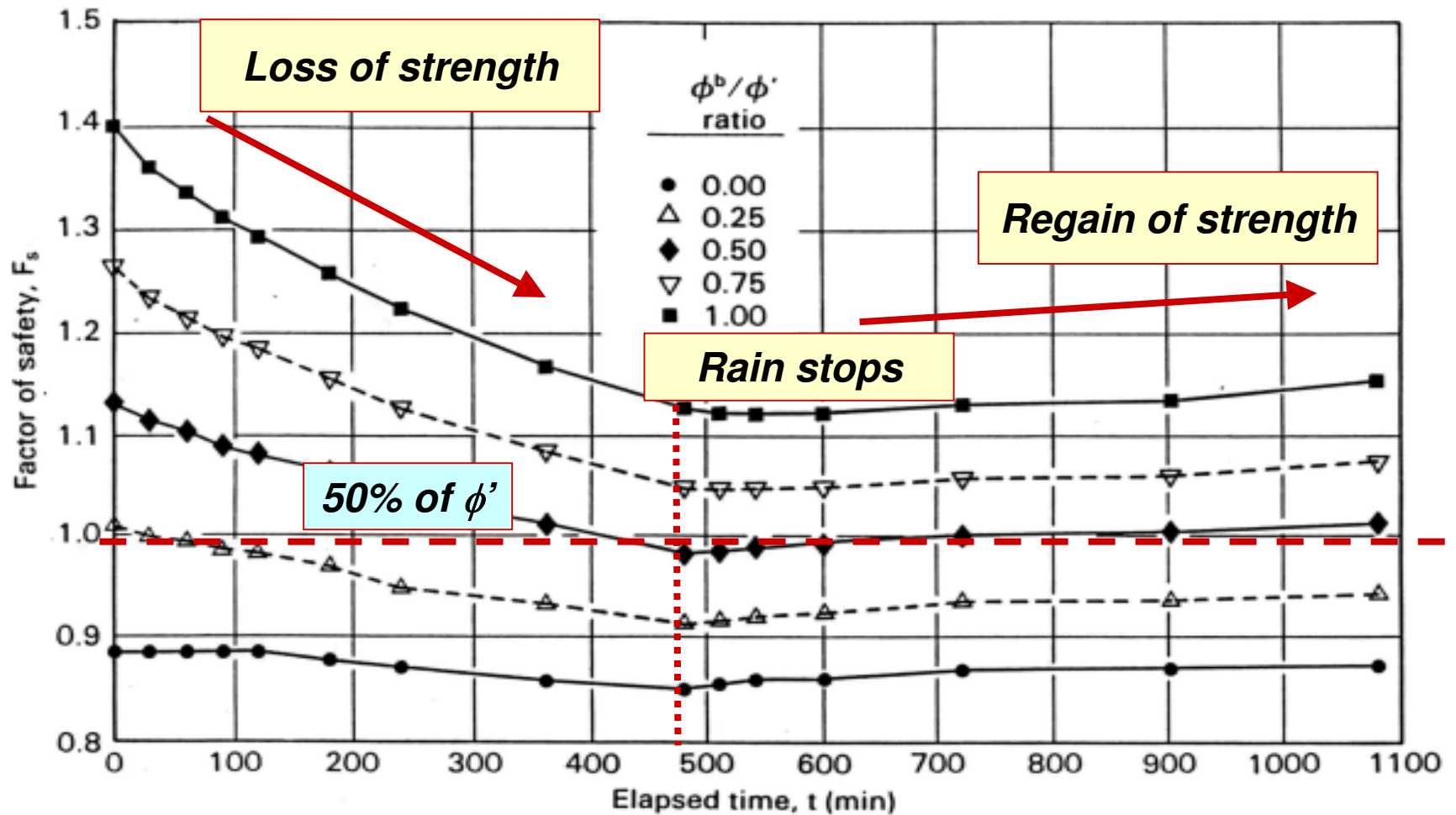


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Matric suction profiles for section X-X at various elapsed times





Factors of safety with respect to elapsed time from the beginning of rainfall



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