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A national laboratory of the U.S. Department of Energy
Office of Energy Efficiency & Renewable Energy

National Renewable Energy Laboratory
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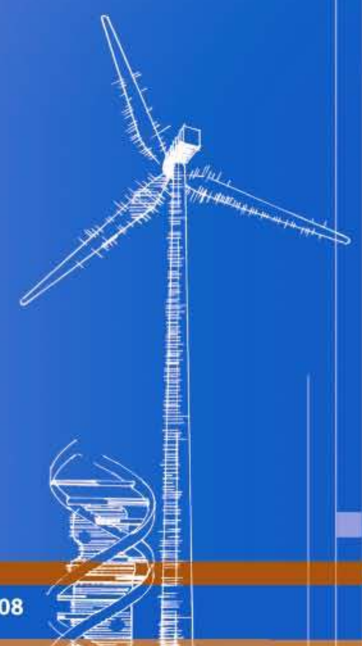

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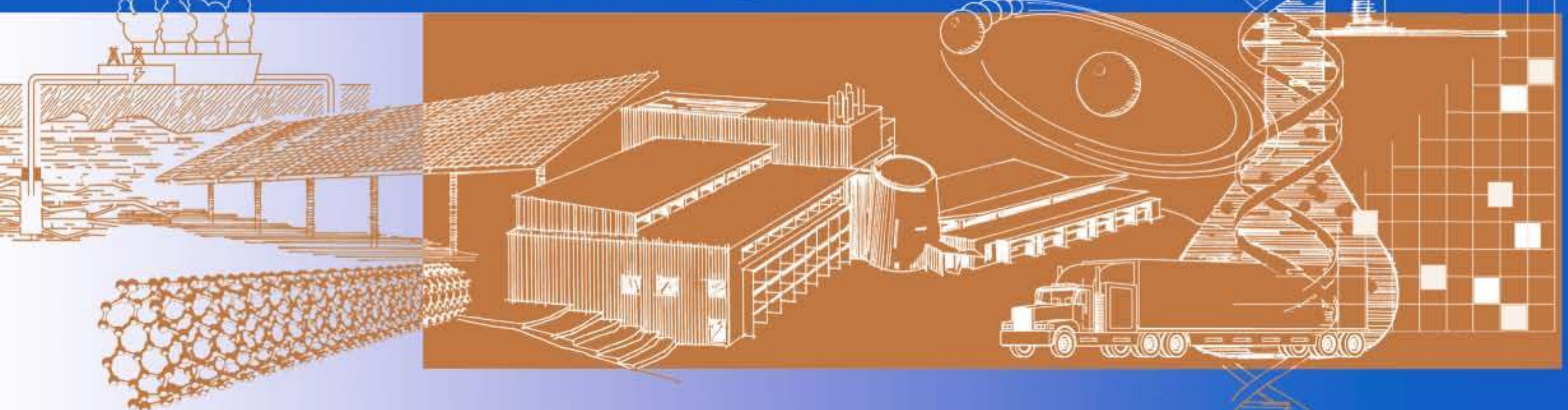

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Acronyms

AA	*Astronomical Almanac*
AU	astronomical unit
JC	Julian century
JD	Julian day
JDE	Julian ephemeris day
JME	Julian ephemeris millennium
MPA	moon position algorithm
SAMPA	solar and moon position algorithm
SPA	solar position algorithm
SUL	Sun's unshaded lune
TAI	international atomic time
TDT	terrestrial dynamical time
TT	terrestrial time
UT	universal time
UTC	coordinated universal time

Executive Summary

The Moon has always played an important role in human culture. It forms the basis of Babylonian, Jewish, and Islamic calendars; it determines the timing of the Christian Easter (which is always the first Sunday after the first full Moon after the vernal equinox). It makes an important contribution to the settings in paranormal and romantic legends and poetry; even cartoons, such as Inuyasha where a half-demon becomes human on the night of New Moon, depend for their stories on the lunar cycle. The Harvest Moon, New Moon and Blue Moon are important in agriculture and astronomy, and man has long used the Moon to navigate travel and predict tides.

In recent years the interest in using solar energy as a major contributor to renewable energy applications has increased, and the focus on smart grids to optimize the use of electrical energy based on demand and resources from different locations has strengthened. We thus need to understand the Moon's position with respect to the Sun. For example, during a solar eclipse, the Sun might be totally or partially shaded by the Moon at the site of interest, which can affect the irradiance level from the sun's disk. A resource on the grid might be affected by this. Instantaneously predicting and monitoring a solar eclipse can provide smart grid users with instantaneous information about potential total or partial solar eclipse at different locations so other resources can instantaneously be directed to a specific location. At least five solar eclipses occur yearly, and can last few hours, depending on location. This rare occurrence can have devastating effects on smart grid users.

This report includes a procedure for implementing an algorithm (described by Jean Meeus [1]) to calculate the Moon's zenith angle with uncertainty of ±0.001° and azimuth angle with uncertainty of ±0.003°. The step-by-step format presented here simplifies the complicated steps Meeus describes to calculate the Moon's position, and focuses on the Moon instead of the planets and stars. It also introduces some changes to accommodate for solar radiation applications. These include changing the direction of measuring azimuth angles from north and eastward instead of from south and eastward, and the direction of measuring the observer's geographical longitude to be measured as positive eastward from Greenwich meridian instead of negative. In conjunction with the solar position algorithm that Reda and Andreas developed in 2004 [2], the angular distance between the Sun and the Moon is used to develop a method to instantaneously monitor the partial or total solar eclipse occurrence for solar energy applications and smart grid users. This method can be used in many other applications for observers of the Sun and the Moon positions for applications limited to the stated uncertainty.

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1.0 Introduction

The Moon has always played an important role in human culture. It forms the basis of Babylonian, Jewish, and Islamic calendars; it determines the timing of the Christian Easter (which is always the first Sunday after the first full Moon after the vernal equinox). It makes an important contribution to the settings in paranormal and romantic legends and poetry; even cartoons, such as Inuyasha where a half-demon becomes human on the night of New Moon, depend for their stories on the lunar cycle. The Harvest Moon, New Moon and Blue Moon are important in agriculture and astronomy, and man has long used the Moon to navigate travel and predict tides.

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First, the Moon position is calculated, then, in conjunction with the solar position algorithm that Reda and Andreas developed [2], the angular distance between the Sun and the Moon is used to develop a method to instantaneously monitor the partial or total solar eclipse occurrence for solar energy applications and smart grid users. SPA [2] has the details of calculating the solar position, so only the Moon position algorithm (MPA) is included in this report. When the solar position calculation is included in this report, the SPA report will be the source for the SPA calculation. This method can be used in many other applications for observers of the Sun and the Moon positions for applications limited to the stated uncertainty.

This report:

- Describes the time scales because using the correct time in MPA is important
- Provides a step-by-step procedure to calculate the Moon position
- Evaluates the Moon's position against the *Astronomical Almanac* (AA) [3] data for 1981, and 2004 through 2010.

- Monitors the solar eclipse occurrence using the Solar and Moon position algorithms, (SAMPA)
- Compares SAMPA against historical eclipses
- Describes a method to estimate the irradiance level during the solar eclipse occurrence.

Because the algorithm is complex, an example is included in Appendix A.1, Table A.1.1 to give the users confidence in their step-by-step calculations; SAMPA results are in Table A.1.2.

This report is used to calculate the Moon's position for solar radiation applications. It is purely mathematical and not meant to teach astronomy or to describe the Moon rotation around the Earth. For more information about the astronomical nomenclature that is used throughout the report, review the definitions in AA or another astronomy reference.

2.0 Time Scale

Following are the internationally recognized time scales:

- Universal time (UT), or Greenwich civil time, is based on the Earth's rotation and referenced to 0-hour at midnight; the unit is mean solar day [3]. UT is used to calculate the solar position in the described algorithm. It is sometimes referred to as UT1.
- International atomic time (TAI) is the duration of the System International Second (SI-second) and based on a large number of atomic clocks [3].
- Coordinated universal time (UTC) is the basis of most radio time signals and legal time systems. It is kept to within 0.9 seconds of UT1 (UT) by introducing one-second steps to its value (leap second); to date the steps are always positive.
- Terrestrial dynamical or terrestrial time (TDT or TT) is the time scale of ephemerides for observations from the Earth's surface.

The following equations describe the relationship between the above time scales (in seconds):

$$TT = TAI + 32.184 \quad , \tag{1}$$

$$UT = TT - \Delta T \quad , \tag{2}$$

where:

ΔT is the difference between the Earth's rotation time and TT. It is derived from observation only and reported yearly in the AA [5].

$$UT = UT1 = UTC + \Delta UT1 \quad , \tag{3}$$

where:

ΔUT1 is a fraction of a second, positive or negative value that is added to the UTC to adjust for the Earth's irregular rotational rate. It is derived from observation, but predicted values are transmitted in code in some time signals; e.g., weekly by the U.S. Naval Observatory [4].

3.0 Moon Position Algorithm

3.1. Calculate the Julian and Julian Ephemeris Day, Century, and Millennium:

The Julian date starts on January 1, in the year –4712 at 12:00:00 UT. The Julian Day (JD) is calculated using UT and the Julian Ephemeris Day (JDE) is calculated using TT. In the following steps, there is a 10-day gap between the Julian and Gregorian calendars where the Julian calendar ends on October 4, 1582 (JD = 2299160), and after 10 days the Gregorian calendar starts on October 15, 1582.

3.1.1 Calculate the Julian Day

$$JD = INT(365.25 * (Y + 4716)) + INT(30.6001 * (M + 1)) + D + B - 1524.5 \quad , \tag{4}$$

where:

INT is the integer of the calculated terms (8.7 = 8, 8.2 = 8, and -8.7 = -8, etc.).

Y is the year (2001, 2002, etc.).

M is the month of the year (1 for January, etc.). If $M > 2$, then Y and M are not changed, but if $M = 1$ or 2, then $Y = Y$-1 and $M = M + 12$.

D is the day of the month with decimal time (e.g., for the second day of the month at 12:30:30 UT, $D = 2.521180556$).

B is equal to 0, for the Julian calendar {i.e. by using $B = 0$ in Equation 4, $JD <$ 2299160}, and equal to (2 - A + INT (A/4)) for the Gregorian calendar {i.e. by using $B = 0$ in Equation 4, and if $JD >$ 2299160}; A = INT(Y/100).

Users who wish to use their local time instead of UT should change the time zone to a fraction of a day (by dividing it by 24), then subtract the result from JD and note the change of the local date. The fraction is subtracted from JD calculated before the test for $B <$ 2299160 to maintain the Julian and Gregorian periods.

3.1.2. Calculate the Julian Ephemeris Day

$$JDE = JD + \frac{\Delta T}{86400} \quad . \tag{5}$$

3.1.3. Calculate the Julian Century and the Julian Ephemeris Century for the 2000 Standard Epoch

$$JC = \frac{JD - 2451545}{36525} \quad , \tag{6}$$

$$JCE = \frac{JDE - 2451545}{36525} \quad . \tag{7}$$

3.1.4. Calculate the Julian Ephemeris Millennium for the 2000 Standard Epoch

$$JME = \frac{JCE}{10} \quad . \tag{8}$$

3.2. Calculate the Moon Geocentric Longitude, Latitude, and Distance Between the Centers of Earth and Moon (*λ, β, and Δ*)

"Geocentric" means that the Moon position is calculated with respect to Earth's center.

3.2.1. Calculate the Moon's Mean Longitude, L' (in degrees)

$$L' = 218.3164477 + 481267.88123421 * T - 0.0015786 * T^2 + \frac{T^3}{538841} - \frac{T^4}{65194000}, \tag{9}$$

where:

T is JCE from Equation 7.

3.2.2. Calculate the Mean Elongation of the Moon, D (in degrees)

$$D = 297.8501921 + 445267.1114034 * T - 0.0018819 * T^2 + \frac{T^3}{545868} - \frac{T^4}{113065000}. \tag{10}$$

3.2.3. Calculate the Sun's Mean Anomaly, M (in degrees)

$$M = 357.5291092 + 35999.0502909 * T - 0.0001536 * T^2 + \frac{T^3}{24490000}. \tag{11}$$

3.2.4. Calculate the Moon's mean anomaly, M' (in degrees)

$$M' = 134.9633964 + 477198.8675055 * T + 0.0087414 * T^4 - \frac{T^4}{14712000}. \tag{12}$$

3.2.5. Calculate the Moon's Argument of Latitude, F (in degrees)

$$F = 93.2720950 + 483202.0175233 * T - 0.0036539 * T^2 - \frac{T^3}{3526000} + \frac{T^4}{863310000}. \tag{13}$$

3.2.6. Use Table 1 To Calculate the Term l (in 0.000001 degrees)

Table 1. Moon's Periodic Terms for Longitude and Distance

d	m	m'	f	l	r
0	0	1	0	6288774	–20905355
2	0	–1	0	1274027	–3699111
2	0	0	0	658314	–2955968
0	0	2	0	213618	–569925
0	1	0	0	–185116	48888
0	0	0	2	–114332	–3149
2	0	–2	0	58793	246158
2	–1	–1	0	57066	–152138
2	0	1	0	53322	–170733
2	–1	0	0	45758	–204586
0	1	–1	0	–40923	–129620
1	0	0	0	–34720	108743
0	1	1	0	–30383	104755
2	0	0	–2	15327	10321
0	0	1	2	–12528	
0	0	1	–2	10980	79661
4	0	–1	0	10675	–34782
0	0	3	0	10034	–23210
4	0	–2	0	8548	–21636
2	1	–1	0	–7888	24208
2	1	0	0	–6766	30824
1	0	–1	0	–5163	–8379
1	1	0	0	4987	–16675
2	–1	1	0	4036	–12831
2	0	2	0	3994	–10445
4	0	0	0	3861	–11650
2	0	–3	0	3665	14403
0	1	–2	0	–2689	–7003
2	0	–1	2	–2602	
2	–1	–2	0	2390	10056
1	0	1	0	–2348	6322
2	–2	0	0	2236	–9884
0	1	2	0	–2120	5751
0	2	0	0	–2069	
2	–2	–1	0	2048	–4950
2	0	1	–2	–1773	4130
2	0	0	2	–1595	
4	–1	–1	0	1215	–3958
0	0	2	2	–1110	
3	0	–1	0	–892	3258
2	1	1	0	–810	2616
4	–1	–2	0	759	–1897
0	2	–1	0	–713	–2117
2	2	–1	0	–700	2354
2	1	–2	0	691	
2	–1	0	–2	596	

d	m	m'	f	l	r
4	0	1	0	549	–1423
0	0	4	0	537	–1117
4	–1	0	0	520	–1571
1	0	–2	0	–487	–1739
2	1	0	–2	–399	
0	0	2	–2	–381	–4421
1	1	1	0	351	
3	0	–2	0	–340	
4	0	–3	0	330	
2	–1	2	0	327	
0	2	1	0	–323	1165
1	1	–1	0	299	
2	0	3	0	294	
2	0	–1	–2		8752

$$l = \sum_{i=1}^{n} l_i * \sin(d_i * D + m_i * M + m'_i * M' + f_i * F) \quad , \qquad (14)$$

where:

l_i, d_i, m_i, m'_i, and f_i are the i^th^ term in columns l, d, m, m', and f in the table.

The terms in column m depend on the decreasing eccentricity of the Earth's orbit around the Sun; therefore, when the term in column m = (1 or –1) or (2 or –2), multiply l_i in Equation 14 by E or E^2, respectively, where:

$$E = 1 - 0.002516 * T - 0.0000074 * T^2 \quad . \qquad (15)$$

3.2.7. Use Table 1 To Calculate the Term r (in 0.001 kilometers)

$$r = \sum_{i=1}^{n} r_i * \cos(d_i * D + m_i * M + m'_i * M' + f_i * F) \quad . \qquad (16)$$

Similar to step 3.2.6, when m = (1 or –1) and (2 or –2), multiply r_i in Equation 16 by E or E^2.

3.2.8. Use Table 2 To Calculate the Term b (in 0.000001 degrees)

Table 2. Periodic Terms for the Moon's Latitude

d	m	m'	f	b
0	0	0	1	5128122
0	0	1	1	280602
0	0	1	–1	277693
2	0	0	–1	173237
2	0	–1	1	55413
2	0	–1	–1	46271
2	0	0	1	32573
0	0	2	1	17198
2	0	1	–1	9266
0	0	2	–1	8822

d	m	m’	f	b
2	–1	0	–1	8216
2	0	–2	–1	4324
2	0	1	1	4200
2	1	0	–1	–3359
2	–1	–1	1	2463
2	–1	0	1	2211
2	–1	–1	–1	2065
0	1	–1	–1	–1870
4	0	–1	–1	1828
0	1	0	1	–1794
0	0	0	3	–1749
0	1	–1	1	–1565
1	0	0	1	–1491
0	1	1	1	–1475
0	1	1	–1	–1410
0	1	0	–1	–1344
1	0	0	–1	–1335
0	0	3	1	1107
4	0	0	–1	1021
4	0	–1	1	833
0	0	1	–3	777
4	0	–2	1	671
2	0	0	–3	607
2	0	2	–1	596
2	–1	1	–1	491
2	0	–2	1	–451
0	0	3	–1	439
2	0	2	1	422
2	0	–3	–1	421
2	1	–1	1	–366
2	1	0	1	–351
4	0	0	1	331
2	–1	1	1	315
2	–2	0	–1	302
0	0	1	3	–283
2	1	1	–1	–229
1	1	0	–1	223
1	1	0	1	223
0	1	–2	–1	–220
2	1	–1	–1	–220
1	0	1	1	–185
2	–1	–2	–1	181
0	1	2	1	–177
4	0	–2	–1	176
4	–1	–1	–1	166
1	0	1	–1	–164
4	0	1	–1	132
1	0	–1	–1	–119
4	–1	0	–1	115
2	–2	0	1	107

$$b = \sum_{i=1}^{n} b_i * \cos(d_i * D + m_i * M + m'_i * M' + f_i * F) \quad (17)$$

Similar to step 3.2.6, when m = (1 or –1) and (2 or –2), multiply b_i in Equation 17 by E or E^2.

3.2.8. Calculate a_1

$$a_1 = 119.75 + 131.849 * T \quad (18)$$

3.2.9 Calculate a_2

$$a_2 = 53.09 + 479264.29 * T \quad (19)$$

3.2.10. Calculate a_3

$$a_3 = 313.45 + 481266.484 * T \quad (20)$$

3.2.11. Calculate Δl,

$$\Delta l = 3958 * \sin(a_1) + 1962 * \sin(L' - F) + 318 * \sin(a_2) \quad (21)$$

3.2.12. Calculate Δb,

$$\Delta b = -2235 * \sin(L') + 382 * \sin(a_3) + 175 * \sin(a_1 - F) + 175 * \sin(a_1 + F) + 127 * \sin(L' - M') - 115 * \sin(L' + M') \quad (22)$$

3.2.13. Calculate the Moon's Longitude, λ' (in degrees)

$$\lambda' = L' + \frac{l + \Delta l}{1000000} \quad (23)$$

3.2.14. Calculate the Moon's Latitude, β (in degrees)

$$\beta = \frac{b + \Delta b}{1000000} \quad (24)$$

3.2.15. Limit λ and β to the Range of 0° to 360°

3.2.16. Calculate the Moon's Distance From the Center of Earth, Δ (in kilometers)

$$\Delta = 385000.56 + \frac{r}{1000} \quad (25)$$

3.3. Calculate the Moon's Equatorial Horizontal Parallax, π

$$\pi = a\sin\left(\frac{6378.14}{\Delta}\right) \quad (26)$$

3.4. Calculate the Nutation in Longitude and Obliquity ($\Delta\psi$ and $\Delta\varepsilon$)

3.4.1. Calculate the Mean Elongation of the Moon From the Sun, X_0 (in degrees)

$$X_0 = 297.85036 + 445267.111480 * JCE - 0.0019142 * JCE^2 + \frac{JCE^3}{189474} \quad . \tag{27}$$

3.4.2. Calculate the Mean Anomaly of the Sun (Earth), X_1 (in degrees)

$$X_1 = 357.52772 + 35999.050340 * JCE - 0.0001603 * JCE^2 - \frac{JCE^3}{300000} \quad . \tag{28}$$

3.4.3. Calculate the Mean Anomaly of the Moon, X_2 (in degrees)

$$X_2 = 134.96298 + 477198.867398 * JCE + 0.0086972 * JCE^2 + \frac{JCE^3}{56250} \quad . \tag{29}$$

3.4.4. Calculate the Moon's Argument of Latitude, X_3 (in degrees)

$$X_3 = 93.27191 + 483202.017538 * JCE - 0.0036825 * JCE^2 + \frac{JCE^3}{327270} \quad . \tag{30}$$

3.4.5. Calculate the Longitude of the Ascending Node of the Moon's Mean Orbit on the Ecliptic, Measured From the Mean Equinox of the Date, X_4 (in degrees)

$$X_4 = 125.04452 - 1934.136261 * JCE + 0.0020708 * JCE^2 + \frac{JCE^3}{450000} \quad . \tag{31}$$

3.4.6. For Each Row in Table 3, Calculate the Terms $\Delta\psi_i$ and $\Delta\varepsilon_i$ (in 0.0001of arc seconds)

Table 3. Periodic Terms for the Nutation in Longitude and Obliquity

Coefficients for Sin Terms					Coefficients for Δψ		Coefficients for Δε	
Y0	**Y1**	**Y2**	**Y3**	**Y4**	**a**	**B**	**c**	**d**
0	0	0	0	1	–171996	–174.2	92025	8.9
–2	0	0	2	2	–13187	–1.6	5736	–3.1
0	0	0	2	2	–2274	–0.2	977	–0.5
0	0	0	0	2	2062	0.2	–895	0.5
0	1	0	0	0	1426	–3.4	54	–0.1
0	0	1	0	0	712	0.1	–7	
–2	1	0	2	2	–517	1.2	224	–0.6
0	0	0	2	1	–386	–0.4	200	
0	0	1	2	2	–301		129	–0.1
–2	–1	0	2	2	217	–0.5	–95	0.3
–2	0	1	0	0	–158			
–2	0	0	2	1	129	0.1	–70	
0	0	–1	2	2	123		–53	
2	0	0	0	0	63			
0	0	1	0	1	63	0.1	–33	
2	0	–1	2	2	–59		26	
0	0	–1	0	1	–58	–0.1	32	
0	0	1	2	1	–51		27	
–2	0	2	0	0	48			
0	0	–2	2	1	46		–24	
2	0	0	2	2	–38		16	
0	0	2	2	2	–31		13	
0	0	2	0	0	29			
–2	0	1	2	2	29		–12	
0	0	0	2	0	26			
–2	0	0	2	0	–22			
0	0	–1	2	1	21		–10	
0	2	0	0	0	17	–0.1		
2	0	–1	0	1	16		–8	
–2	2	0	2	2	–16	0.1	7	
0	1	0	0	1	–15		9	

Coefficients for Sin Terms					Coefficients for Δψ		Coefficients for Δε	
Y0	**Y1**	**Y2**	**Y3**	**Y4**	**a**	**B**	**c**	**d**
–2	0	1	0	1	–13		7	
0	–1	0	0	1	–12		6	
0	0	2	–2	0	11			
2	0	–1	2	1	–10		5	
2	0	1	2	2	–8		3	
0	1	0	2	2	7		–3	
–2	1	1	0	0	–7			
0	–1	0	2	2	–7		3	
2	0	0	2	1	–7		3	
2	0	1	0	0	6			
–2	0	2	2	2	6		–3	
–2	0	1	2	1	6		–3	
2	0	–2	0	1	–6		3	
2	0	0	0	1	–6		3	
0	–1	1	0	0	5			
–2	–1	0	2	1	–5		3	
–2	0	0	0	1	–5		3	
0	0	2	2	1	–5		3	
–2	0	2	0	1	4			
–2	1	0	2	1	4			
0	0	1	–2	0	4			
–1	0	1	0	0	–4			
–2	1	0	0	0	–4			
1	0	0	0	0	–4			
0	0	1	2	0	3			
0	0	–2	2	2	–3			
–1	–1	1	0	0	–3			
0	1	1	0	0	–3			
0	–1	1	2	2	–3			
2	–1	–1	2	2	–3			
0	0	3	2	2	–3			
2	–1	0	2	2	–3			

$$\Delta \psi_i = (a_i + b_i * JCE) * \sin\left(\sum_{j=1}^{4} X_j * Y_{i,j}\right) \quad , \tag{32}$$

$$\Delta \varepsilon_i = (c_i + d_i * JCE) * \cos\left(\sum_{j=1}^{4} X_j * Y_{i,j}\right) \quad , \tag{33}$$

where:

a_i , b_i , c_i , and d_i	are the values listed in the ith row and columns a, b, c, and d in Table 5.
X_j	is the jth X calculated by using Equations 15 through 19.
$Y_{i,j}$	is the value listed in ith row and jth Y column in Table 5.

3.4.7. Calculate the Nutation in Longitude, Δψ (in degrees)

$$\Delta \psi = \frac{\sum_{i=1}^{n} \Delta \psi_i}{36000000} \quad , \tag{34}$$

where:

n is the number of rows in Table 5 (n equals 63 rows in the table).

3.4.8. Calculate the Nutation in Obliquity, Δε (in degrees)

$$\Delta \varepsilon = \frac{\sum_{i=1}^{n} \Delta \varepsilon_i}{36000000} \quad . \tag{35}$$

3.5. Calculate the True Obliquity of the Ecliptic, ε (in degrees)

3.5.1. Calculate the Mean Obliquity of the Ecliptic, ε_0 (in arc seconds)

$$\begin{aligned} \varepsilon_0 = {} & 84381.448 - 4680.93U - 1.55U^2 + 1999.25U^3 - \\ & 51.38U^4 - 249.67U^5 - 39.05U^6 + 7.12U^7 + \\ & 27.87U^8 + 5.79U^9 + 2.45U^{10} \quad , \end{aligned} \tag{36}$$

where:

U is $JME/10$.

3.5.2. Calculate the True Obliquity of the Ecliptic, ε (in degrees)

$$\varepsilon = \frac{\varepsilon_0}{3600} + \Delta \varepsilon \quad . \tag{37}$$

3.6. Calculate the Apparent Moon Longitude, *λ* (in degrees)

$$\lambda = \lambda' + \Delta \psi \ . \tag{38}$$

3.7. Calculate the Apparent Sidereal Time at Greenwich at Any Given Time, *v* (in degrees)

3.7.1. Calculate the Mean Sidereal Time at Greenwich, v_0 (in degrees)

$$\nu_0 = 280.46061837 + 360.98564736629 * (JD - 2451545) + 0.000387933 * JC^2 - \frac{JC^3}{38710000} \ . \tag{39}$$

3.7.2. Calculate the Apparent Sidereal Time at Greenwich, v (in degrees)

$$\nu = \nu_0 + \Delta \psi * \cos(\varepsilon) \ . \tag{40}$$

3.7.3. Limit v to the Range of 0° to 360°

3.8. Calculate the Moon's Geocentric Right Ascension, *α* (in degrees)

3.8.1. Calculate the Moon's Right Ascension, α (in radians)

$$\alpha = Arc\tan 2 \left(\frac{\sin \lambda * \cos\varepsilon - \tan \beta * \sin \varepsilon}{\cos\lambda}\right) \ , \tag{41}$$

where:

Arc*tan*2 is an arctangent function that is applied to the numerator and the denominator (instead of the actual division) to maintain the correct quadrant of *α*, where *α* is in the rage of –π to π.

3.8.2. Calculate α in Degrees, Then Limit It to the Range of 0° to 360°

3.9. Calculate the Moon's Geocentric Declination, *δ* (in degrees)

$$\delta = Arc\sin(\sin \beta * \cos\varepsilon + \cos\beta * \sin \varepsilon * \sin \lambda) \ , \tag{42}$$

where:

δ is positive or negative if the Sun is north or south of the celestial equator, respectively. Then change δ to degrees.

3.10. Calculate the Observer Local Hour Angle, *H* (in degrees)

$$H = \nu + \sigma - \alpha \ , \tag{43}$$

where:

σ is the observer geographical longitude, positive or negative for east or west of Greenwich, respectively.

Limit *H* to the range from 0° to 360° and note that it is measured westward from south in this algorithm.

3.11. Calculate the Moon's Topocentric Right Ascension *α'* (in degrees)

"Topocentric" means that the Moon's position is calculated with respect to the observer local position at the Earth's surface.

3.11.1. Calculate the Term u (in radians)

$$u = Arc\tan\ (0.99664719 * \tan\varphi) \quad , \tag{44}$$

where:

ϕ is the observer's geographical latitude, positive or negative if north or south of the equator, respectively. The 0.99664719 number equals $(1 - f)$, where f is the Earth's flattening.

3.11.2. Calculate the Term x

$$x = \cos u + \frac{E}{6378140} * \cos\varphi \quad , \tag{45}$$

where:

E is the observer's elevation (in meters). Note that x equals $\rho * \cos\ \phi'$ where ρ is the observer's distance to the center of the Earth, and ϕ' is the observer's geocentric latitude.

3.11.3. Calculate the Term y

$$y = 0.99664719 * \sin u + \frac{E}{6378140} * \sin\varphi \quad , \tag{46}$$

note that y equals $\rho * \sin\ \phi'$,

3.11.4. Calculate the Parallax in the Moon's Right Ascension, Δα (in degrees)

$$\Delta\alpha = Arc\tan 2\ \left(\frac{-x * \sin\ \pi * \sin\ H}{\cos\delta - x * \sin\ \pi * \cos H}\right) \quad . \tag{47}$$

then change $\Delta\alpha$ to degrees.

3.11.5. Calculate the Moon's Topocentric Right Ascension α' (in degrees)

$$\alpha' = \alpha + \Delta\alpha \quad . \tag{48}$$

3.11.6. Calculate the Topocentric Moon's Declination, δ' (in degrees)

$$\delta' = Arc\tan 2\,\left(\frac{(\sin\delta - y * \sin\pi) * \cos\Delta\alpha}{\cos\delta - y * \sin\pi * \cos H}\right) \quad . \tag{49}$$

3.12. Calculate the Topocentric Local Hour Angle, *H'* (in degrees)

$$H' = H - \Delta\alpha \quad . \tag{50}$$

3.13. Calculate the Moon's topocentric zenith angle, θ_m (in degrees)

3.13.1. Calculate the Topocentric Elevation Angle Without Atmospheric Refraction Correction, e_0 (in degrees)

$$e_0 = Arc\sin\,(\sin\varphi * \sin\delta' + \cos\varphi * \cos\delta' * \cos H') \quad . \tag{51}$$

then change e_0 to degrees.

3.13.2. Calculate the Atmospheric Refraction Correction, Δe (in degrees)

$$\Delta e = \frac{P}{1010} * \frac{283}{273 + T} * \frac{1.02}{60 * \tan\,\left(e_0 + \frac{10.3}{e_0 + 5.11}\right)} \quad , \tag{52}$$

where:

- P is the annual average local pressure (in millibars).
- T is the annual average local temperature (in °C).
- e_0 is in degrees. Calculate the tangent argument in degrees, then convert to radians.

3.13.3. Calculate the Topocentric Elevation Angle, e (in degrees)

$$e = e_0 + \Delta e \quad . \tag{53}$$

3.13.4. Calculate the Topocentric Zenith Angle, θ (in degrees)

$$\theta_m = 90 - e \quad . \tag{54}$$

3.14. Calculate the Moon's Topocentric Azimuth Angle, Φ_m (in degrees)

3.14.1. Calculate the Topocentric Astronomers' Azimuth Angle, Γ (in degrees)

$$\Gamma = Arc\tan 2\,\left(\frac{\sin H'}{\cos H' * \sin\varphi - \tan\delta' * \cos\varphi}\right) \quad , \tag{55}$$

Change *Γ* to degrees, then limit it to the range of 0° to 360°. *Γ* is measured *westward* from *south*.

3.14.2. Calculate the Topocentric Azimuth Angle, Φ for Navigators and Solar Radiation Users (in degrees)

$$\Phi_m = \Gamma + 180 \quad , \tag{56}$$

Limit Φ_m to the range from 0° to 360° using step 3.2.6. Φ_m is measured *eastward* from *north*.

4.0 Moon Position Algorithm Validation

The solar zenith and azimuth angles are not reported in AA, so the Moon's declination and equatorial horizontal parallax are used for the evaluation. Exact trigonometric functions are used with the AA reported Moon parameters to calculate the solar zenith and azimuth angles; therefore, it is adequate to evaluate the MPA uncertainty with these parameters. To evaluate the uncertainty of the MPA, arbitrary dates, January 17 and October 17, are chosen from each of the years 2004 to 2010, and 1981, at 0-hour TT. Figure 1 shows that the maximum difference between the AA and MPA is 0.00055° for the Moon's declination, and Figure 2 shows that the maximum difference is 0.00003 for the equatorial Moon parallax. This implies that the MPA is well within the stated uncertainty of ±0.001° and ±0.003° in the zenith and azimuth angles, respectively.

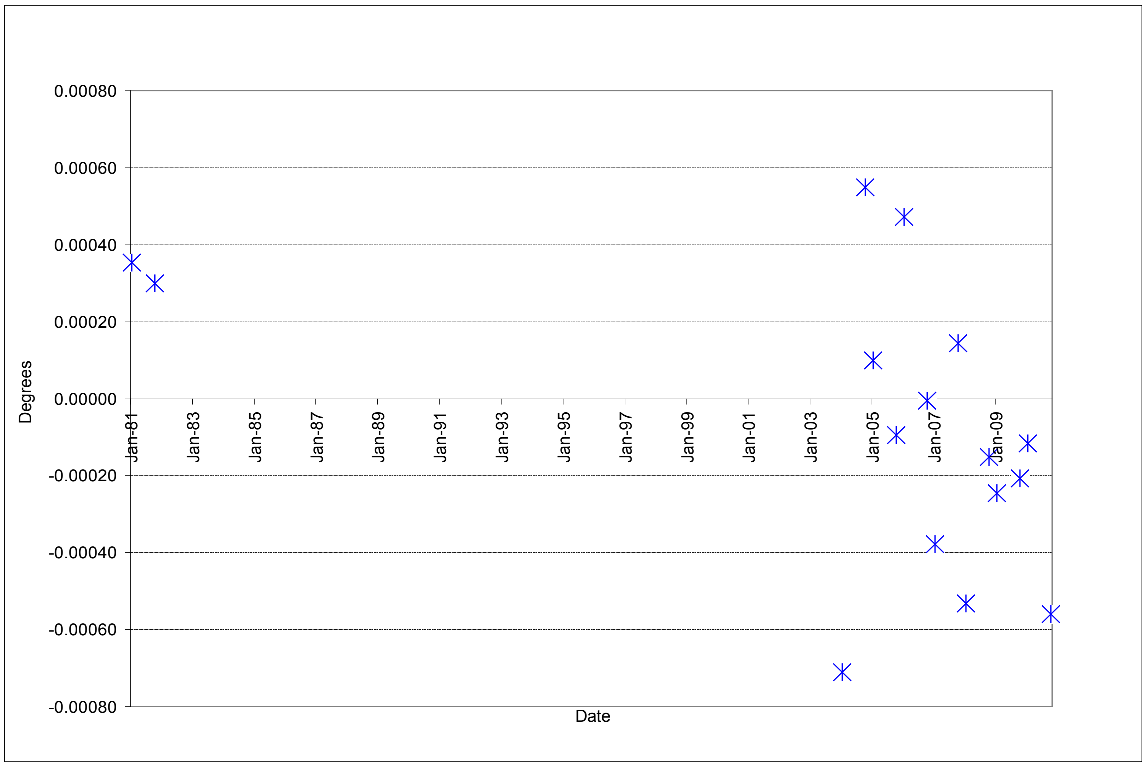

Figure 1. Difference between the AA and MPA for the Moon's declination

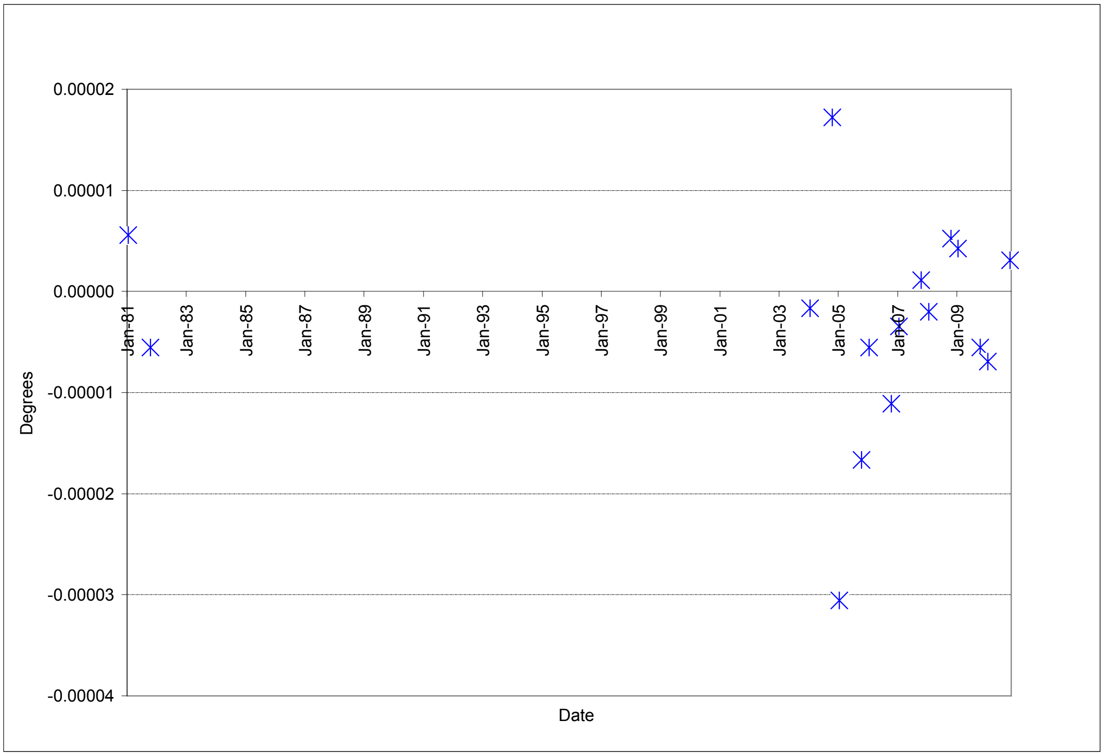

Figure 2. Difference between the AA and MPA for the Moon's horizontal parallax

5.0 Predicting and Monitoring the Solar Eclipse Occurrence

The full astronomical nomenclature for eclipse monitoring is beyond the scope of this report, so only the total and partial solar eclipse nomenclatures are used. In this section, the zenith and azimuth angles of the Sun and the Moon are calculated continuously with SAMPA. The SPA is described in detail in Reda and Andreas [2]; the MPA is described above. A solar eclipse can then be monitored as follows:

5.1. Calculate the Local Observed, Topocentric, Angular Distance Between the Sun and Moon Centers, E_{ms} (in degrees)

$$E_{ms} = \cos^{-1}[\cos\theta_s * \cos\theta_m + \sin\theta_s * \sin\theta_m * \cos(\phi_s - \phi_m)], \quad (57)$$

where:

θ_s and Φ_s are the zenith and azimuth angles of the sun, calculated using SPA [2].

5.2. Calculate the Radius of the Sun's Disk, r_s (in degrees)

$$r_s = \frac{959.63}{3600 * R_s}, \quad (58)$$

where:

R_s is the Sun's distance from the center of the Earth, in astronomical units (AUs). This distance is calculated using SPA [2].

5.3. Calculate the Radius of the Moon's Disk, r_m (in degrees)

$$r_m = \frac{358473400 * (1 + \sin e * \sin \pi)}{3600 * \Delta}, \quad (59)$$

where:

e, π, and Δ   are calculated in Section 3.

5.4. Set the Boundary Conditions for the Solar Eclipse

5.4.1. No Eclipse

$E_{ms} > (r_m + r_s)$, where r_m and r_s are the Sun and Moon radii

5.4.2. Start and End of Eclipse

$E_{ms} = (r_m + r_s)$

5.4.3. Solar Eclipse

$E_{ms} < (r_m + r_s)$

5.4.4. Sun unshaded area during Solar Eclipse

If $E_{ms} \leq abs(r_m - r_s)$, it is a total eclipse where the Sun and Moon disks (circles) completely overlap; therefore, if $r_s > r_m$, the unshaded Sun area by the Moon will equal the area of the Sun disk minus the area of the moon disk. Moreover, if $r_s \leq r_m$, the unshaded area of the Sun equals zero.

To monitor the solar eclipse, a criterion where E_{ms} equals $(r_m + r_s)$ is set at the beginning and end of the eclipse. At this moment the time is advanced; e.g., in one-second increments, for at least three hours. The distance E_{ms} can then be plotted to show the eclipse, and from the data, one might calculate the duration of a partial or total solar eclipse by calculating the time when minimum E_{ms} occurs, T_{min} , then as the eclipse ends, the time when maximum E_{ms} occurres, T_{max} , is calculated. The total duration for the eclipse occurrence will equal $T_{min} + T_{max}$. Figure 3 shows E_{ms} versus time for the central solar eclipse on July 22, 2009. Using this method, the eclipse duration equals 1.97 hours and the minimum E_{ms} equals 0.0001°, which is well within the uncertainty of calculating the Sun and Moon positions. To verify this method, E_{ms} is calculated for some historical total solar eclipses at different locations. Table 4 shows that $E_{ms} < 0.0011$ for the listed seven solar eclipses, which is within the stated uncertainty of ±0.003° in the azimuth angle.

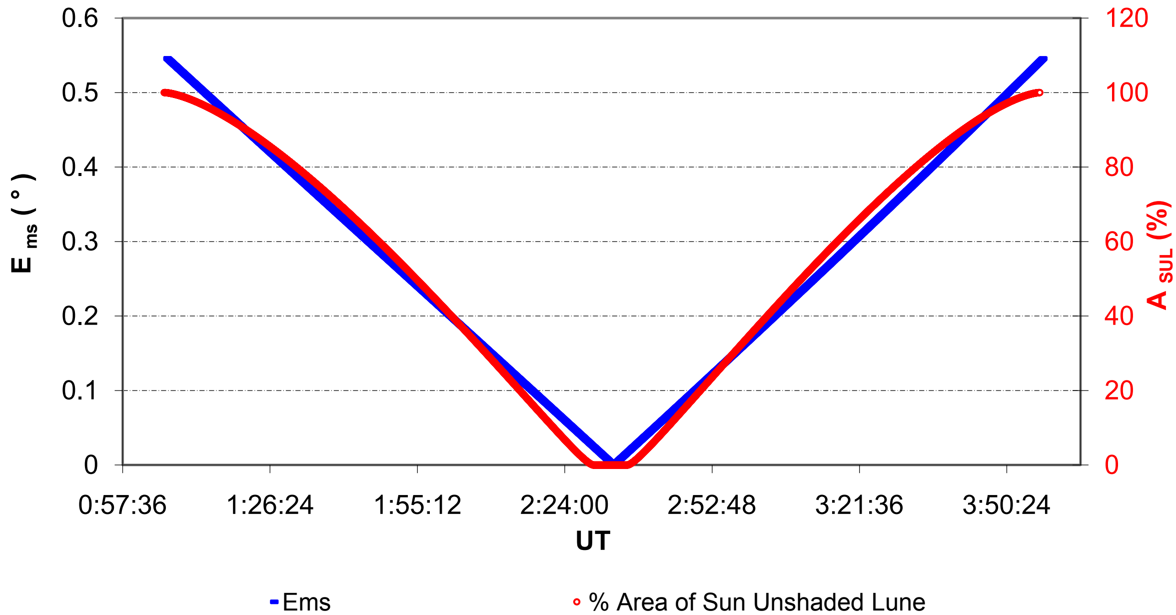

Figure 3. Distance between the Sun and Moon centers and the percentage of the SUL for the July 22, 2009 solar eclipse

Table 4. Historical Solar Eclipses Versus SAMPA Eclipse Monitor, E_{ms}

Historical Eclipse Dates	UT	Observer's Longitude °	Observer's Latitude °	Delta T (Sec)	SAMPA, $E°_{ms}$
7/22/2009	2:33:00	143.3617	24.6117	66.4	0.0001
8/1/2008	9:47:18	34.7417	81.1133	65.8	0.0002
3/29/2006	10:33:18	22.8867	29.6200	64.9	0.0005
4/8/2005	20:15:36	–123.4817	–15.7883	64.8	0.0011
12/4/2002	7:38:42	62.8383	–40.5283	64.4	0.0005
6/21/2001	11:57:48	0.9867	–11.5950	64.2	0.0003
2/4/1981	21:57:36	–145.9033	–45.8883	51.5	0.0004

6.0 Estimating the Solar Irradiance During a Solar Eclipse

When a solar eclipse occurs, the observed Moon's disk will start shading the Sun's disk; the shaded area will change as time progresses; therefore, the unshaded area of the Sun disk is called the Sun's Unshaded Lune (SUL), which will also change with time. The percentage of the SUL from the total Sun's disk area is then calculated, Figure 3 shows the change of SUL during the July 22, 2009 solar eclipse. The percentage area might then be multiplied by an estimated direct beam irradiance to calculate the irradiance during the eclipse. A spectacular phenomena occurs during the solar eclipse, therefore, the spectral distribution of the irradiance from the Sun changes. The method described below illustrates how the irradiance might be estimated during the solar eclipse, yet it does not account for the significant change in the spectral distribution of the sun irradiance during the eclipse occurrence. In the future, when significant changes in the instrumentation and spectral measurement are achieved, users might use other methods or models to achieve smaller uncertainty for such estimates.

6.1. Calculate the Area of SUL, A_{SUL}

To calculate this area, draw two intersecting circles, with two different radii of the Sun and the Moon, r_s and r_m. An illustration is shown in Appendix A.2, Figure A.2.1.

$$A_{SUL} = \pi * r_s^{\,2} - A_i \,, \tag{60}$$

where:

A_i is the area of the Sun's disk that is shaded by the Moon. A step-by-step method to calculate A_{SUL} is described in Appendix A.2.

6.2. Calculate the Percentage Area of the SUL With Respect to the Area of the Sun's Disk, $\%A_{SUL}$

$$\% A_{SUL} = \frac{A_{SUL} * 100}{\pi * r_s^{\,2}} \,. \tag{61}$$

6.3. Calculate the Direct Beam Irradiance Using the Appropriate Model for the Required Uncertainty, in W/m^2

The Bird and Hulstrom simple model [5] is used as an illustration.

6.4. Calculate the Irradiance (W/m^2) During the Eclipse, I_e

$$I_e = \frac{I * \% A_{SUL}}{100} \,. \tag{62}$$

where:

I is the estimated direct beam irradiance calculated by the model.

To evaluate the described method, the calculated irradiance is compared against the irradiance measured at the University of Oregon, Eugene, Solar Radiation Monitoring Laboratory. The irradiance was measured during the June 10, 2002 partial eclipse, using a pyrheliometer model NIP, manufactured by the Eppley Laboratory, Inc. Figure 4 shows the difference between the measured irradiance and the calculated irradiance using the method described above. The figure shows that the difference between the measured irradiance at the University of Oregon and the calculated irradiance by SAMPA is about 8% when the partial eclipse starts, 4% at the maximum eclipse, and 6% as the eclipse ends. These differences are expected, because the measuring instruments (estimated $U_{95} = \pm 3\%$) and the model used above (estimated $U_{95} = \pm 5\%$) do not account for the significant change in the spectral distribution of the irradiance during the solar eclipse occurrence. In the future, with the advancement in pyrheliometer design and spectral measurement technology, advanced models might be used to improve the uncertainty in measuring such significant change in the spectral distribution during the eclipse occurrence.

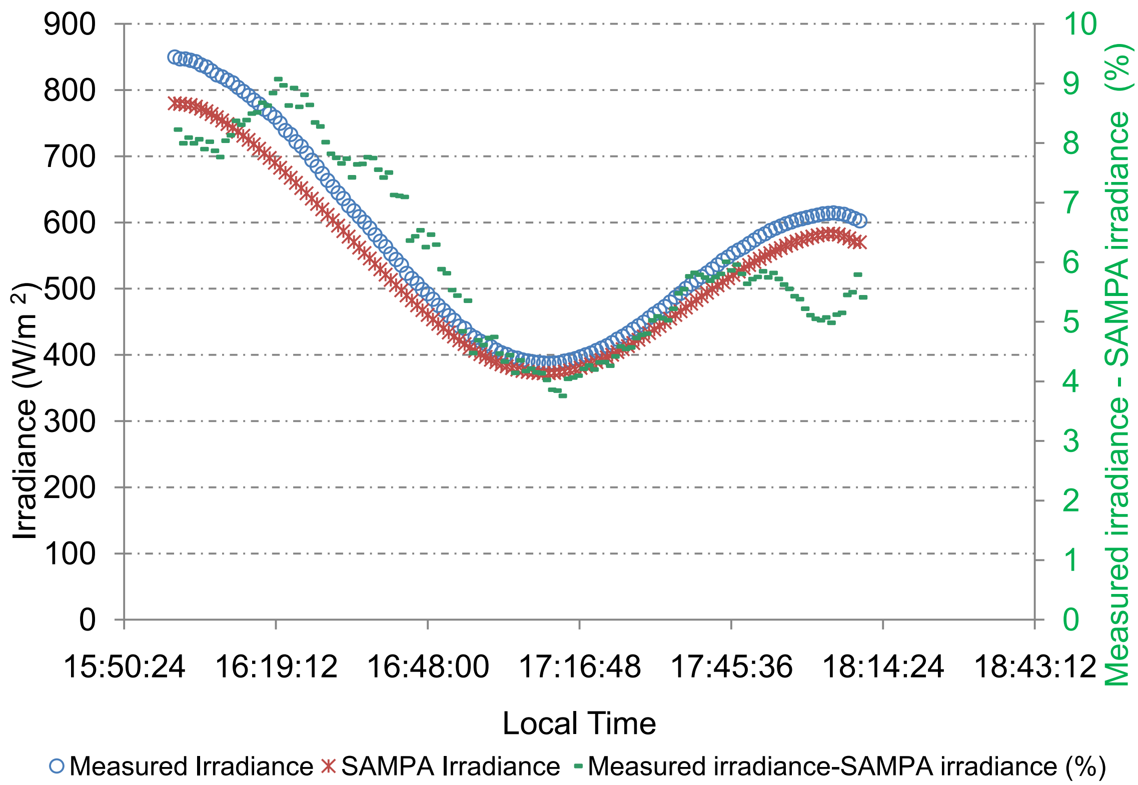

Figure 4. Measured irradiance versus calculated irradiance using SAMP during the June 10, 2002 partial solar eclipse

7. Conclusions

The MPA achieves uncertainties of ±0.001° and ±0.003° in calculating the zenith and azimuth angles of the Moon (see Figures 1 and 2). Using MPA in conjunction with the SPA (uncertainty of ±0.0003°) to monitor solar eclipses, is consistent with the historical eclipses to within the stated uncertainty of the MPA see Table 5. Section 6 and Figure 4 show that the direct beam irradiance from the sun during a solar eclipse is estimated within 4% to 8% from measured irradiance that was collected during the eclipse of June 10, 2002. This implies that smart grid users might be able, with the current technology, to use this information to manage the grid's solar resources during eclipses to within 8%. Improved uncertainties might be achieved by developing advanced models that include the change of the spectral distribution during the spectacular solar eclipse. Figure 4 also shows that the partial solar eclipse of June 10, 2002 in Eugene, Oregon, lasted longer than two hours, which might have devastating effect on the smart grid if solar energy is used as a grid resource.

References

1. Meeus, J. *Astronomical Algorithms*. Second edition 1998, Willmann-Bell, Inc., Richmond, Virginia, USA.

2. Reda, I.; Andreas, A. "Solar Position Algorithm for Solar Radiation Applications." *Solar Energy* 76(5), 2004; pp. 577_589; NREL Report No. JA-560-35518. doi:10.1016/j.solener.2003.12.003

3. *The Astronomical Almanac*. Norwich: 2004.

4. The U.S. Naval Observatory. Washington, D.C., www.usno.navy.mil/.

5. Bird, R.E.; Hulstrom, R.L. *Simplified Clear Sky Model for Direct and Diffuse Insolation on Horizontal Surfaces*, Technical Report No. SERI/TR-642-761, Golden, CO: Solar Energy Research Institute, 1981.

Appendix

Some symbols used in the appendix are independent from those used in the main report.

A.1. Example

Table A.1.1. Example to Compare Users' Calculations to SAMPA

Inputs for SAMPA	Value	Unit	Inputs for SAMPA	Value	Unit
Date	7/22/2009		**Latitude**	24.61167	°
UT	1:33:00		**Average Pressure**	1000	Mb
Δ T	66.4	Seconds	**Average Temperature**	11	°C
Δ UT1	0	Seconds	**Location Elevation**	0	Km
Longitude	143.36167	°			

Table A.1. 2. SAMPA Results

θ_s +refraction	14.50514	°	**M'**	6.110197	°
Φ_s	104.38792	°	**E**	0.9997596	Unitless
r_s	0.26236	°	**∑ l + additive**	575973.275	0.000001 °
A_{SUL}	0.169478	$°^2$	**∑ b + additive**	131572.571	0.000001 °
%A_{SUL}	78.3733	%	**∑ r**	–27486437.833	0.001 Km
θ_m +refraction	14.13343	°	**β**	0.131573	°
Φ_m	104.19314	°	**Δ**	357514.1221	Km
E_{ms}	0.37481367	°	**λ**	118.7934477	°
L'	118.2130333	°	**α'**	121.202944	°
D	358.2658977	°	**δ'**	20.448307	°
M	196.845702	°			

A.2. Calculating the Areas of the Lunes When Two Circles With Different Diameters Intersect

The following steps are intended for calculating the area of the Sun Unshaded Lune, A_{SUL}, during solar eclipses. The Sun and Moon radii are not equal and change with the day of year. Also, during the solar eclipse, as the Moon starts to shade the Sun disk, the intersection area changes with time. In Figure A.2.1, the circles with centers C_s and C_m represent the Sun and Moon disks, respectively. Refer to the figure to calculate A_{SUL}, bounded by sector aebq:

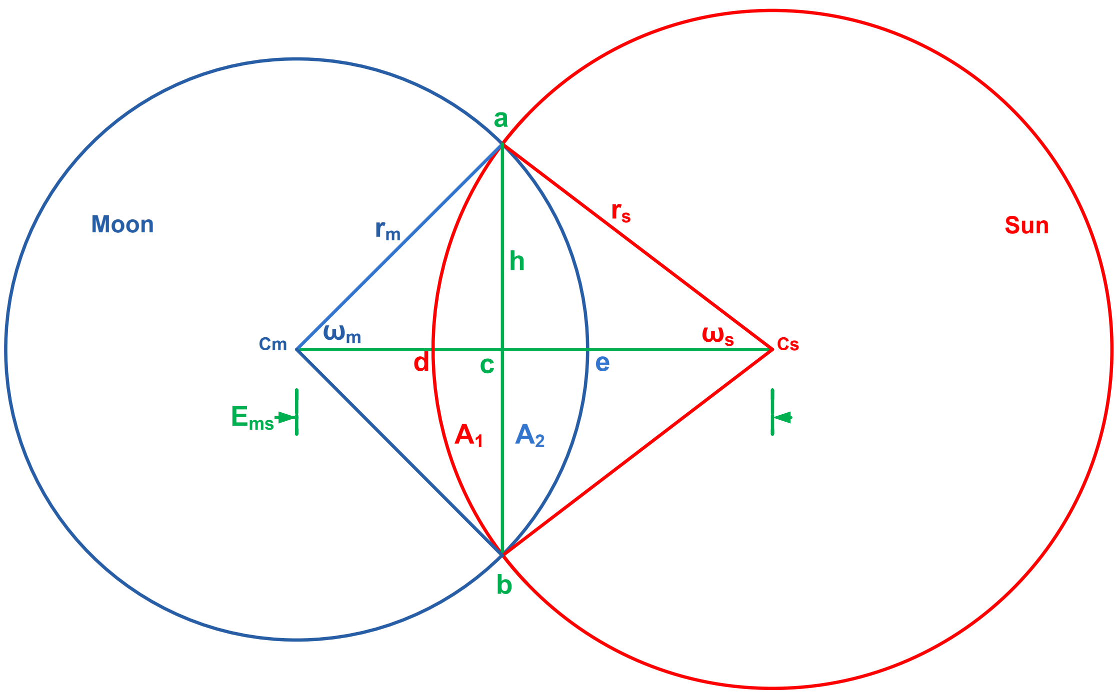

Figure A2.1. Intersecting circles to calculate the areas of the Sun and Moon lunes

1. Use the procedure described in this article to calculate the angular distance between the Sun and Moon centers, E_{ms}
2. Write the following equation:

$$E_{ms} = m + s\ , \tag{A1}$$

where:

m is the distance cC_m

s is the distance cC_s.

Note that m and s are the heights of the two triangles, in degrees.

3. Using the Pythagorean theorem:

$$h^2 = r_s^{\ 2} - s^2 = r_m^{\ 2} - m^2\ , \tag{A2}$$

where:

r_s and r_m are the Sun and Moon radii, calculated using the procedure described in this report, in degrees

h is half the base of the two triangles abC_s and abC_m , in degrees.

Therefore:

$$r_s^{\ 2} - s^2 = r_m^{\ 2} - m^2\ . \tag{A3}$$

4. Solve Equations A1 and A3 with two unknowns to calculate s and m:

$$s = \frac{E_{ms}^2 + r_s^2 - r_m^2}{2 * E_{ms}}, \qquad \text{(A4)}$$

and,

$$m = \frac{E_{ms}^2 - r_s^2 + r_m^2}{2 * E_{ms}}. \qquad \text{(A5)}$$

5. Use Equations A2 and A4 to calculate h:

$$h = \frac{\sqrt{4 * E_{ms}^2 * r_s^2 - (E_{ms}^2 + r_s^2 - r_m^2)^2}}{2 * E_{ms}}. \qquad \text{(A6)}$$

6. Calculate the area of triangle abC$_s$, T$_s$:

$$T_s = h * s. \qquad \text{(A7)}$$

7. Calculate the area of triangle abC$_m$, T$_m$:

$$T_m = h * m. \qquad \text{(A8)}$$

8. Calculate the area of sector adbC$_s$ in the Sun's circle, A$_s$:

$$A_s = \pi * r_s^2 * \frac{2 * \omega_s}{360} = r_s^2 * \cos^{-1}\frac{s}{r_s}, \qquad \text{(A9)}$$

where:

ω_s is half the central angle of sector adbC$_s$ in the Sun's circle.

9. Calculate the area of section abd in the Sun's circle, A$_1$:

$$A_1 = A_s - T_s. \qquad \text{(A10)}$$

10. Similar to Equation A9, calculate the area of sector aebC$_m$ in the Moon's circle, A$_m$,

$$A_m = r_m^2 * COS^{-1}\frac{m}{r_m}. \qquad \text{(A11)}$$

11. Calculate the area of section abe in the Moon's circle, A$_2$:

$$A_2 = A_m - T_m. \qquad \text{(A12)}$$

12. Calculate the area of the Sun's circle shaded by the Moon's circle, A_i:

$$A_i = A_1 + A_2 \,. \tag{A13}$$

13. Calculate the Sun's Unshaded Lune, A_{SUL}:

$$A_{SUL} = \pi * r_s^{\,2} - A_i \,. \tag{A14}$$

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14. ABSTRACT *(Maximum 200 Words)*
This report includes a procedure for implementing an algorithm (described by Jean Meeus [1]) to calculate the Moon's zenith angle with uncertainty of ±0.001° and azimuth angle with uncertainty of ±0.003°. The step-by-step format presented here simplifies the complicated steps Meeus describes to calculate the Moon's position, and focuses on the Moon instead of the planets and stars. It also introduces some changes to accommodate for solar radiation applications.

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