NATURAL RESOURCES JOURNAL

Volume 10
Issue 2 *Spring 1970*

Spring 1970

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Recommended Citation

Richard A. Tybout & George O. Lof, *Solar House Heating*, 10 Nat. Resources J. 268 (1970).
Available at: https://digitalrepository.unm.edu/nrj/vol10/iss2/3

SOLAR HOUSE HEATING*

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Contemporary standards of living have been realized through technologies that depend especially on the conventional mineral fuels, coal, oil and gas. These minerals account for three quarters of all energy utilized throughout the world.[1] Another fifteen percent comes from vegetable matter,[2] which (in the form of wood) was the most important source of energy a century ago. The remainder of the energy is derived from falling water, atomic energy (through conversion to electricity), direct muscular efforts of man and draft animals and from other unclassified, largely noncommercial, sources.

Direct uses of solar energy are limited to recovery of salt from brines, drying of food products, scattered practical applications in hot water heating and salt water distillation, and a wide range of semi-experimental applications, of which space heating is one. Other uses take the form of focusing systems for high temperature research, satellite power units, terrestial power units and home cookers. The United States space program has provided the financial support for research on very small power units, and spin-offs for terrestial applications can be expected in the communications field. Whether large scale terrestrial uses of solar power will eventually materialize is more conjectural. Financial support for research on this topic is almost nonexistent, though practical solar power would be at least as important to the underdeveloped sun-lit regions of the world as civilian atomic power, which we have already spent billions of dollars to develop.[3]

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† The authors are, respectively, Professor of Economics, The Ohio State University and Professor of Civil Engineering, Colorado State University. Professor Löf is also a Consultant, Resources for the Future, Inc., and is the resident-owner of a solar heated house of his own design. The authors are jointly conducting a research program to evaluate the economic potential of several major solar energy applications. Financial assistance from Resources for the Future, Inc., and The Ohio State University is gratefully acknowledged.

[1] United Nations, Department of Economic and Social Affairs, *World Energy Requirements in 1975 and 2000,* 1 Peaceful Uses of Atomic Energy, Table 22, U.N. Doc. A/Conf. 8/3 (1956).

[2] *Id.*

[3] A cumulative total expenditure of approximately $5½ billion has been assigned to reactor development by the U.S. Atomic Energy Commission through calendar year 1967. *See Major Activities in the Atomic Energy Programs* (annual). This figure includes reactor development for military as well as civilian applications, though low cost civilian power has always been a major mission of the AEC's reactor development

The potential of solar energy for carrying a part of the world's bulk energy load must be judged in the light of two properties of the energy resource: (1) its sheer abundance and (2) its low intensity and interruptibility. Solar energy falls on the land areas of the world at roughly one thousand times the annual rate of total world energy consumption estimated for year 2000.[4] But because of its low intensity and interruptibility, there is no prospect for its direct use in such small-unit high-intensity applications as required for transportation (which accounts for almost one quarter of all energy consumption in the United States). For other applications, house heating in particular, low intensity does not preclude its use; interruptibility can be offset by heat storage, and compactness of energy supply system is not the crucial concern that it is in mobile vehicles. Moreover, space heating is an important energy consumer. Household energy consumption in the United States (excluding transportation) accounts for 17 to 18% of total energy consumption,[5] slightly over half of which is for space heat.[6] In addition, at least as important a part of commercial energy consumption (8 to 9% of total) is for space heat, and some industrial energy consumption (approximately 40 percent of total) falls in the same category.[7]

A rough calculation will illustrate the possibilities of solar house heating. Assume a typical American middle-income house with 1000 square feet of floor area. Such a house will have a space heat consumption in the neighborhood of 15,000 British Thermal Units per Degree Day (hereafter referred to as BTU/DD), depending on insulation, fenestration and other architectural properties.[8] Further, a representative American climate will have about 4000 degree days in a year. This means the house will have a heat demand of 15,000 times 4000, or 60 million BTU/yr. Now, the average annual U.S. solar radiation intensity is 1400 BTU/ft^2 per day. On 1,000 square feet of surface (the horizontal projection of the roof area of the house) over 365 days, simple multiplication shows that ap-

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program. In addition, the above figure does not include certain other (social) costs. For a discussion of the nature of the latter, see Tybout, *Atomic Power and the Public Interest,* 34 Land Econ. 281 (1958). A final sorting out of properly attributable costs is less important here than the order of magnitude of the public investment in civilian power development, which is clear from the above.

[4] R.A. Tybout, Atomic Power and Energy Resource Planning 23 (Business Monograph 94, Ohio State Univ., 1958).

[5] S.H. Schurr, Energy in the American Economy 1850-1975, at 264 (1960).

[6] U.S. Federal Power Commission, All Electric Homes in the United States, Table 2 (FPC R-70, 1967).

[7] Percentages of total energy consumption are from Schurr, *supra,* note 5.

[8] The number of degree days is calculated as the product $(65-t_a)$, where t_a is atmospheric average daily temperature on the Fahrenheit scale, times the number of days t_a holds. When t_a is above 65°F., it is assumed no house heating is needed.

proximately 510 million BTU will be received. In other words, the solar energy annually falling on the roof of a typical American house is nearly ten times as great as the annual space heat demand of that house. Many refinements of this calculation will be found necessary in the course of later analysis. But for the purpose at hand, it is sufficient to note that the solar energy resource is, on the average, sufficiently abundant to encourage a deeper inquiry into the costs and practicality of its utilization. Such an inquiry is conducted in this article.

The organization of the present article follows the sequence from solar design, to performance, to costs, to cost comparisons with conventional space heat. The authors have made almost a hundred runs on a high speed computer calculating hour-by-hour performance over one-year periods for a wide variety of solar space heat equipment designs in eight different world climates. The latter comprehensively represent the climates of the temperate and semi-tropical regions of the world. On the basis of these computer runs, optimal solar heating system designs have been established. Costs of solar heat in these systems have been projected from engineering information, combined with manufacturers' cost data for various individual components. The results show promise for solar space heat costs that are usually lower than electric space heat and fall in the range of heat costs from conventional fuels in regions of the world where fuel prices are high.

The potential of solar space heating has been appraised by putting together with the help of a modern computer approximately 400,000 weather observations and existing engineering knowledge about solar heating design. The design data were drawn from experience with the few solar space heating systems in existence.[9] The operating results of these systems are unique to the particular design and location of each. But the results of the present analysis extend more generally to other designs and other climates, making possible generalizations inherent in the data but heretofore unrecognized.

I

DESIGN

The essential parts of a solar house heating system are shown in Figure 1. Pumps and blowers for transferring heat from the collector to storage and from storage to either living space or hot water

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[9] There have been twelve to fifteen buildings heated by solar energy systems. For a summary of operating performance of most of these buildings, see Löf, *Use of Solar Energy for Heating Purposes: Space Heat,* 5 New Sources of Energy 114, U.N. Doc. E/Conf. 35/6 (1961).

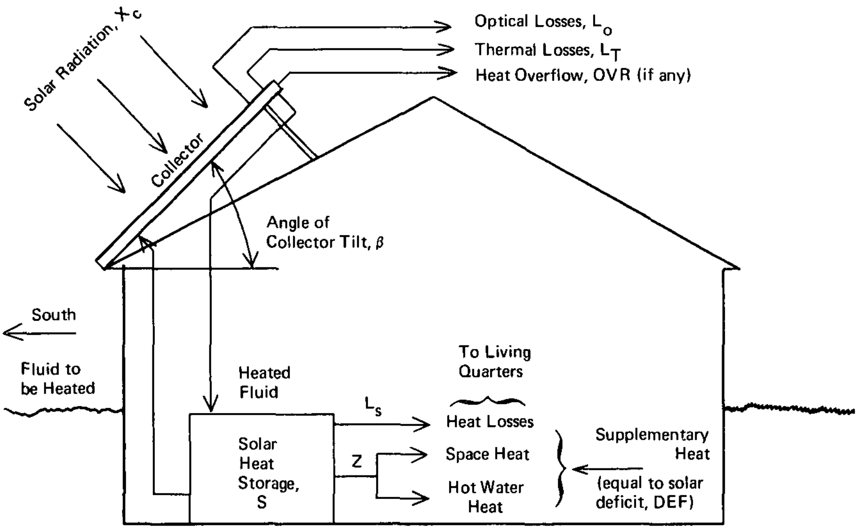

Figure 1
Solar House Heating System

are omitted. A supplementary heat source is included because, as will be seen in succeeding analysis, it is very costly per unit of heat received to provide for extremely high heating demands through solar equipment alone. The cost minimizing calculation, where solar heat can be used, is to determine the optimal mix of solar and conventional heat (unless a variable indoor temperature is acceptable). No heat transfer equipment from either solar heat storage or supplementary heat source to living quarters is shown. The reason is that conventional duct work and other equipment is used. The use of solar heat requires no important changes in the heat distribution system.

All solar heat goes through the storage unit, which may take the form of a large hot water tank but can be a bin of dry crushed rock when air is used as the heat transfer fluid. Heat losses from storage go to heat the inside of the house and hence are not losses at all when indoor heating is desired. They are true losses only when outside temperatures are high enough that there is no need for indoor heat.

A schematic solar heat flow diagram appears as Figure 2. Variables are also there identified for use in the equations below. These equations represent the essential features of the computer model. All design equations are written on the basis of one square foot of collector area so that the results can be adapted to collectors of dif-

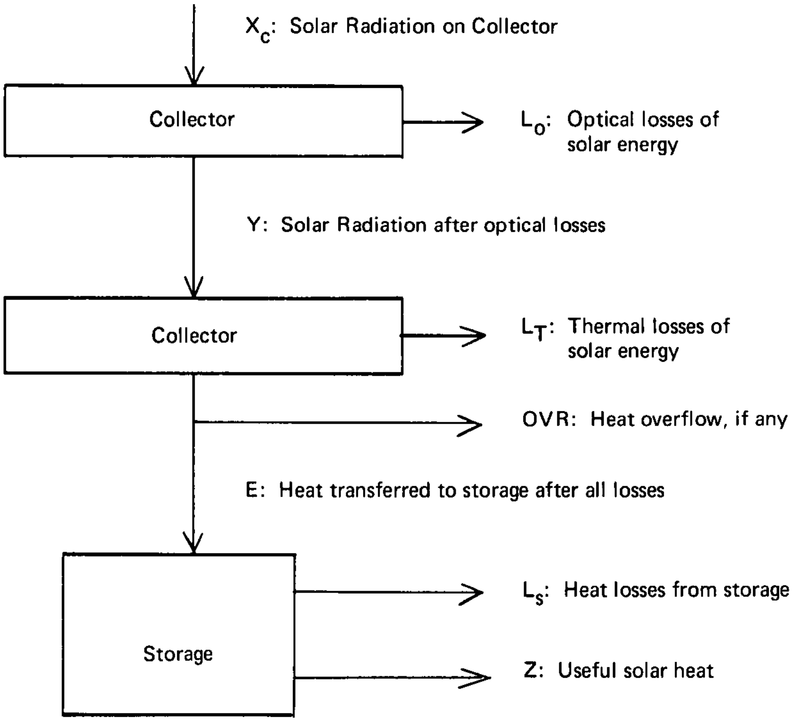

Figure 2
Solar Heat Flow

ferent sizes. The purpose of the following equations is to give a sufficiently exact statement of physical relationships so that the effects of design on performance are understood. This is all that is necessary for the interpretation we shall make of performance data. Full details, of interest mainly to the specialist, are not reported.

Solar energy phenomena in the collector have been divided into two parts, optical and thermal, though in fact the collector is but one unit, as shown in Figure 1. The general nature of the phenomena is suggested by identifying independent variables:

X_c, radiation falling on the collector surface, is a function of

(1) X_h, radiation on a horizontal surface,

(2) angle at which the sun strikes the collector, which is determined from two angles: (a) α, the variable angle the sun makes with a horizontal plane (a climate variable); and (b) β, the fixed angle of collector tilt (a design variable),

(3) Cloud cover, CC, which in turn determines the direct and diffuse components of solar radiation.

L_o, optical loss (primarily reflection) from the collector, is a function of

(1) Solar angle, α,
(2) Angle of collector tilt, β,
(3) Number of glass panes on the collector, N, and
(4) Other optical properties of the collector.

Y, radiation after optical loss, is then

$$Y = X_c - L_o \qquad (1)$$

Both collector and storage may be thought of as heat reservoirs. The collector is a heat reservoir by necessity; storage is a heat reservoir by design. The system operates in such a way that temperature of collector, t_c, must exceed temperature of storage, t_s, before heating fluid is circulated. In the computer model, a temperature differential $t_c - t_s = 10$ was set. Obviously, it would do no good to circulate fluid from storage to the collector until the latter has reached a temperature at least as high as that of storage. The ten degree differential was set to account for heat losses in transit and to justify the expense of operating the circulating pump or blower. Thus, the collector accumulates energy up to the point where its temperature is sufficient to provide heat for storage. It follows that the higher the temperature of storage (*i.e.*, the more heat is already in storage), the more difficult it will be to get the collector hot enough to spill heat over into storage. Conversely, when storage temperature is low, it is relatively easier to add heat to storage from the collector since the latter will not need to be raised to as high a temperature before heat can be removed from it. The heat required to bring the collector up to operating temperature is the collector's "heat storage capacity," defined by

$$Q = (J)\ (0.6t_c + 0.4t_a) \qquad (2)$$

where J represents the relation between heat added and the temperature, t_c, of the collector. Atmospheric temperature, t_a, is also included to take account of the fact that the exterior parts of the collector will be at temperatures nearer that of the atmosphere than that of the heat transfer fluid. Implicitly, Q is measured above a reference level of zero degrees (Fahrenheit). The net heat added as storage in collector in any one hour is

$$Q - QP \gtreqless O \qquad (3)$$

where QP represents Q in the previous hour. The quantity (Q − QP) may be either positive or negative, depending on whether t_c is higher or lower than it was in the previous hour (for a given t_a). In an operating system, t_c would be changing continuously. In the computer model, an effective average t_c was calculated for each hour.

One additional point should be noted. If collector temperature reaches 210°F. (and the corresponding storage temperature reaches 200°F.), the circulation of fluid from storage to collector is stopped. It is assumed that water is used as the heating fluid and any further temperature increase would lead to vaporization of the water. Hence, $t_s = 200$°F. was used as the upper limit on the ability of the system to store heat. In the computer model, the amount of heat lost by virtue of this upper limit was tabulated and recorded as heat overflow, OVR.

The amount of heat transferred to storage in any given hour is then

$$E = Y - L_T - (Q - QP) - OVR \tag{4}$$

L_T, the thermal loss rate from the collector, is a function of

(1) Temperature of the collector, t_c,

(2) Atmospheric temperature, t_a,

(3) Wind speed,

(4) Various heat transfer coefficients, which depend on the number of glass panes on collector, N, and

(5) Thermal radiation coefficient.

The amount of heat removed from storage, Z, depends on atmospheric temperature, t_a, which determines demand for space heat, and storage temperature, t_s. The latter determines how much heat can be removed.

$$Z = A(65 - t_a) + HW - L_s \text{ if } t_a \leqq 65°\text{F. and } t_s > 85°\text{F.} \tag{5a}$$

$$Z = HW \text{ if } t_a > 65°\text{F. and } t_s > 85°\text{F.} \tag{5b}$$

$$Z = 0 \text{ if } t_s \leqq 85°\text{F.} \tag{5c}$$

The three possibilities are conditional on the two temperatures. If the storage temperature is 85°F. or less, no heat is removed from storage, no matter how much it is needed. This is where supplementary heat is required. If storage temperature is above 85°F., then the amount of heat removed depends on whether atmospheric temperature is less than 65°F. $A(65-t_a)$ is the space heat demand. The term "A" is an architectural index of space heating requirements per square foot of collector per degree-hour (DH).[10] Practical values of A range from 1.0 to 10.0. The higher the value of A, the greater the heat requirements (due, for example, to poor insulation of the house) with a given collector and house size. Or, A can be used to optimize the ratio of collector size to house size, given insulation, fenestration and other architectural properties, as will be seen in later discussion. HW represents domestic hot water heat demand and L_s is heat loss from storage. L_s is subtracted from energy withdrawn when $t_a \leqq 65$°F. because the heat is not truly lost when space heat is demanded. As previously explained, it enters directly from the heat storage unit into the house.

L_s is a function of

(1) Temperature of storage, t_s,

(2) Temperature of interior of house, held constant at 70°F., and

(3) Heat transfer coefficient, h_s, which depends on thickness of insulation around the storage tank.

There is an important distinction between heat delivered, Z, and heat demanded, D. As noted above, the former is conditional on $t_s > 85$°F. The latter is independent of temperature of storage:

$$D = A(65-t_a) + HW-L_s \text{ if } t_a \leqq 65°\text{F.} \tag{6a}$$

$$D = HW \text{ if } t_a > 65°\text{F.} \tag{6b}$$

If $Z = D$, heat demands have been satisfied. If $Z < D$, they have not. The computer program cumulates $D - Z$, which is called DEF, heat deficit. Optimal heating design consists in finding the right value of DEF (supplied by a nonsolar heat source) along with the solar heating system parameters.

Energy in storage at the end of each hour is

$$S = SP + E - L_s - Z \tag{7}$$

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[10] A degree-hour is, of course, one twenty-fourth of a degree-day. Space heat requirements are normally rated in degree-days for purposes of house design, but the degree-hour is preferred here because all calculations were made on an hourly basis, as previously noted. Moreover, the calculation of space heat demand by degree hours is more accurate. Thus, the average daily temperature might be 65°F., which would give no degree-day demand. But some hours during the day with such an average would be below 65° and demand during these hours would be properly reported at nonzero values when records are kept on a degree-hour basis.

The term SP, analogous to QP, represents energy in storage at the end of the previous hour. To this is added E and from it is subtracted L_s and Z. Temperature in storage, t_s, satisfies the equation

$$S = K(t_s) \tag{8}$$

The term K relates energy in storage to t_s and hence is the heat capacity of storage. Since S is measured in BTU of storage per square foot of collector area (all variables are related to unit collector area), K is in pounds of water (or its thermal equivalent) per square foot of collector area. It follows that parameterization of K affords the means by which storage capacity is optimally adjusted to collector area.

The above eight equations represent the basic structure of the computer model. The functional relationships controlling X_c, L_o, L_T and L_s have not been reported because they add complexity without changing structure. Needless to say, problems of solution of the equations cannot be discussed here.

II

PERFORMANCE

Solar heating system performance was determined using the fully detailed version of the above model with climate data from the U.S. Weather Bureau for eight different world climates, all of which were represented by observation stations in the United States. A full year of hour-by-hour observations was obtained for each of the following: X_h, solar radiation on a horizontal surface; t_a, atmospheric temperature; CC, cloud cover; and wind speed. The years to which the data apply were, in general, different for each station and were selected on the advice of the Weather Bureau to be "most typical" for the period of record. The data are not averages but actual observations at each time and place. The use of averages was rejected because averages fail to reveal the patterns of sunny and cloudy weather that follow from the serial correlation of climate over time. It is essential that the latter be taken into account in the optimization of the collector to storage ratio.

The Weather Bureau stations, together with the year of observation and the corresponding climate classification (Trewartha classification) are as follows. Cities in parentheses represent the actual sites at which solar and other climate observations were made in cases where these differ by a few miles from the city with which the observations will be identified for our purposes.

Year	Site	Climate Classification (Trewartha, including alphabetic code)
1955	Miami	Aw: Tropical Savannah
1959	Albuquerque	BS: Tropical and Subtropical Steppe
1956	Phoenix	BW: Tropical and Subtropical Desert
1955	Santa Maria	Cs: Mediterranean or Dry Summer Subtropical
1955	Charleston	Ca: Humid Subtropical
1960	Seattle-Tacoma	Cb: Marine West Coast
1959	Omaha (North Omaha)	Da: Humid Continental, Warm Summer
1958	Boston (Blue Hill)	Db: Humid Continental, Cool Summer

A sample run showing performance of most of the previously identified variables for a typical winter day (January 1, 1959) in Albuquerque is shown in Figure 3. Values of the design parameters are also given in the figure. The quantity of heat in storage is represented by the storage temperature, t_s.

It will be noted in Figure 3 that the quantity of radiation received, X_h, appears to be less than the quantity of radiation remaining after optical losses, Y. This is not a correct interpretation because radiation received is measured on a horizontal surface, whereas radiation after optical losses is computed for the surface of the collector, which is tilted. The tilt is such as to favor winter collection, as will be explained. The reader will note the effect of solar radiation and atmospheric temperature in controlling the interplay of system components. Solar radiation received during the sunlit hours increases heat in storage. Storage is able to satisfy demand for the entire day, as indicated by the absence of a deficit. A final point of interest is that the cloud cover that appears at ten o'clock does not reduce radiation received, X_h. Indeed a close study of the graph will show that X_h tends to be augmented at ten o'clock. This is a correct interpretation. Apparently the clouds did not obscure the sun at the observation site, but did cause reflection of additional radiation to that site. In general, the reflection effect augments diffuse radiation. Whether total radiation is increased (in a case such as the present, where the clouds do not cross the face of the sun) depends on the extent to which direct radiation is reduced by any decline in atmospheric clarity.

The output of an annual run is shown in Figure 4. The heating

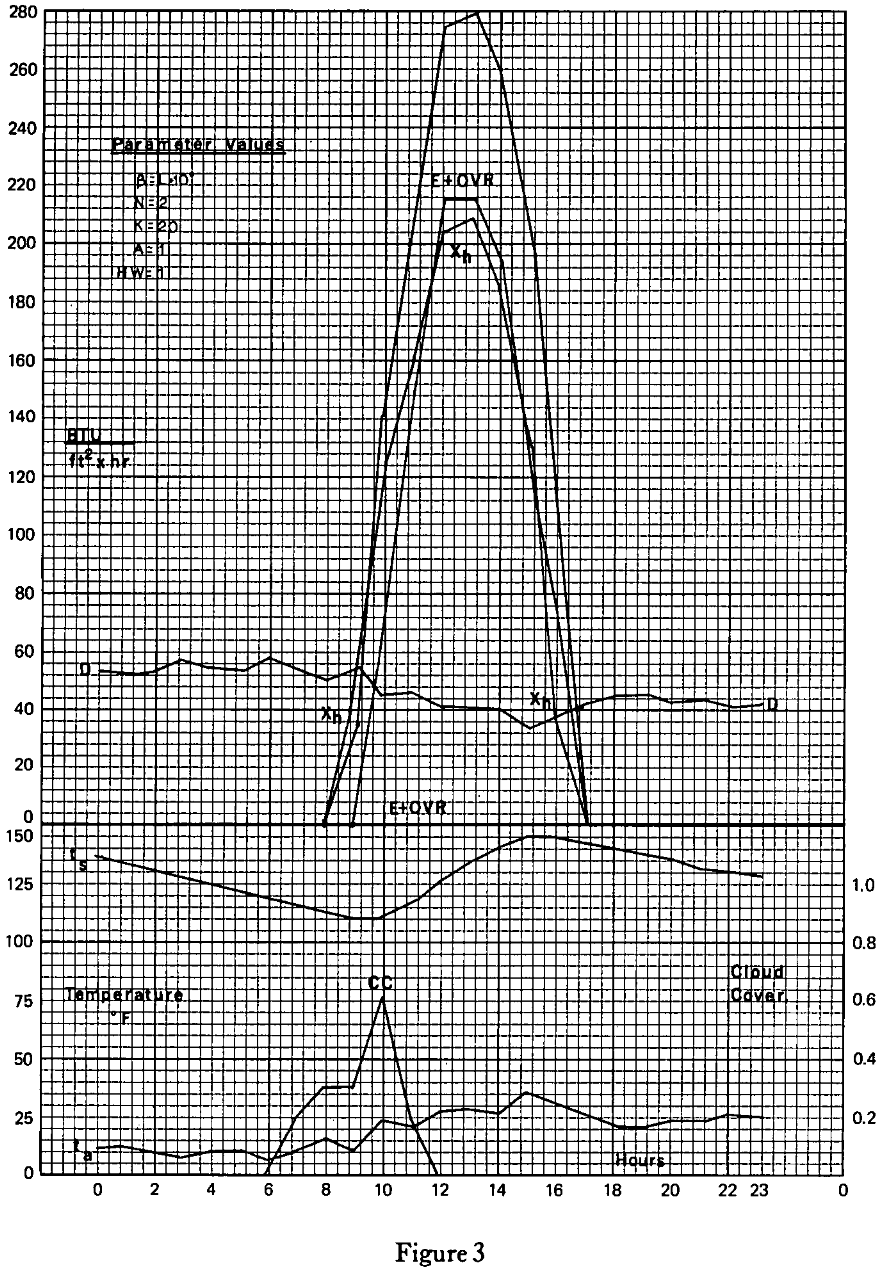

Figure 3
Sample Winter Day Solar Heat Performance
(Albuquerque, January 1, 1959)

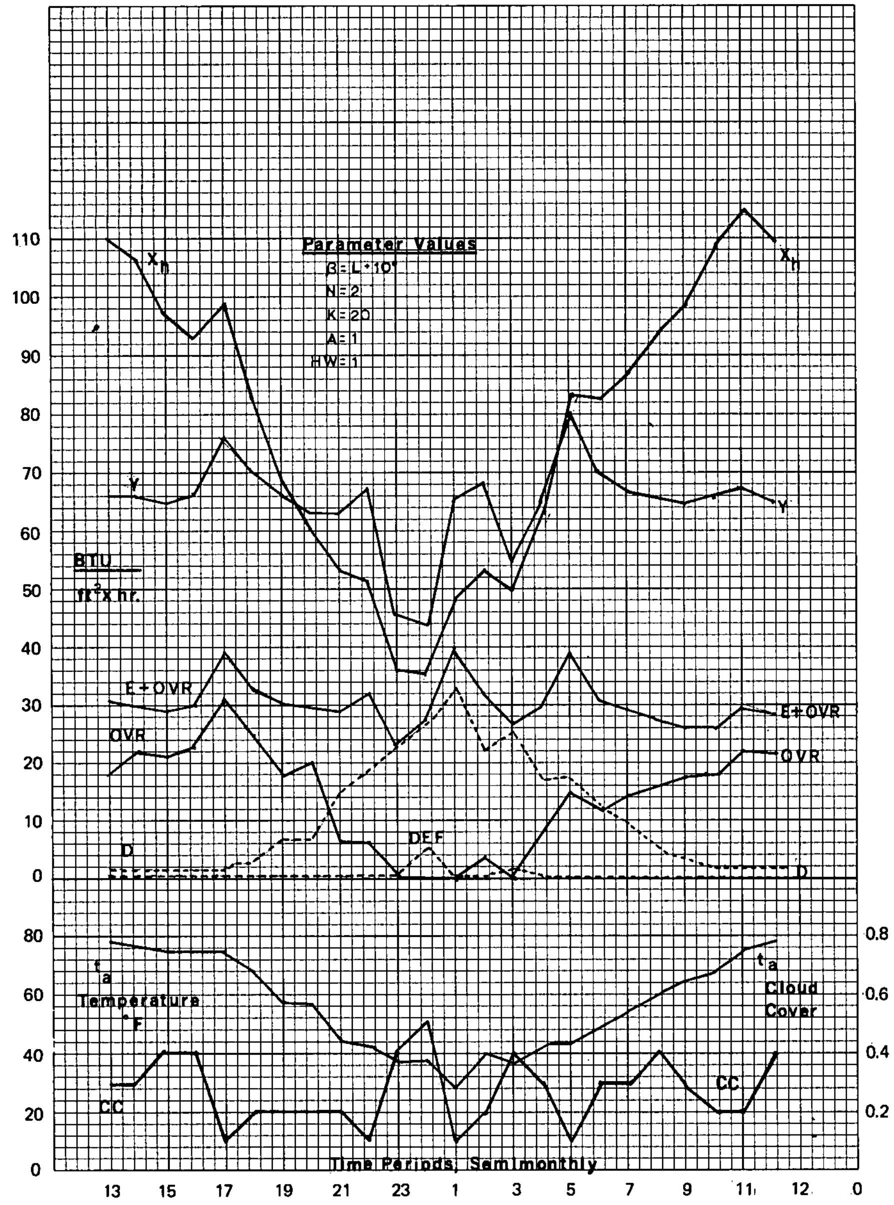

Figure 4
Sample Solar Performance
(Albuquerque, annual, 98% solar heat)

system was assumed to go into operation on July 1 and to run until June 30 of the following year. In this way, start-up conditions were confined to an unimportant summer month. All results are reported in average hourly values for the time periods identified as one of twenty-four half-month intervals numbered in succession from the beginning of the year. The run refers to Albuquerque and includes the day shown in Figure 3.

Several points of interest in Figure 4 should be noted. As previously pointed out, solar radiation, X_h, is measured on a horizontal surface. Hence despite optical losses, a higher value Y (radiation after optical losses) is achieved in the winter months on the collector surface tilted at 45.1° from the horizontal and, of course, facing due south. The latitude of Albuquerque is 35.1° north; hence this tilt represents an angle 10° greater than the latitude. Since demand is highest in winter (note dotted curve D), still further tilt seems desirable in an effort to get Y as high as possible at this time of year. The effect of average cloud cover can be seen in the irregularities of both X_h and Y.

Net output from the collector, E, can be calculated by subtracting overflow, OVR, from gross output, E + OVR. During the summer months, OVR is very high. Little heat is used, hot water being the only demand and this at a low level relative to the collector's capacity. Hence storage stays close to its ceiling temperature of 200°F. Even in the last half of January (time period #2), Figure 4 shows a positive value of OVR. Inspection of the temperature and cloud cover graphs show that the former was unusually high and the latter relatively low for this two week period. The greatest net heat output, E, was, of course, in the winter. But somewhat more surprisingly, the peak value of (E + OVR) is found when OVR = 0. See time period #1, corresponding to highest demand, D. This same phenomenon has been observed in many runs. Note also that E tends to be closer to Y in the winter. The reason is that collector efficiency is higher when the heat transfer fluid is supplied to the collector at lower temperatures. The collector does not lose as much heat to the atmosphere if storage temperature (and hence collector fluid temperature) is low. One might expect that lower atmospheric temperature would cause more heat loss, and it would if collector fluid temperature stayed high. But storage temperature, heat transfer fluid temperature and atmospheric temperature are all going down at the same time. The average temperature difference between collector and atmosphere decreases faster than the atmospheric temperature itself, thereby decreasing the heat transfer rate. We shall find this phenomenon significant in later generalization of results.

The run shown in Figure 4 was based on typical parameter values, except for space heat demand factor and hot water demand, A and HW, which were set at the low end of their scales. Hence D was relatively low (per square foot of collector area) and the solar heating system had a high capability. Indeed, for the run shown, in only 93 of the total annual 8760 hours did the solar unit fail to provide all heat demanded. Note the low DEF.

By way of contrast, Figure 5 shows a run at the same site with values of A and HW at levels that will later be shown to come close to giving least cost solar energy. The climatological variables in the lower part of Figure 5 are the same as in Figure 4. So also is X_h. But Y is slightly different, due to a different tilt angle. OVR is lower and E is higher, indicating a considerably greater system output per square foot of collector. E is quite close to Y during the high demand months, for reasons already explained in connection with Figure 4. Since A in Figure 5 is five times A in Figure 4, the collector is one fifth as large and values of D and DEF are several times larger (per square foot of collector). The D's do not differ by an exact multiple of 5 between the two runs because, as will be seen in equation (6a), the nonparametric term L_s enters the calculation of D. There is no reason to expect any simple relation between the DEF's in the two figures. The very high DEF in Figure 5 is a consequence of the low capability (45 percent of total heat demanded was supplied by the solar heat system) reported in the figure.

Performance of the solar heating system in a very different and less suitable climate is shown in Figure 6. The run applies to Seattle-Tacoma with a parameter combination that will later be shown to give the least-cost output of solar energy at that site. It is immediately obvious that high cloud cover helps explain the low levels of X_h. OVR is almost non-existent. The high correlation of E and Y is apparent during the high demand months.

III

COSTS

Solar heat equipment costs can be estimated as a result of experience with: (1) experimental solar space heat systems, (2) commercial solar hot water heat systems, and (3) commercial equipment components generally available for nonsolar applications. Fortunately for the costing problem, most of the solar heat system components fall in the last class. Included are such common items as motors, pumps, blowers, water storage tanks of various sizes, control instruments, and so on. The most costly item, the collector,

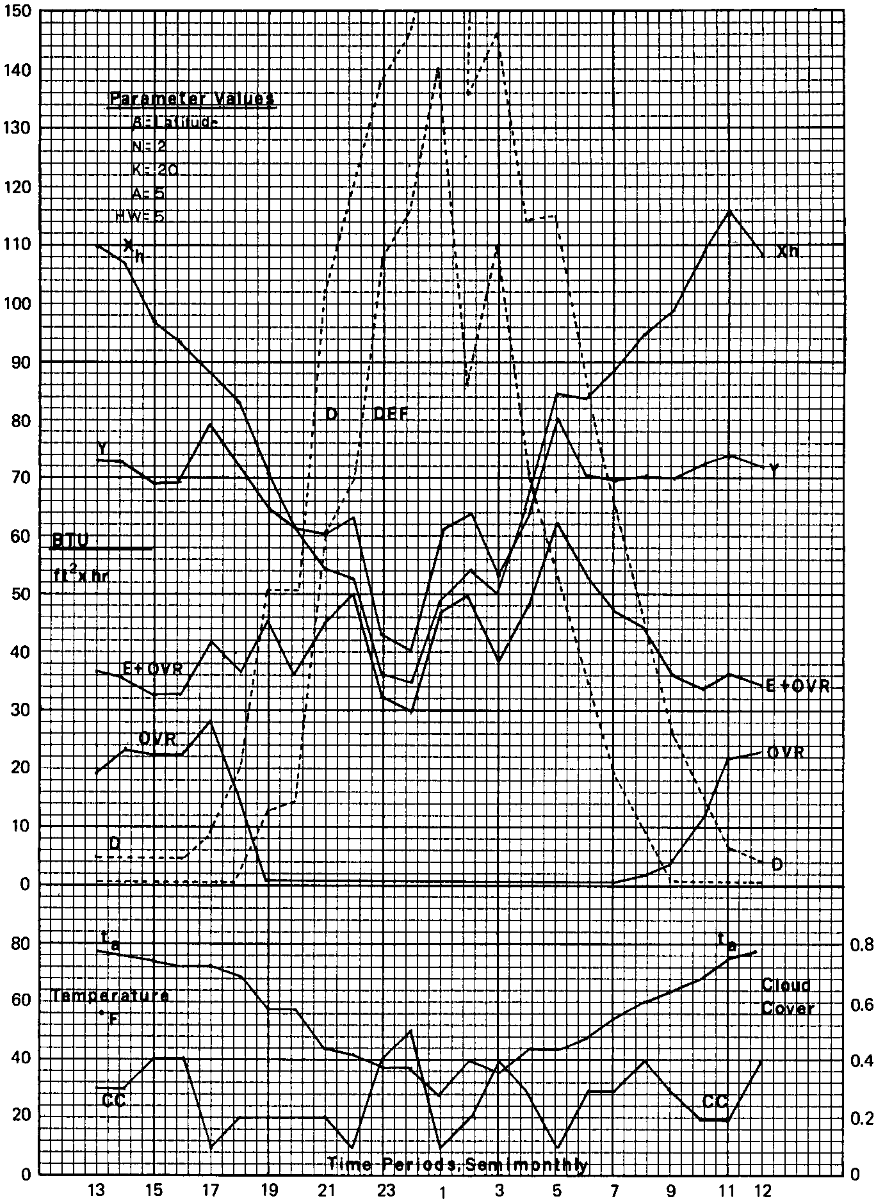

Figure 5
Sample Solar Performance
(Albuquerque, annual, 45% solar heat)

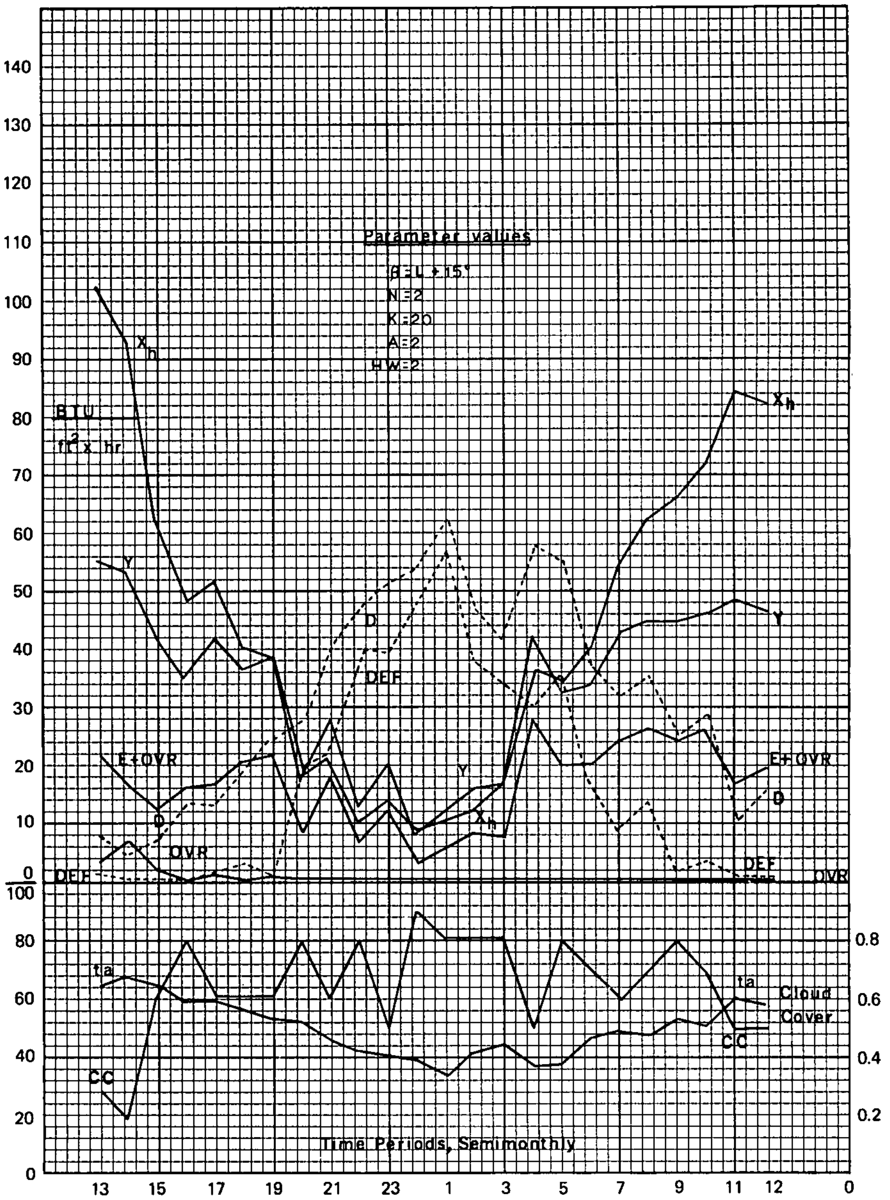

Figure 6
Sample Solar Performance
(Seattle, annual, 48% solar heat)

presents real costing problems; but even for this, experience is available in the commercial production of collectors for solar water heating.

The authors made a world-wide survey of solar equipment designers by mail questionnaire in 1961-62. This survey was conducted for the purpose of determining solar equipment costs. It immediately followed the United Nations Conference on New Sources of Energy (Rome, 1961), where the majority of the respondents were present and the purpose of the questionnaires had been explained. A high response rate was experienced. Because the survey was conducted in 1961-62, costs are at the levels prevailing then, primarily in 1961. These costs will be used in all future analysis. As we shall see, adjustments for price level changes do not significantly affect our conclusions.

On the basis of details tabulated from the questionnaires and analyzed for five solar heated buildings, capital costs for noncollector components were derived. Expressed as functions of collector area, the resulting relationships give mean values of conventional equipment, including costs of installation, in the five heating systems. The physical relationships previously reported for equipment design give performance results representative of these systems. By the terms of the questionnaire survey, individual responses are confidential.

Storage = \$0.05 per pound of water stored
= \$0.05 K (collector area, ft^2), where K is a parameter equal to pounds of water storage per square foot of collector.

Controls = \$150.00 regardless of system size.

Pipes, fittings = \$100.00 + \$0.10 (collector area, ft^2).

Motors, pumps = \$50.00 + \$0.20 (collector area, ft^2).

Heat exchangers = \$75.00 + \$0.15 (collector area, ft^2).

The capital cost appropriately assigned the collector is, of course, more difficult to determine. For the purpose of judging the potential impact of solar energy, it is inappropriate to use the collector costs experienced with any existent solar heated buildings. The buildings are experimental and the collectors on them have been fabricated by hand according to a variety of different specifications. In contrast to the noncollector equipment, which is commercially available, collectors for space heating have yet to be manufactured in volume. It would understate the possibilities of solar heating to use their experimental costs.

The nearest thing to a solar collector for space heating is one for domestic water heating, if the latter is of the circulating type. Some hot water collectors are not. For example, the simple box-type hot water collector with inlet and outlet water lines is popular in Japan.[11] A plastic version can be purchased for the U.S. equivalent of $1.00 per square foot of surface area. It has, however, no resemblance to the forced circulation, tilted, insulated collector of the type relevant for our comparison. A second caution to be noted in making analogies with hot water collectors is that the latter are generally small (10 to 50 square feet in area) as compared with the larger collectors needed for space heat. The higher edge-to-surface ratio of hot water collectors raises the average cost per square foot for the same design. Finally, it is doubtful that the full economies of scale potentially possible in the manufacture of hot water heaters of sophisticated design have been realized. Comparatively speaking, production volumes are still small for collectors of this type. Nevertheless, it is worth noting certain results.

Four questionnaire respondents manufactured solar hot water collectors of the type usable for space heat in sufficient volume to establish (small scale) manufacturing costs. Unfortunately, however, their price lists were for all components, including storage tank, installed. System prices expressed per square foot of collector area were found to range from $5.95 to $7.90 for a combined total production of 1620 units by two Israeli manufacturers and $6.40 to $8.90 for a combined total production of 4000 units by two U.S. manufacturers. Most of the variation in price is due to collector size, the smaller systems having the higher prices per unit collector area.

Hot water heating systems, in contrast to space heating systems, do not have controls, heat exchangers and most of the other components separately detailed above. To the small extent that ancilliary equipment (such as piping or electrical immersion heater) is used, its cost is included with storage. Therefore, collector plus installation cost estimates can be obtained by subtracting storage costs from the above cited totals. The storage costs for this purpose range from 10-15¢ per pound of water, reflecting the higher unit cost of a smaller sized tank than envisaged above for space heat storage and the auxiliary equipment. Costs of collector (plus installation) so estimated fall in the neighborhood of $4.00 to $4.50 per square foot at the lower end of the above ranges for Israeli and U.S. manufacturers. The lower end is the relevant end since it applies to the

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[11] There were in 1960 an estimated total of 350,000 solar hot water heaters of all types in use in Japan and 10,000 in Israel. *See* Oshida, *Use of Solar Energy for Heating Purposes: Water Heating,* 5 Conference on New Sources of Energy 3, U.N. Doc. E/Conf. 35/6 (1961). Solar hot water heaters are known also to be in use in other parts of the Mediterranean, in Australia, Florida and elsewhere.

larger hot water collectors (which are still nowhere near as large as space heat collectors). Further confirmation of a "now available" cost in the same range is given by an Australian government study of solar hot water heating in which collector costs of two Australian pounds per square foot were listed for the city of Melbourne.[12] This corresponds to approximately $4.50 per square foot. In view of economies of size manifest in comparing small with large hot water collectors, it seems appropriate to take on the basis of the foregoing evidence the estimate of $4.00 per square foot of collector (installed) as a cost that present manufacturing methods could easily make available for the very much larger collectors needed for house heating. This we consider the upper limit of our estimated range of collector costs.

The lower limit is simply not based on empirical evidence beyond the observation that manufacturing experience typically brings radical cost reduction over the first decade of production in volume of a new piece of equipment such as space heat collectors. On this ground, we make the judgment that half of the high estimate collector price is a sufficiently likely figure to justify an exploration of the potential of solar space heat. That is to say, we hypothesize that collector costs could reach a level as low as $2.00 per square foot simply as a result of manufacturing improvements and without any change in design. If substitute materials and/or new designs are devised, lower prices are possible, though such prices are too speculative to base predictions on them.

That our $2.00 per square foot figure, or perhaps one below that, is not out of the question is suggested by two pieces of information currently available. First, the price increments on the U.S. manufacturers' solar hot water systems cited above imply cost increments of $1.30 per square foot in collector cost (installed). This incremental cost was calculated in the same way as the above averages, *i.e.*, by subtracting storage (in this case incremental storage) costs. Second, a review of hot water collectors in current use in Australia of the design desired for space heat collectors gave a total of $1.90 per square foot for materials alone; however, a list of material substitutions was suggested that would reduce the cost to $0.90, again for materials alone.[13] At least it is clear that on a marginal cost basis or a materials-only basis, a future cost of $2.00 per square foot of collector (installed) for space heat is not unreasonable.

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[12] Commonwealth Scientific Industrial Research Organization, Division of Mechanical Engineering, *Solar Water Heaters* 11 (Circular Number 2, 1964).

[13] Czarnecki, *Economics of Solar Water Heating*, 7 Solar Energy Progress in Australia and New Zealand 22 (1968).

The sum of the preceding cost estimates is represented in (9a) with low (future) collector costs and in (9b) with high (present capability) collector costs. Both (9a) and (9b) are capital costs only.

$$\text{Total cost} = \$375 + (\$2.45 + \$0.05K)\ (\text{collector area, ft}^2) \quad (9a)$$

$$\text{Total cost} = \$375 + (\$4.45 + \$0.05K)\ (\text{collector area, ft}^2) \quad (9b)$$

A final point to be noted is that collectors with double glazing (two panes of glass) have been assumed in the above estimates. We shall find that two panes of glass give least-cost soar heat in most locations, but for very warm areas, one pane is better and for very cold areas, three panes are better. The exact effects of varying the number of panes of glass will be noted in subsequent discussion. For present purposes, however, it is necessary that cost adjustments for such variation be included. For this purpose, an independent analysis (not based on questionnaire results) by the authors suggests that the unit collector costs should be adjusted by 40¢ per square foot for each pane of glass added to or subtracted from the above collector estimates. The figure is, of course, approximate. In a finer analysis, different figures would be used for the high and low cost estimates. However, a finer analysis was not judged desirable in view of the data currently available. Hence equations (9a) and (9b) are replaced by:

$$\text{Total Cost} = \$375 + [\$2.45+(N\text{-}2)\$0.40+\$0.05K]\,(\text{collector area, ft}^2) \quad (10a)$$

$$\text{Total Cost} = \$375 + [\$4.45+(N\text{-}2)\$0.40+\$0.05K]\,(\text{collector area, ft}^2) \quad (10b)$$

For the purpose of reducing capital costs to annual equivalents, a discount rate of 6% was used with sinking fund depreciation over an expected life of twenty years. The expected life is based on questionnaire results and other information regarding the collectors of the design here considered (different expected lives are applicable for other, less durable, collectors). The discount rate is typical, though perhaps low in contemporary money markets.

Costs will later be calculated for a range of different sizes and designs of solar heating units in each of two typical houses rated by degree days of space heating demand: (1) 15,000 BTU/DD and (2) 25,000 BTU/DD. The two houses may be thought of as (1) middle income and (2) upper middle income houses, respectively, in the United States. It is readily seen that when $A = 1$ in equation (6a), 1 BTU of heat is demanded per degree hour per square foot of collector, or 24 BTU/DD per ft^2 of collector. The necessary collector areas are 15,000/24 or 625 ft^2 and 25,000/24 or 1041 ft^2.

If $A = 2$, the demand per square foot of collector is doubled and, of course, this is the case when collectors are only half as big in the same two houses. Thus, collector areas for these two houses are calculated in general as $(625/A)$ and $(1041/A)$.[14]

The effects described in the two preceding paragraphs are combined in the following set of equations, which give annual equivalent capital costs as a function of the design parameter A. The first two equations, (11a) and (11b), give low cost estimates for the two houses. The second two give high cost estimates. Only capital costs are involved.

$$\text{Annual Cost} = \$35.80 + [\$0.213 + (N-2)\$0.035 + \$0.00436K]\frac{625}{A} \tag{11a}$$

$$\text{Annual Cost} = \$35.80 + [\$0.213 + (N-2)\$0.035 + \$0.00436K]\frac{1041}{A} \tag{11b}$$

$$\text{Annual Cost} = \$35.80 + [\$0.388 + (N-2)\$0.035 + \$0.00436K]\frac{625}{A} \tag{11c}$$

$$\text{Annual Cost} = \$35.80 + [\$0.388 + (N-2)\$0.035 + \$0.00436K]\frac{1041}{A} \tag{11d}$$

Operating and occasional maintenance costs are also involved. It is not possible to generalize about the latter on the basis of experience to the present time, but there is no reason to think they will be any more serious with a solar heat system than with any conventional home appliance. Operating costs consist of electric power to drive motors and pumps to circulate collector heating fluid. It can be shown that the cost of power for this purpose is quite small. Our solar energy heat equations assume a ten degree temperature increment in water heated by the collector. From this, it is possible to calculate how much water must be circulated to satisfy the computer-

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[14] In calculating collector areas in this way, only space heat demand is used. It is recognized that a hot water demand is also present, as given by (6a) and (6b). An increment could be added to collector area to account for hot water demand, but it is not worth complicating the formulas to do so. The important point is that no error is introduced. However collector area is calculated, performance from computer runs based on equations (6a) and (6b) is compared with system costs for that collector area.

calculated heat production of the system and, knowing electric power consumption for pumping, the amount of electricity consumed. The result can be shown to range from 1½ to 2½ kilowatt-hours per million BTU of solar heat, which corresponds to electric power costs of two to five cents per million BTU. Compared with annual equivalent capital expenses that will later be seen to fall in the range of two to three dollars per million BTU for most efficient applications of solar energy, electric power costs are so small that they can be ignored within the accuracy of the estimates.

IV

OPTIMIZATION

Optimization in the present context means cost minimization. Some trade-off with aesthetic considerations is involved in house design and community planning, as will be noted. But these matters are introduced as possible constraints in the same context as the advantages of solar energy for its clean heating properties. In the absence of unanimity on how such intangibles might be quantified, the approach will be to recognize their possible significance while concentrating on costs in the usual market sense.

The search for an optimum will be conducted in two stages. First, attention will be given to solar design parameters in an effort to find least-cost combinations of these. Second, solar equipment will be combined with conventional space heating systems to obtain the least-cost combination of both. Since some conventional heat will be needed to offset solar deficit, the second step is necessary before any final conclusions are reached.

The approach will be to use the lower cost estimates of solar heat as given by equations (11a) and (11b) in the search for best solar design, but to calculate total costs with both low and high collector costs in tabulating results. The reason for this approach is that the authors judge the high cost estimates to be overly conservative for any large volume production of solar collectors. If solar house heating by one means or another passes the "infant industry" stage, the authors hold it will be better represented by costs at the low end of the above range.

A. Solar Equipment

Seven design and demand parameters were varied so as to achieve least-cost solar space heat at each of the eight stations. Each parameter was varied while keeping all other parameters constant at levels thought at the time of the run to be nearest their optimal values.

Needless to say, the search process could lead to a very large number of runs if all possible combinations of parameters were tried for all weather stations. This was not done. Most of the runs were confined to two or three stations and the results confirmed at other stations after the properties of the solar least-cost point and the influence of climate on them were better known. Similarly, in the choice of which parameters to vary, ways were found to narrow the search by giving most attention to parameters to which optimal design was most sensitive.

Two parameters had practically no effect on system performance. These were: (1) heat transfer coefficient, h_s, of the insulation on the storage tank; and (2) heat capacity of collector, J. The unimportance of the latter has generally not been appreciated among solar design engineers. It is one of the interesting findings of the computer runs. In both cases, these results mean that within the range of common practice (which is the range explored by the parameterizations), costs can be minimized without worrying about the effect of design on the efficiency of the system.

The effects of each of the other five parameters will be briefly summarized in the following paragraphs, together with some evaluation of their significance for the system as a whole.

1. Angle of Collector Tilt, β

The parameterization of collector tilt is not relevant to costs insofar as equations (11a) and (11b) can show. This, however, is a simplification in the analysis. In point of fact, special mountings or the integration of the collector with the roof design of the house can have cost effects, as will be noted. Equations (11a) and (11b) should be interpreted as including average mounting costs, which may be higher or lower than actual mounting costs in a particular situation.

At highest physical efficiency, the most radiation after optical losses, Y, is received for a given solar radiation input, X_c. As noted in the discussion of Figures 3 and 4, the object in selecting a collector tilt is to make the most solar energy available during the winter when it is needed most. High optical losses in summer are not important because hot water demand is small.

Figure 7 shows the effect of varying collector tilt in three cities: Albuquerque, Boston and Santa Maria. The values of the various parameters other than β are tabulated on the graph. Numbers along the left hand side of the graph refer to costs of solar heat for the entire system as indicated by equations (11a) and (11b). These costs are different at different β values because useful solar heat

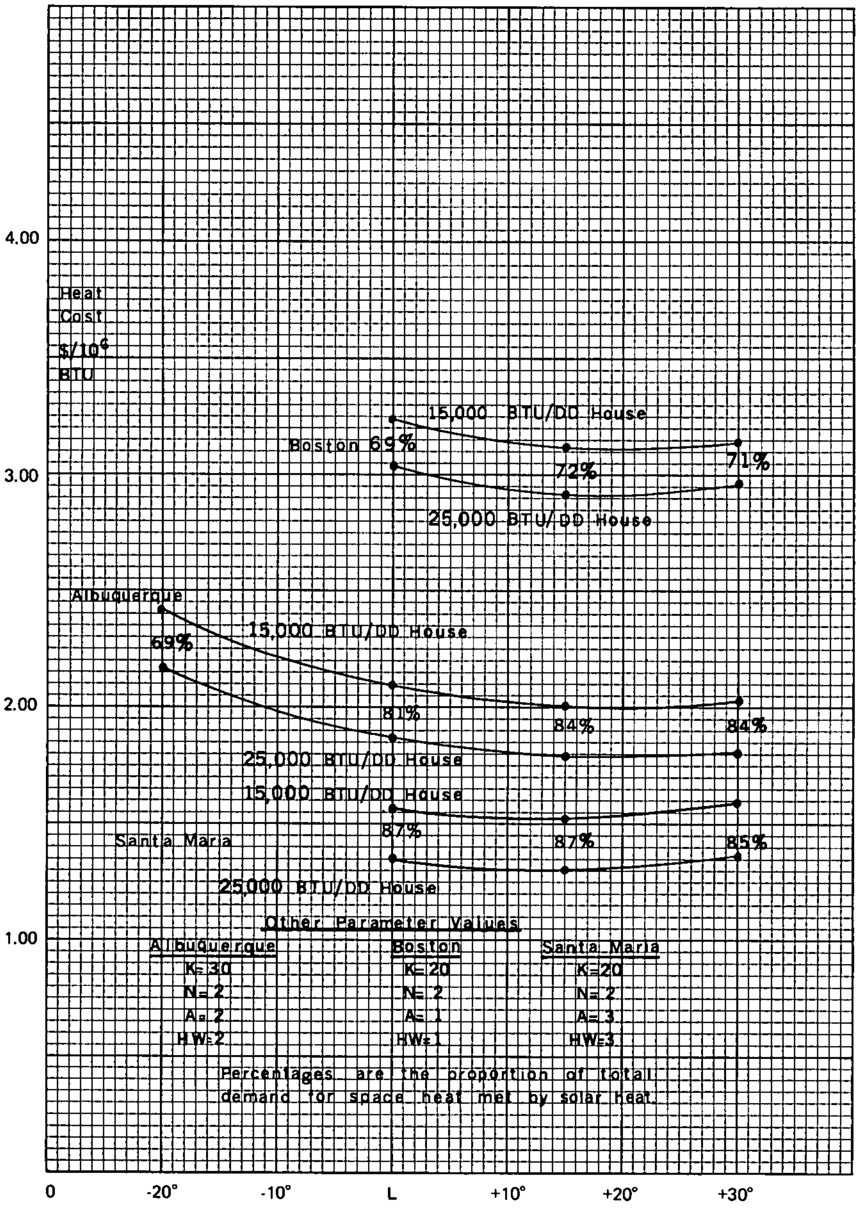

Figure 7
Influence of Collector Tilt
(β Parameterizations)

output is different. Values of β along the horizontal axis are expressed in relation to latitude, L, so as to put them on a common basis for the three cities. Two curves are given for each city, corresponding to the two house sizes. At each point on the curves appear percentage statements. These refer to the percentages of total heat demand supplied by the solar heating system. Each point represents the results of a computer run on one year of data. It will later be seen that the parameter values other than β are not themselves exactly at their least-cost levels, but within the range of their deviation from these levels, they do not have any significant effect on the β optimization.

Two important conclusions follow from Figure 7: (1) Optimum tilt is 10-20° greater than latitude at sites of widely different latitude; and (2) Collector tilt does not make much difference in solar heat costs over a wide range of angles from, say, latitude to latitude plus thirty degrees. These angles, however, are relatively high. If median latitude for the United States population is 40° north, a median collector tilt of 40-70° is implied. The roof pitch of a modern house is not generally this steep. If the collector is not built on the same slope as the roof, then the possibility of some common costs of collector and shelter is lost and some structural costs are incurred by a separate collector mounting.

Other constraints from an architectural standpoint arise from the desirability of the collector facing south (north in the southern hemisphere). No computer runs were made with collectors with other orientation, but it can be shown that any important deviation from south (or north) reduces the energy received during the important four hours centered on solar noon and drastically curtails either mid-morning or mid-afternoon collection, depending on whether the bias is to west or east, respectively.

A possible conflict of aesthetics and costs appears in the orientation of the house. By suitable orientation, one roof can be made to face south, but this is not always the best orientation from other standpoints. The problem can be avoided in a house with a flat roof by orienting the collector frame but not the house. Further implications of the problem extend to aesthetic aspects of community planning as affected by any sameness of house orientation on a large scale. The requirement that each collector have an unblocked southern exposure would likewise force all houses to similar heights and play havoc with the location of trees where lots are small and houses close together. These are costs not included in the calculation and may or may not be important in the eyes of particular consumers. On the other hand, we note that other costs, such as air pollution, have not always been included in the calculation for solar energy's competitors.

2. Number of Glass Plates, N.

The number of glass cover plates enters directly into the calculation of costs in equations (11a) and (11b). Physically, there is a trade-off between optical and thermal losses. Each additional plate reduces thermal loss at the collector and increases optical loss. Where thermal loss is high, because of low atmospheric temperatures, as at Boston or Omaha, a better case can be made for more than one glass plate than at Phoenix or Miami, where thermal loss is relatively low.

The results of the N parameterizations are shown in Figure 8. For two extreme cases, Boston and Phoenix, three plate runs were carried out with the same values of the other parameters as used in the one- and two-plate runs. At Omaha, the three-plate runs were made at other parameter values that differ slightly from those for the runs reported. This fact is suggested by the dotted lines. However, it is clear by analogy with the Boston results—representative of the coldest climates among the sites tested—that three-plate collectors will not produce least solar heat costs in any other of the eight cities. One-plate collectors are clearly optimal in Phoenix. Two-plate collectors are clearly optimal in Boston and Omaha. Either one or two plates produce about the same costs of solar heat in Albuquerque and Santa Maria. The improved collection of energy at these two sites is just offset by the increased cost of the collector due to the addition of the second plate. In later parameterizations, we have chosen to use the two-plate collectors at these last two sites.

3. Heat Storage, K.

The cost of heat storage per square foot of collector enters directly into equations (11a) and (11b). There are, in addition, certain minor architectural considerations in the location of storage. Thus, with $A = 1$ and $K = 10$, the 25,000 BTU/DD house will have 10,410 pounds or 1,250 gallons of water storage.[15] It will be seen in future discussion of collector area that these figures tend toward the upper limit of storage size for a least-cost solar heating system. A tank that would hold the above volume of water would occupy approximately 170 cubic feet, or ten to twenty times the volume of a normal household hot water tank. Equations (11a) and

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[15] As previously noted, collector area is given by the expression 1041/A for the 25,000 BTU/DD house. With $K = 10$, there will be ten pounds of water in storage for each square foot of collector area.

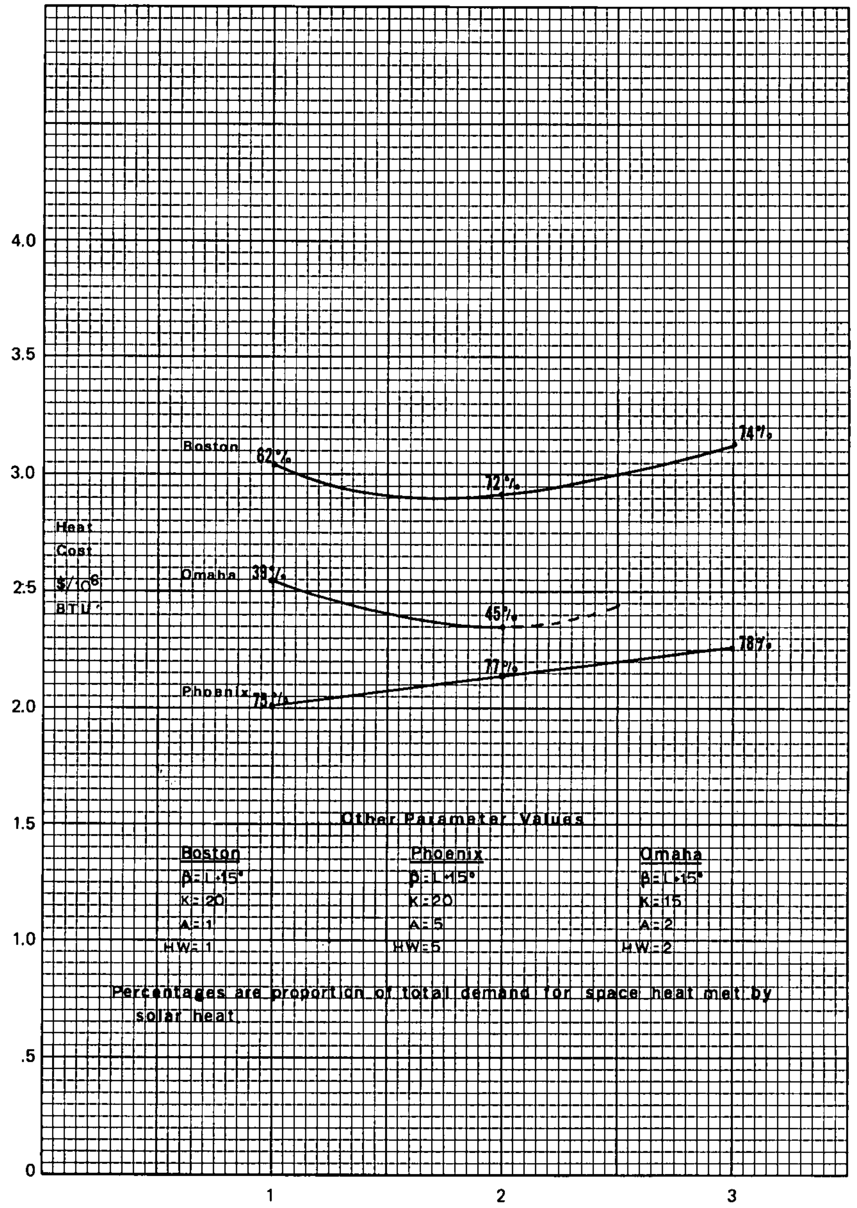

Figure 8
Influence of Number of Plates
(N Parameterizations)
(25,000 BTU/DD House)

(11b) include the costs of the tank, heat exchangers and pumps, but not the value of basement space. This may or may not be important to the individual home owner. If basement space is valued at $4 to $5 per square foot of floor area, this space might come to about $100. The figure is low compared with total costs of solar energy (as previously reported) and hence would not change the annual cost significantly. Moreover, the effective volume is small compared to the old coal bin which for so many years was tolerated as a basement space consumer (and source of coal dust).

The results of the K parameterizations are shown in Figure 9. As with Figure 8, the parameterizations were carried out with relatively high capability systems (low values of the parameter A). Note the percent solar heat given for the points on the graph. For this reason, the absolute costs shown on the left-hand axis do not represent general least-cost values. They are, however, quite adequate for the relative comparisons that are required for optimization.

Figure 9 shows distinct optima in all three climates. The optima are more precisely determined for Albuquerque and Santa Maria because more runs were made for these stations than for Boston. In all cases, it appears that the optimum is in the neighborhood of $K = 10$ or 15, with very little cost difference perceptible in this range. It is remarkable that as much similarity obtains among these three climates. To understand the meaning of this result, remember that K is storage capacity per unit collector area; and hence for a given value of K, storage varies in absolute size with collector area, which we shall find differs significantly among the same sites. The finding of nearly the same optima for K tells us that larger collector areas (required by colder climates) must be accompanied by larger storage (required by higher variance in solar energy). Greater differences among optimal K values will be apparent in results for other sites, to be reported later.

A finding of more general importance for solar heating design is that storage is relatively small compared to radiation. Thus, the maximum usable storage temperature range is 85°F. to 200°F. This is a span of 115°F, or 1150 BTU per square foot of collector when $K = 10$. Average daily heat delivered in winter in Albuquerque is 1000 to 1200 BTU per square foot of collector. (Multiply values of E in Figure 5 by 24 to convert from an average hourly to an average daily basis.) This means that in Albuquerque, optimal storage will hold, at the most, about one winter day's heat delivery. The phrase "at the most" is relevant since, as a practical matter, heat losses become severe where storage temperatures get relatively high. Perhaps the workable range is closer to 70 or 80

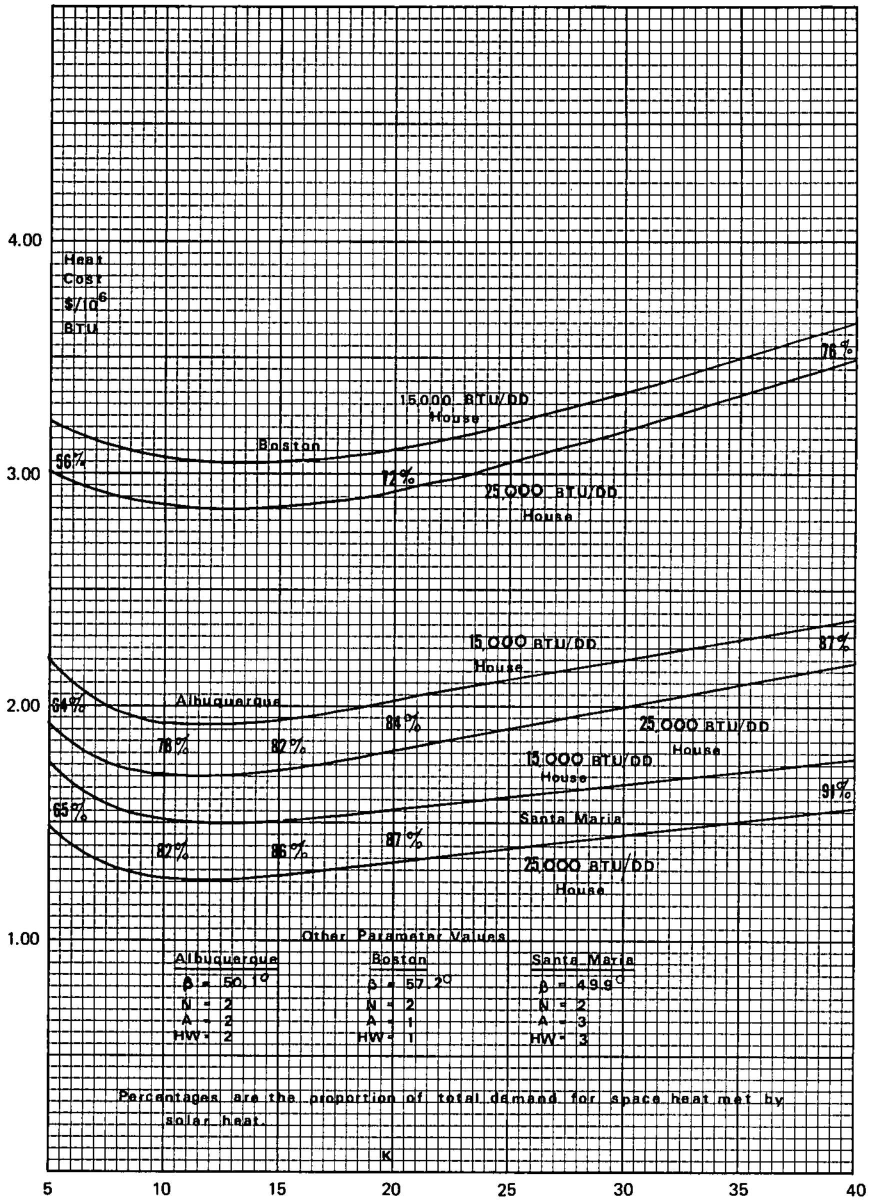

Figure 9
Influence of Storage Capacity
(K Parameterizations)

degrees rather than 115. Comparable winter heat delivery in Boston is about 500 BTU/ft^2 per day, which, by similar calculations implies that with $K = 10$, storage will hold about two days' collected heat. Thus, with $K = 10$ or 15, optimum storage capacity is of one to three winter days' heat delivery. It is definitely not on the order of a week's heat delivery or more, as can be seen by noting the cost at $K = 40$. The question of optimal storage size is one that has occupied a good deal of attention in solar heat design, but until the calculation of results could be made for many different systems in different climates, evidence one way or the other was inconclusive.

4. Collector Area.

All the foregoing results have been expressed per square foot of collector area. It is now appropriate to relate collector area to the size of the house. This is done by considering variations in the reciprocal of A. The variable A measures heat demand per square foot of collector. If heat demand is fixed by house characteristics, as it is for our 15,000 BTU/DD and 25,000 BTU/DD houses in climates with given numbers of degree days (or degree-hours), then any variation in A implies an inverse variation in collector area, as noted in the derivation of equations (11a) through (11d).

Since a "collector size" interpretation is made of A, it is necessary for the purpose of conducting parameterizations that comparable adjustments be made in HW. See equations (5a), (5b), (6a) and (6b). Remember that HW is expressed as hot water demand per square foot of collector. If collector area is cut in half, hot water demand per square foot of collector must be doubled in order to end up with the same hot water demand as before. The computation is similar for any other adjustment of collector area. Thus, in (5a) and (6a), HW was parameterized simultaneously with A, being assigned an identically equal numerical value to A at all times. This approach implicitly assumes that average hourly hot water demand is equal to one degree-hour of space heat demand, and such an assumption is reasonable. In the United States, a rough estimate is that the typical middle-upper income family uses 20 gallons of water at 120°F. per day per person. If this water is heated from 60°F., it can be shown that the equivalent household demand for three people is 1250 BTU/hour.[16] As we have seen, a space heat demand of 25,000 BTU/DD converts to 1041 BTU/DH, which in view of the approximate nature of the estimated hot water demand, is close enough to 1250 BTU/hr to justify the numerical convenience

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[16] Sixty gallons of water weigh 500 pounds, which when multiplied by 60°F. gives 30,000 BTU required per day, or 1250 BTU/hr.

$A = HW$. In the 15,000 BTU/DD house, the space heat demand is $15{,}000/24 = 625$ BTU/DH, which is exactly half the above hot water demand. Nevertheless, the same parameterization practice $A = HW$ will be followed for this house as well, partly because of the crudity of the hot water demand estimate, and partly because hot water demand is lower in a smaller and less expensive house and in overseas areas to which the results will be applied.[17]

The numerical approximation $A = HW$ makes it convenient to define a modified climate index per hour:

$$U = (65 - t_a) + 1 \text{ when } t_a \leqq 65 \qquad (12a)$$

$$U = 1 \qquad \text{when } t_a > 65 \qquad (12b)$$

In cases where the $A = HW$ convention is adopted, modifications of (5) and (6) are possible. Thus (6) becomes

$$D = AU - L_s \quad \text{when } t_a \leqq 65 \qquad (6a')$$

$$D = AU \qquad \text{when } t_a > 65 \qquad (6b')$$

The convenience afforded by the modified climate index, U, will become apparent in later climate comparisons.

Figures 10 through 13 show the results of collector size variation with $A = HW$ increased to get smaller collector areas. Triangles on the Albuquerque, Boston, Santa Maria and Phoenix curves mark output levels at which previous parameterizations (for other variables) were conducted. The numbers appearing at each point on the curves show corresponding $A = HW$ values. The solid lines refer to runs carried out at all stations with the same set of parameter values: $\beta = L + 15$, $N = 2$, $K = 20$. Subsequent to most of these runs, it was found that the optimum K values should be lower at most stations and the optimum N value should be 1 at Phoenix and Miami. Single runs taking advantage of these findings were made and dotted lines have been added to show the effect on the curves in the neighborhood of the supplementary runs. Where no dotted line appears, supplementary runs were unnecessary. For the time being, ignore the dot-dash lines in Figure 13.

One obvious conclusion from Figures 10 through 13 is that least-

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[17] For example, J.T. Czarnecki uses 45 gallons per day heated to 135°F. for five people in calculating hot water demand in various Australian cities. *Supra*, note 13, at 12. This corresponds to 28,200 BTU required per day, or 1175 BTU/hr., but note that five rather than three people are thought to have this demand. If Czarnecki's demand is used with three people, approximately 700 BTU/hr. is required and our assumption $A = HW$ is approximately correct for the 15,000 BTU/DD house with three people but overstates HW for the 25,000 BTU/DD house.

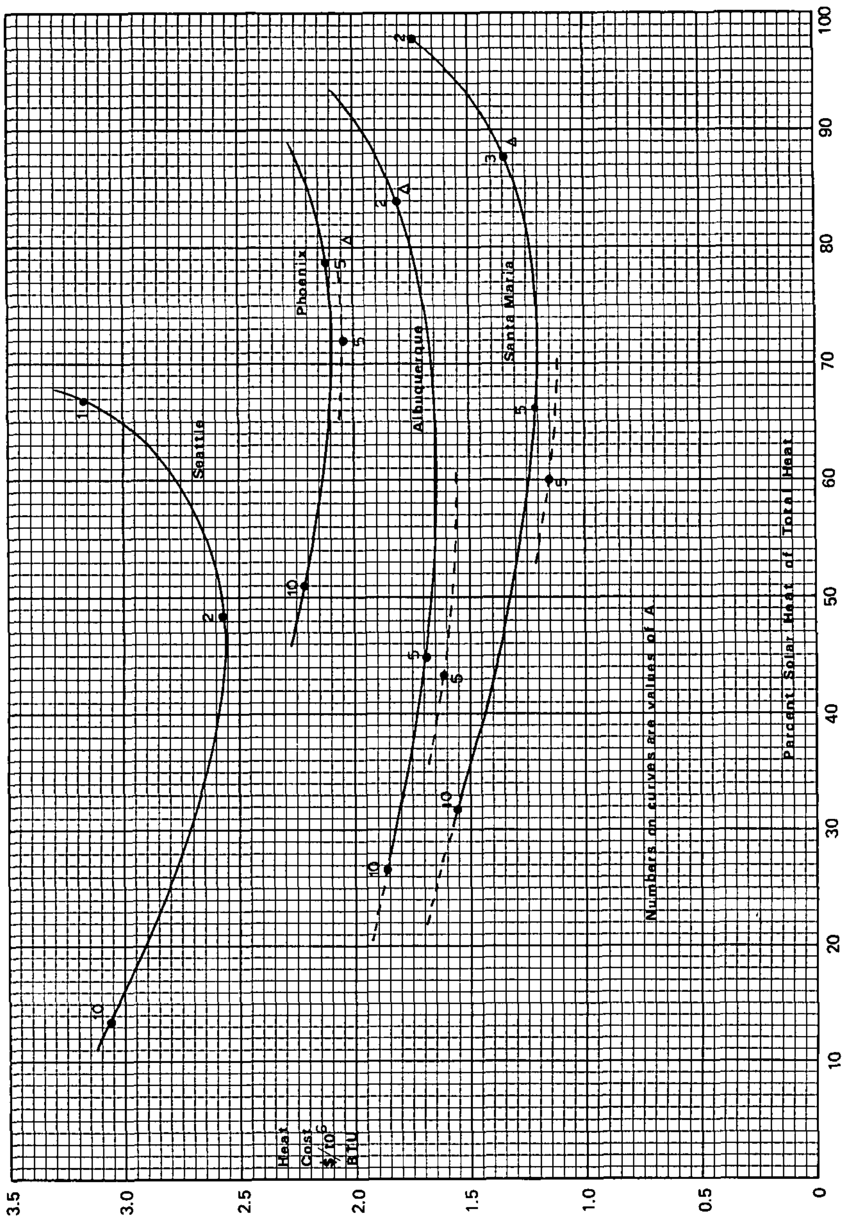

Figure 10
Influence of Collector Size on Solar Heat Costs
(25,000 BTU/DD House)

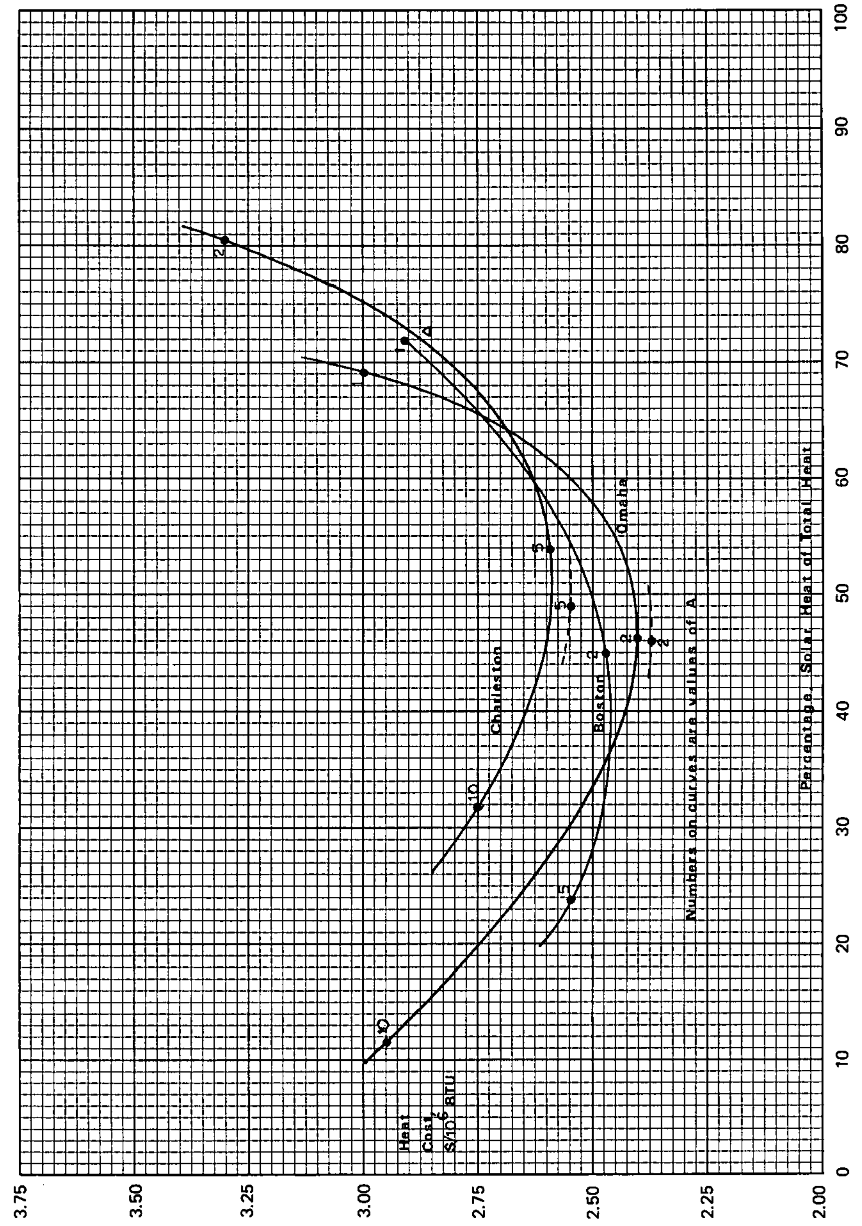

Figure 11
Influence of Collector Size on Solar Heat Costs
(25,000 BTU/DD House)

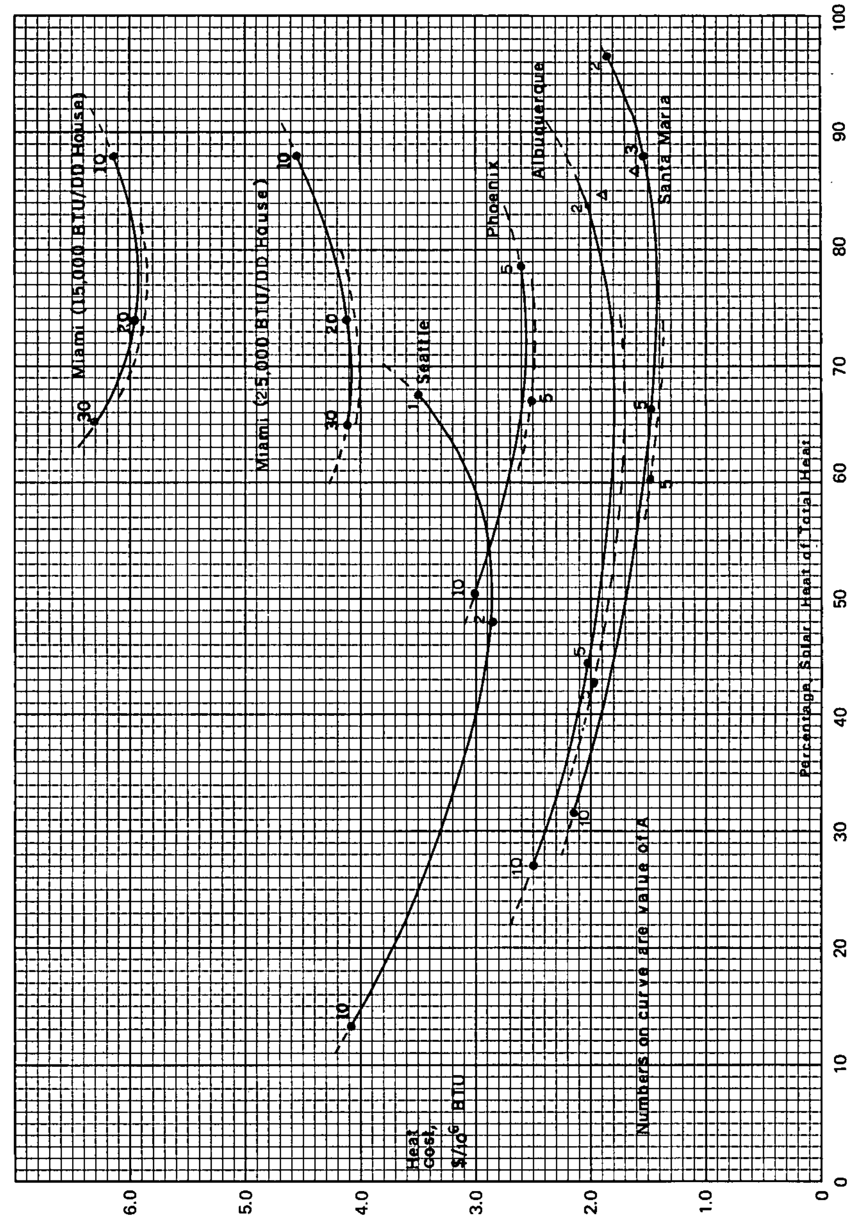

Figure 12
Influence of Collector Size on Solar Heat Costs
(15,000 BTU/DD House plus Miami 25,000 BTU/DD House)

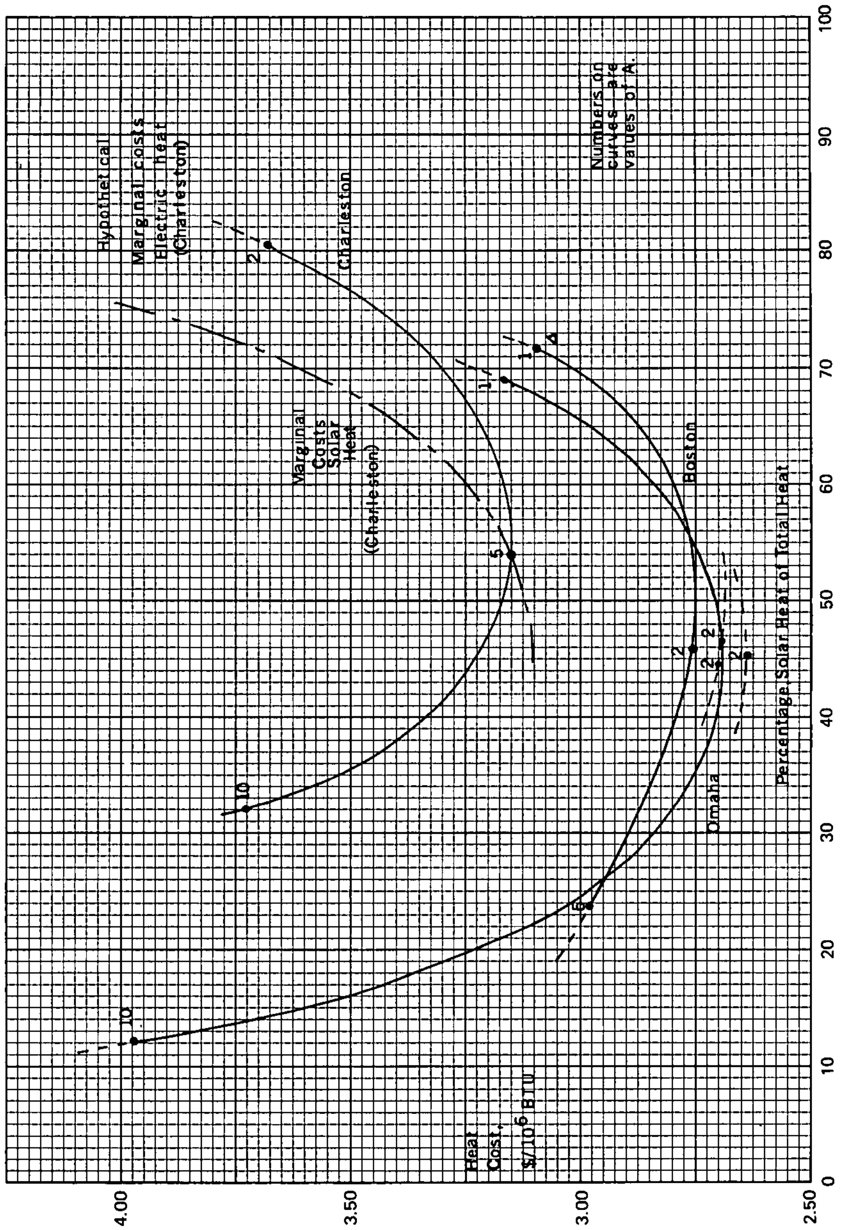

Figure 13
Influence of Collector size on Solar Heat Costs
(15,000 BTU/DD House)

cost solar heating system size in all cities provides a good deal less than total heat supply. If the solar heating system were designed to supply 90 or 95 percent of total heat, average costs would be quite high, as the graphs indicate. The reason is that the cost of solar heating is almost all in fixed capital investment. Additional capacity beyond the least-cost levels would be used with ever-decreasing frequency if system capability were expanded to higher and higher percentages because the more extreme the demand the less frequently it is experienced. The incremental investment needed to satisfy the last degree-hour of space heat demand would be chargeable against that degree-hour alone, making the cost of heat for that degree-hour quite high indeed. The situation becomes even worse when it is recognized that climate is a stochastic variable. We do not know exactly what the maximum demand will be over the life of the system. It is better to recognize from the beginning that 100% solar heating is not a rational objective. The question then becomes that of finding the optimal mix of solar and conventional heat. To this question we shall soon turn.

The average cost curves likewise become high at small capability levels. This reflects the effect on costs of controls and other components that are independent of system size. The cost of such components remains the same for small as well as for large systems; hence, this cost is a lump sum spread over smaller and smaller outputs as system usage is reduced.

Inspection of the graphs shows that the lower average cost points are obtained over a wide range of solar system size. Thus, for the 25,000 BTU/DD house in Santa Maria, there is a 10¢ per million BTU differential over the range 65-85% solar. In Albuquerque, the same differential extends from 35-80% solar; in Boston, it is 20-60%. Other minima are somewhat narrower, but the general conclusion is that the size variable is not critical within a reasonable range of the least-cost level. Solar costs will not vary widely if an error is made in the choice of system size, at least within the above ranges.

A considerable difference in the absolute level of costs is apparent among the eight cities. Absolute minima have been obtained by interpolation of the curves for the 25,000 BTU/DD house and are tabulated in Table 1. Where relevant, the dotted-line curves were used. Also shown is the effect on system average total costs of the previously derived high collector costs. These costs are, indeed, higher than they would have been if the design optimization runs had been carried out on the basis of equation (11d) rather than, as they were, on the basis of (11b). Thus, the higher collector cost

Table 1
LEAST COST SOLAR HEAT
(25,000 BTU/DD HOUSE)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Design Parameters[a]				Least Cost Solar Energy		Climate Variables[c]		
	N	K	A	Collector Area, ft^2	Output, % Solar	Cost $\$/10^6$ BTU[b]	X_h Annual av. BTU/ft^2 hr	t_a, Annual Av. °F.	$\sum^{8760} U$ (thousands)[d]
Santa Maria	2	10	4	261	75	1.10	75.8	54	112.0
Albuquerque	2	10	4	261	60	1.60	79.2	57	119.2
Phoenix	1	10	5	208	72	2.05	81.3	70	55.4
Omaha	2	15	2	521	47	2.45	55.9	52	161.2
Boston	2	15	3	347	40	2.50	49.6	50	158.7
Charleston	2	10	5	208	55	2.55	63.6	64	65.9
Seattle-Tacoma	2	20	2	521	45	2.60	45.9	50	145.4
Miami	1	20	25	52	70	4.05	71.0	75	15.3

[a] At all sites, a tilt of Latitude plus 15 degrees was used.
[b] Low estimate, based on equation (11b).
[c] Climate variables are, of course, independent of the size of the house.
[d] There are 8760 hours in a year. The modified weather variable U is defined by equations (12a) and (12b), text.

would have shifted the optimal design toward a higher investment in storage, to name the most obvious result.

Some approximate cause and effect relationships among system characteristics and climate are seen by reference to the design variables and climate characteristics also listed in Table 1. It is emphasized that the effects to be described are approximate since a complete interpretation could be obtained only by tracing through the workings of the solar energy house heating equations. To implement discussion, the sites are listed according to increasing cost levels, the least cost appearing first.

The sites fall into three groups, according to their relative positions in Table 1. The first three cities are in the sunlit southwest and far west. Differences among them in costs of solar energy are considerable. The next four, Omaha, Boston, Charleston and Seattle, represent higher latitudes and/or more humid climates. Costs are bunched closely. If solar energy is adopted at the cost levels represented by these four, the greatest part of the population of the United States will be covered. The last location, Miami, Florida, is a poor site for solar house heating. Miami has not enough demand to use the solar equipment effectively, though a high percentage of its small annual demand is satisfied by solar energy. Note the contrast with Seattle in column (5). Seattle has a high demand but limited sunlight; its utilization level is the lowest of all utilization levels. Note also column (4) for Miami and Seattle. Charleston is a less extreme case of Miami, and Boston is a less extreme case of Seattle.

Comparisons of demand and solar energy availability likewise help explain the relative standing of other cities. Omaha has slightly higher demand and a good deal more solar energy than Boston. There is a quantum jump in solar energy as we move on up the list to Phoenix and Albuquerque, but the latter has over twice the demand of the former. The reasoning breaks down, however, when we try to explain the difference between the number one and two sites, Santa Maria and Albuquerque. For this comparison, additional considerations are relevant. The first four columns of Table 1 show that the same design of solar heating system is used in both cities. But column (5) shows a significantly higher proportion of total heat load supplied by solar in Santa Maria. The reason will be clear from Figure 14. As there indicated, Santa Maria has a year-round relatively stable demand for solar heat in modest amounts. Albuquerque, with approximately the same annual total heat demand, has it bunched in the winter months. Herein lies the difference. In the language of conventional electric power systems, Santa Maria has a higher load factor.[18] Stated differently, less collected solar heat is wasted (OVR) in Santa Maria in the summer than in Albuquerque. The same consideration of load factor is relevant, of course, for comparisons among other sites, but its importance is masked by other variables.

A similar analysis applies to solar heat costs in the smaller (15,000 BTU/DD) house. Costs of heat are, of course, higher in the smaller house because the constant term, $35.80, in equations (11a) and (11c) is relatively more important for the same value of A than it is in equations (11b) and (11d). This also shifts the optimal percentage solar and, of course, the optimal A, to higher values. Compare Figures 10 and 11 with 12 and 13, and note the comparison for the two houses in Miami in Figure 12. As previously noted, solar heating designs for our "middle four" cities (Omaha, Boston, Charleston, and Seattle) are not markedly different and lead to least costs for the 25,000 BTU/DD house that differ by only 15¢ per million BTU. In the 15,000 BTU/DD house, the spread is increased but principally because demand in Charleston is

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[18] The load factor is defined as the ratio of total power produced over a given period of time to total power that could have been produced with full use of existing plant capacity over that same period of time. The greater the utilization of plant capacity, the higher the load factor and, of course, the larger the volume of output over which fixed plant costs can be spread. Since solar heating system costs are almost entirely fixed plant costs, the load factor is an important consideration.

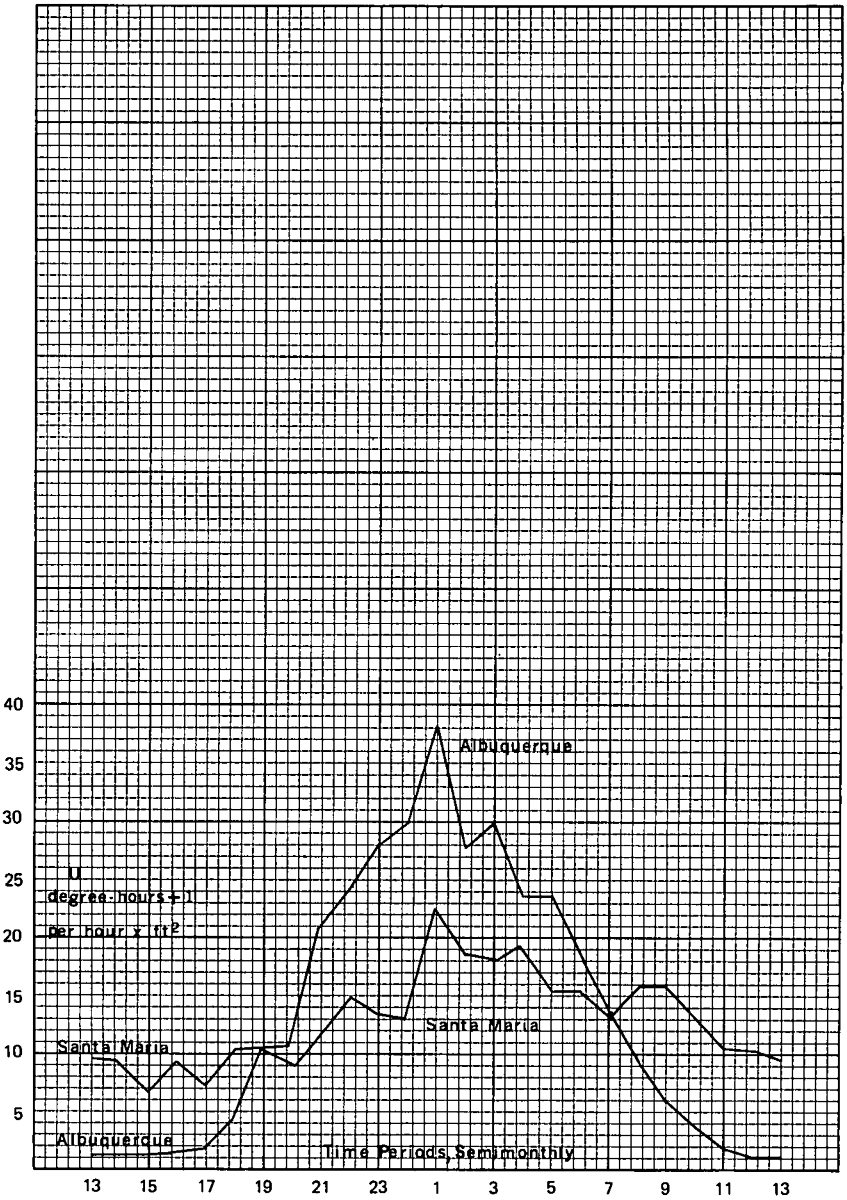

Figure 14
Solar Climate Comparison

so low. The other three cities have a spread of only 20¢ per million BTU.

A final comment refers to solar space cooling. Some experiments with day-night heat exchange have been conducted in solar heated houses, but they have not yielded sufficient cooling, nor have they utilized conventional cooling cycles. It is possible to design a solar cooling machine that operates on the heat provided by the solar collector and, indeed, laboratory devices of this sort have been built. They differ from commercially available heat-operated cooling machines in that they are designed to use the low-intensity heat provided by solar energy.[19]

The prospects for space cooling cannot be explored in the present article, which is already long in its description of solar heating. But at least two advantages would follow from the development of fully practical solar space cooling. First, the same solar collector could be used for both heating and cooling, with corresponding improvements in the load factor and lower unit output costs of collector. Second, space cooling may well be of high value for productivity in many underdeveloped countries that are plentifully endowed with solar energy. The foregoing conclusions on the merits of space heating in different climates would be modified if combination heating-cooling systems were being considered. The same, of course, is true of optimum collector size and perhaps other design parameters.

B. Solar-Conventional Heat Comparisons and Combinations

The levels of least cost design of solar heating systems in each of the two houses are tabulated in Table 2, along with costs of space heat from other sources. It will be noted that the latter are reported for electricity and fuel costs only, excluding the cost of heating elements and/or furnaces. The reason for this exclusion is that roughly the same cost of a conventional furnace (or electric heating element) will be required whether or not a solar heating system is used. The only saving offered by solar energy, where it is of low enough cost, is in the fuel or energy input used as an alternative heat source.

The reason the owner of a solar-heated home must provide almost equal furnace capacity for conventional heat is simply that it will be needed in extreme weather. Column (5) of Table 1, it will be recalled, reports the fraction of annual total heat supplied by solar. The detailed calculations for runs with optimum design variables (or near-optimum, when the optimum required interpolation)

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[19] For further discussion, see papers presented in *Use of Solar Energy for Cooling Purposes* 6, Conference on New Sources of Energy, U.N. Doc. E/Conf. 35/6 (1961).

Table 2

COSTS OF SPACE HEAT (UNITED STATES)

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*		*(7)*	*(8)*
	Least Cost Solar Heat, $\$/10^6$ BTU (1961)				*Electric Heat, Electricity Cost Only, $\$/10^6$ BTU (1967)*			*Conventional Heat, Fuel Cost Only, $\$/10^6$ BTU (1962)*	
	15,000 BTU/ DD House		*25,000 BTU/ DD House*		*20,000 kwhr/yr*	*30,000 kwhr/yr*			
	Low	*High*	*Low*	*High*				*Gas*	*Oil*
Santa Maria	1.35	1.84	1.10	1.59	4.51[a]	4.36[a]	California	1.42	1.62
Albuquerque	1.70	2.31	1.60	2.32	4.89	4.62	New Mexico	0.89	2.07
Phoenix	2.55	3.55	2.05	3.09	4.56	4.25	Arizona	0.79	1.60
Omaha	2.65	3.16	2.45	2.98	3.30	3.24	Nebraska	1.05	1.32
Boston	2.70	3.15	2.50	3.02	5.49	5.25	Massachusetts	1.73	1.76
Charleston	3.15	4.16	2.55	3.56	4.50	4.22	South Carolina	0.96	1.55
Seattle-Tacoma	2.85	4.05	2.60	3.82	2.26[b]	2.31[b]	Washington	1.83	2.00
Miami	5.85	6.48	4.05	4.64	5.16	4.90	Florida	2.81	1.73

a Electric power costs are for Santa Barbara. Electric power data for Santa Maria were not available.

b Electric power costs are for Seattle.

Source: Solar heat costs are from optimal design systems by interpolation of Figures 10-13. Electric power heat costs are from U.S. Federal Power Commission, All Electric Homes, Table 1 (1967). Conventional heat fuel costs are derived from prices per million BTU reported in P. Balestra, The Demand for Natural Gas in the United States, Tables 1.2 and 1.3 (North Holland Publishing Co., 1967). Fuel prices were converted to fuel costs by dividing by the following national average heat (combustion) efficiencies: gas, 75%; oil, 75%. Heat efficiencies are from American Society of Heating, Refrigerating and Air Conditioning Engineers, Guide and Data Book 692-694 (1963 ed.).

showed that in the fortnightly periods of poorest solar performance, the supplementary heat source furnished 72.5% of total heat in Albuquerque (compare Figure 5), 77.8% in Boston and 75.5% in Omaha. On the poorest days, still higher percentages were required. A second point is that furnace prices change only slightly with capacity variations of the magnitude here involved. As a practical matter, any possible cost saving is small enough that it can be ignored within the accuracy of the present cost comparisons.

Now compare the figures in the first four columns of Table 2 with those in columns (5) and (6). The latter are actually-experienced costs for all-electric homes, which include electric space heating, in the indicated cities. Although costs were reported by city, the number of all-electric homes was not reported by city in the source used for columns (5) and (6) but by electric power company. For example, the Los Angeles Department of Water and Power served 10,300 all-electric customers on January 1, 1967.[20] The Federal

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[20] *See* note 6, *supra.*

Power Commission's survey of all-electric houses was based on reports from 180 electric utilities serving between 45 and 46 million customers. Of these utilities, 162 reported serving 1,459,251 completely electrified residences.[21] There is a tendency for these residences to be found more frequently in the low cost electric power areas of the Tennessee Valley and the Northwest or in mild climates where space heating demand is not great. Nevertheless, the number of all-electric homes reported for utilities serving the eight cities listed in Table 2 was typically in the hundreds or thousands. It is clear that the costs of space heating shown in columns (5) and (6) are widely experienced.

Two minor qualifications need be made to ensure comparability of the data in columns (5) and (6) with (1) through (4). First, 20,000 kilowatt hours corresponds to 68.2 million BTU. This is the amount of heat that the 15,000 BTU/DD house would require with a U value of 12.5, corresponding to 109,000 a year. It will be seen that this value of U falls somewhere near the middle of the range reported in Table 1, column (9). Where the small house is in a severe climate, it requires more than 20,000 kwhr; in a mild climate, it needs less. A similar calculation shows that 30,000 kwhr would be required for the 25,000 BTU/DD house in a climate where $U = 11.2$, corresponding to 98,000 a year. Again, compare Table 1, column (9). Thus, the heat loads for which electric power costs are reported are quite comparable to those of our solar heated houses. Moreover, it will be noted that unit electric power costs do not change significantly in a given city between 20,000 kwhr and 30,000 kwhr. Hence, differences in climate that would change the quantity of electricity consumed in our two houses would not importantly change the electric power cost per million BTU, except possibly in Miami, where a very low space heat demand is experienced and higher electric power rates apply.

The second qualification needed to ensure comparability arises from the fact that the electric power cost data are in 1967 dollars; whereas the solar heat data are in 1961 dollars. This difference, however, is of no practical importance. The national index for wholesale electric power rates stood at 100.6 (1958 = 100) in January, 1967,[22] the exact date to which the electric power cost figures apply. It follows that the electricity costs would have been practically identical in 1961. On the other hand, if the 1961 solar equipment costs were raised to 1967 using the U.S. wholesale price index, they would be increased in the ratio of 106.1 (1967) to 100.3

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[21] *Id.* at 1.

[22] U.S. Bureau of Labor Statistics, Monthly Labor Review Table D-4 (1967).

(1961), or approximately six percent.[23] The results are different depending on the way the price adjustment is made, but the magnitude of the difference is small and can be ignored within the accuracy of the estimates of solar heating costs.

In every city except Seattle and Miami, solar heat with either low or high cost estimates in either of our houses is cheaper than electric space heat. Seattle is lower because of unusually low electric power rates and relatively high solar costs. Miami is lower only for the 15,000 BTU/DD house.

Presumably the attraction of electric heat is its cleanliness and sometimes the lack of availability of gas. A common situation in which electric space heat has an appeal is in the modern semi-rural home, one step beyond the suburbs, where the alternatives are often oil or propane,[24] gas pipelines not being available. It would appear that a considerable market for solar heat must exist where electric heat is used. Bear in mind that for the upper limit of the solar cost estimates, our calculations were based on manufacturers' system components that exist today, integrated into designs of established performance. The low cost solar designs postulated improvements in collector manufacturing costs, but not changes in design.

It remains to select optimum combinations of solar and electric heat. Enough has been said to establish that solar heat cannot be expected at a reasonable cost to provide 100% of total space plus hotwater heat. Yet, with electric heat at the costs observed, solar heat can economically provide a good deal more of the total annual heat than corresponds to its least-cost design levels.

The optimum combination is found where the marginal, or incremental, costs of solar and electric heat are equal. Our previous comparisons were in terms of averages. Let us illustrate the calculation using marginal costs at Charleston. Marginal costs of solar energy are shown graphically in Figure 13 by the broken line through the Charleston average cost curve (low estimate). As long as average costs are declining, marginal costs must be below average. For symmetrical reasons, the marginal cost curve must lead the average up if the latter is to rise. This establishes the general shape of the marginal cost curve shown for solar heat.

Intersection with marginal cost of electric power is assumed to take place at $4.00 per million BTU. Four dollars is probably too low a figure, even with step rates, in view of the averages shown in

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[23] *Id.*

[24] Propane prices are roughly the same as oil prices per million BTU of delivered heat.

Table 2 and the low level of electricity that would be consumed. With electricity furnishing only the deficit heat, a good deal less than 20,000 kwhr/yr would be used, even in our larger house. But the purpose of the analysis is to illustrate a method of calculation. If realistic marginal electric power costs are higher, the intersection will be found at a greater solar heat output.

The intersection of marginal costs takes place at a solar output equal to 75% of total annual space heat output and an average solar cost level of $3.45. By interpolation on the average cost curve, it appears that optimal solar design for these conditions would lead to a value of A of approximately 3. The comparison of an expenditure on fixed capital with an expenditure on a variable input implicitly assumes that the variable input can be treated (at the margin) as constant at $4.00 over the lifetime of the fixed capital. If the price of electric power is expected to vary over the future life of the solar equipment, it is more accurate to make the comparison on the basis of present values (at the margin) of the expected time profile of prices rather than constant annual equivalents, as is done here.[25] This, however, would not change the general nature of the comparison shown diagramatically for Charleston in Figure 13. It would change only the numbers on the vertical axis of that figure.

The same considerations, of course, govern the use of solar heat in conjunction with any other heat source. Table 2, columns (7) and (8), gives the basis for average cost comparisons of gas and oil costs with solar energy. Data for each of the cities were not separately available but were included in state-wide averages, which, of course, give less precision to the comparison. Conventional fuel costs were available only for 1962, but the national price index for all fuels showed practically no change from 1961 to 1962.

A comparison of the data in columns (1) through (4) with those in (7) and (8) suggests the possibility that solar energy cost might be attractive even when compared with gas and oil. It is possible

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[25] The calculation is, of course, made prospectively at the time the solar equipment is installed. Seen from that time, a present value comparison would relate (at the margin) the entire capital expense of the solar equipment to the discounted future stream of electric power costs. In effect, this is what we are doing by the comparison of an assumed $4.00 electric power cost with the annual equivalent capital cost. It can be shown that annual sinking fund depreciation differs from the initial book value of a capital expense by the same factor as a given annual cost ($4.00 in this case) differs from the present value of a constant stream of that cost over the same number of years. *See* R.A. Tybout, *Economic Criteria for Evaluating Power Technologies in Less Developed Countries,* 1 United States Papers Prepared for the United Nations Conference on the Application of Science and Technology for the Benefit of the Less Developed Areas 191-92 (U.S. Dep't of State, 1963).

that in Santa Maria and other parts of California with similar climates, gas prices may be high enough, or perhaps gas not available, so that solar energy might be attractive on the basis of comparisons with oil or propane. In Albuquerque and other parts of New Mexico with similar climates, it would appear that only when gas is not available and the comparison is with fuel oil or electric space heat is there the possibility of finding solar heat attractive from a cost standpoint. In Phoenix and like climates of Arizona, the possibilities are much more limited and more likely to be influenced by the size of house, but variability of oil prices throughout the state keep open the prospect for some use of solar heat. In sparsely settled areas, transportation of small oil shipments can seriously raise local prices. The same observations apply with much diminished force to the other states shown in Table 2. The possibilities of local price variation in conventional fuels remains even when the state average cost comparison is unfavorable to solar heat. Finally, it must be remembered that we are dealing with fuel prices at the time of solar unit installation. As with electric power, it is necessary to think in terms of the conventional fuel costs over the twenty year life of the solar equipment. When the possibilities of conventional fuel price increase are taken into account, the comparison becomes more favorable to solar heat.

C. Hot Water Heat Only

To this point, attention has been directed to space heat, with hot water considered as auxiliary heat demand only. To judge the validity of this view of the problem, remember that the variable U is defined in such a way that annual hot water demand accrues at the rate of 1 BTU per hour per square foot of collector area. See equations (12a) and (12b). This means that 8760 of the annual heat demand in column (9) of Table 1 is for hot water. Hot water demand thus accounts for 57.5% of the total demand in Miami, almost 16% in Phoenix, slightly over 13% in Charleston and percentages a good deal smaller elsewhere. At least in Miami, there is good reason to question whether solar equipment should be designed for space heat at all. The alternative would be to design for solar heating of hot water only. As previously noted (see prior heading "Costs") solar hot water heating is, indeed, practiced on a commercial scale in Florida.

Previously reported questionnaire results can be used to get least-cost estimates for hot water-only systems in Miami. The estimates will be based on more exact cost data than for space heat because commercial production, albeit small scale, is an established fact. They will be more approximate for output, however, because a comprehensive model for hot water heating with simplified equipment has not been developed. For the sake of approximation, overall average annual energy efficiencies can be used. An experimental study by R. N. Morse gave an annual average efficiency of 35% for a tilted, double glazed hot water collector in Melbourne, Australia.[26] Morse reported 30% loss of energy as a result of optical reflection. It is known that optical losses would be cut approximately in half by reducing the glazing to one glass cover and that this will not produce offsetting thermal losses in Miami. The authors estimate a 5% increase in thermal loss in the Miami climate with normal operation of a hot water heater there. The net result, then, of reducing the number of glass covers from two to one would be roughly to increase heat collection efficiency from an annual average of 35% (Melbourne, two covers) to 45% (Miami, one cover).

If we use a representative energy conversion efficiency of 45% with the annual solar radiation at Miami on a surface tilted at the latitude of the site for the year 1955 (for which space heat calculations were made), the annual heat collected as hot water is 324,000 BTU/ft^2 of collector area. The costs of solar water heaters were previously reported at $6.40 to $8.90 per square foot of collector. A fifteen year life was estimated on these systems by some of the respondents.[27] On the same basis as used for space heat systems (6% interest, sinking fund depreciation) annual equivalent costs are 66 to 92¢/ft^2 respectively, which give a hot water cost of $1.97 to $2.84 per million BTU. The advantages of installing a simpler hot water-only system rather than a space heat-hot water combination are obvious for Miami. Indeed, costs of solar heated water in Florida are within the same range as conventional fuel costs, as will be seen from Table 2. In practical applications, supplementary heat is required for hot water, as for space heat, if 100% capability is desired, but previous discussion has indicated the nature of this problem. It does not change our conclusions as far as the cost of solar heat is concerned.

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[26] Morse, *Water Heating by Solar Energy,* 5 Conference on New Sources of Energy 67, U.N. Doc. E/Conf. 35/6 (1961).

[27] The component constraining the life of the system to fifteen years is the water tank. The collector can be made to last more than fifteen years and, indeed, one of the two U.S. manufacturers of the type of solar hot water heater here discussed provides for separate tank replacement on the grounds that the collector will last a "life-time." In our previous estimate of collector costs, the collector life was taken as twenty years. To the extent that the true life of the collector is more than fifteen years, hot water costs are here overstated.

V

EXTENSIONS

The detailed calculations reported above describe solar heating system performance and potential costs in eight different cities of the United States. Wherever similar solar radiation and temperature are experienced, similar results can be expected. But there are obvious limits on the precision afforded by the method of analogy. The purpose of the analysis to follow is to give a better method of forecasting solar heating system performance and costs in other locations, domestic and foreign.

The forecasts are based on statistical simplification of the basic model in two steps, dealing respectively with optical and thermal losses (see Figure 2). The method of analysis in full detail as used heretofore was necessary to optimize design and to determine the magnitude of transient effects. With these results in hand, it is now desirable to relate them back to average climate variables (solar radiation and temperature) prevailing at the time. This will be done using half-month averages as in Figures 4 through 6. Extensions to other locations will then be made on the basis of whole month averages. This will reduce the forecasting job to a magnitude convenient for hand calculation and consistent with the form in which solar radiation and temperature data are most often available. Aggregation beyond the one month limit is not desirable because it would excessively obscure differences in climatic patterns.

The optical and thermal loss regressions are reported in Table 3, equations (13) and (14), respectively. Consider equation (13). The term, I, expresses well-known trigonometric relations between collector tilt and solar radiation, X_h, on a horizontal surface. Direct and diffuse components of solar radiation were not distinguished in the regression. The term Y is a declination index to take account of changes in reflectivity due to changes in the angle at which the sun strikes the collector over the annual cycle. X_h and Y are as previously defined. Similarly, the terms E and t_a in (14) are as previously defined. The regressions were run with values of all variables (per square foot of collector area, as before) from the computer runs for seven of the eight cities with design parameters as listed in Table 1.[28] Only the data for half-month intervals in which there was no overflow ($OVR = 0$) were used. The reason will be apparent on inspecting Figures 5 and 6. High correlations of E and Y are found when the system is operating at capacity, as noted in previous discus-

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[28] Seattle was omitted because the least cost design conditions there had not been determined at the time the regressions were run.

Table 3

SOLAR HEAT REGRESSIONS

Optical loss regression:

$$Y = 0.2097 + 0.7166\ (X_h)(I) - 0.1541\ (X_h)(Y) \qquad (13)$$

standard errors (1.201) (0.01572) (0.00643)

multiple correlation coefficient, $R = 0.963$

standard error of residuals = 3.26

Thermal loss regression (when OVR = 0):

$$E = 0.4186 + 0.7992Y - 0.1274t_a \qquad (14)$$

standard errors (1.781) (0.0283) (0.0295)

multiple correlation coefficient, $R = 0.950$

standard error of residuals = 3.48

Total energy conversion regression (when OVR =0):

$$E = 2.71 + 0.611\ (X_h)(I) - 0.189\ (X_h)(Y) - 0.135t_a \qquad (15)$$

standard errors (2.22) (0.0265) (0.0136) (0.0364)

partial correlation $(X_h)(I)$ vs. $(X_h)(Y) = 0.474$

Definitions (Table 3)

$$I = \frac{2\cos d\cos(L-\beta) + \sin d\sin(L-\beta)}{2\cos d\cos L + \sin d\sin L}$$

$$Y = \sin\left[\frac{\Pi}{12}(TP-6)\right]$$

d = Solar declination, degrees

= 23.45Y

TP = Time period measured in half month intervals from beginning of the year.

sion of these figures. But when there is positive overflow, no such correlations appear, nor would they be expected.

The high correlations in regressions (13) and (14) attest to the similarity in operating performance of solar heat systems of best input-output (least-cost) design in widely different climates. The results imply that our design parameters can be adjusted to fit the different weather conditions in such a way as to produce predictable results of climate variation. Note, for example, the regression coefficient 0.7992 for Y in (14) with extremely low standard error, 0.0283. In combination with the clearly nonsignificant estimate of the intercept[29] and the small coefficient of t_a, the implication is that E varies from 70-80% of Y, depending on t_a. E is higher the lower t_a because, as previously noted, heat losses tend to be lower in colder weather when t_a is lower. Similarly meaningful interpretations of (13) could be made but would lead into details beyond those discussed in the previous statement of the model.

A final step was to regress E as a function of the independent variables $(X_h)(I)$, $(X_h)(Y)$ and t_a, thus substituting Y from (13) into (14). The results of this regression are shown as (15). The

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[29] The intercept is not significantly different from zero according to the T test with a standard error as high as reported (1.781).

small standard error of residuals in (15), 4.11 should be compared with the average value of E, 35.7.

Now, it will be recalled that the observations on which (15) is based represent full capacity (OVR = 0) operation only. To make predictions of solar heat system performance in new locations, one starts with values of U defined in (12 a,b). This definition of U includes hot water demand at levels described in the derivation of (12 a, b). The degree-hour component of U can be obtained from monthly average temperatures only if hourly frequency distributions of temperature are available.[30] Such frequency distributions were constructed from the eight climates observed, and degree-hours for each month were calculated for other locations from these. Knowing U, total demand D was calculated for alternative assumed values of A using (6a′) and (6b′) and assuming $L_s = 0$. The assumption $L_s = 0$ introduced a conservative bias in the sense that the demand the system was called upon to fill was slightly overstated; however, the bias was only on the order of a few percent, judging by the results for our eight U.S. cities.

For a given location and for each assumed value of A, calculations were made month-by-month for D as above described and for E as given by (15). For each month in which $E < D$, the system was working at capacity and system output was equal to E. For each month in which the calculated $E \geqq D$, equation (15) did not give the correct value of E, but that did not matter since useful system output was D. Demand was completely satisfied and there was some overflow. The amount of overflow is unknown, but immaterial. Annual output for each new location was calculated by taking the sum of monthly D or E values, whichever was lower in each separate month.

The locations for which outputs were thus estimated are listed in Table 4. Parameter values were found as follows. In all cases, a tilt of $\beta = L + 15$ degrees was used, because this had been optimal for all U.S. sites and was hence implicit in the calculation of E. Two plates of glass ($N = 2$) were used at all overseas locations except for New Delhi, which was judged to have temperatures close enough to Miami and Phoenix to justify the use of a one-plate collector. Values of K were chosen by analogy with comparable climates in the United States. Inspection of Table 4 will show that they fall into two groups: the relatively cold climates of Belgrade, Paris and Berlin were assigned $K = 10$. A precise determination of

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[30] The reason is, of course, that space heat demand occurs only at $t_a < 65°F$. Monthly mean values do not by themselves tell how many hours there are of such temperatures.

Table 4

LEAST COST SOLAR HEAT IN 25,000 BTU/DD HOUSE (FOREIGN LOCATIONS)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
	Design Parameters[a]				Least Cost Solar Energy		Climate Variables[c]			
	N	K	A	Collector Area, ft²	Output, % Solar	Cost $/10⁶ BTU[b]	X_h Annual av. BTU/ft² x hr	t_a, Annual Av. °F	$\sum^{8760} U$ (thousands)[d]	Climate Class
Melbourne, Australia	2	10	5	208	60	1.90	56.2	59	111.9	Cb (Seattle)
Lisbon, Portugal	2	10	5	208	60	2.05	65.6	62	89.3	Cs (Santa Maria)
Madrid, Spain	2	10	3	347	55	2.05	57.6	58	139.7	BS (Albuquerque)
Buenos Aires, Argentina	2	10	5	208	60	2.10	63.1	62	91.4	Ca (Charleston)
Sydney, Australia	2	10	5	208	70	2.15	58.8	64	74.6	Cb (Seattle)
Paris, France	2	15	3	347	40	2.35	40.4	51	190.4	Cb (Seattle)
Belgrade, Yugoslavia	2	15	3	347	35	2.50	47.0	54	198.6	Da (Omaha)
Berlin, Germany	2	15	3	347	30	2.55	35.3[e]	49	221.8	Db (Boston)
Rome, Italy	2	10	3	347	55	2.55	50.7	60	117.0	Cs (Santa Maria)
Taranto, Italy	2	10	3	347	60	2.70	59.3	63	98.6	Cs (Santa Maria)
Nagasaki, Japan	2	10	4	261	45	2.70	48.2	61	115.0	Ca (Charleston)
Athens, Greece	2	10	4	261	60	2.75	56.8	66	80.0	Cs (Santa Maria)
Tokyo, Japan	2	10	3	347	45	2.85	39.4	59	134.0	Ca (Charleston)
Trieste, Italy	2	10	3	347	40	2.95	41.6	57	141.3	Ca (Charleston)
New Delhi, India	1	10	10	104	70	3.20	74.7	78	33.7	Ca (Charleston)

[a] At all sites, a tilt of Latitude plus 15 degrees was used.

[b] Low estimate, based on equation (11b).

[c] Climate variables are, of course, independent of the size of the house.

[d] There are 8760 hours in a year. The modified weather variable U is defined by equations (12a) and (12b), text.

[e] Solar Observations taken at Lindenberg.

Source: Columns (1), (2), (3), (4), (5), (6), and (9) calculated as described in text. Column (7) is based on George O. G. Löf, John A. Duffie and Clayton O. Smith *World Distribution of Solar Radiation,* University of Wisconsin Engineering Experiment Station Report No. 21 (1966). Column (8) is derived from World Meterological Organization, *World Weather Records* (Washington, 1967).

optimum K values cannot be made without conducting hour-by-hour calculations, as for the eight U.S. sites. But we have already seen that K within the range of ten to twenty is not very cost sensitive (see Figure 9).

The determination of A was made in the same way as for U.S. sites. For each assumed value of A, output was calculated as described above. Least cost values were found by making graphs of costs calculated from equation (11b) (low estimate for 25,000 BTU/DD house), as before. Compare Figures 10 through 13. The results were as reported in Table 4, in increasing order of least solar cost.

An attempt was made to represent overseas every climate that was represented in the United States, but only six of the eight U.S. climates appear in Table 4. No counterpart of Phoenix (BW, desert climate) or of Miami (Aw, tropical savanna) could be found in which there was any significant demand for space heat. The number of sites that could be investigated, however, was limited to the approximately 260 locations for which solar radiation data were available on a month-to-month basis. Only one representative of the Albuquerque (steppe), Omaha (continental, warm), and Boston (continental, cool) climates is included, partly because of the scarcity of meteorological data, but even more as a result of the greater scarcity of reliable price data for conventional fuels (used in later comparisons). These climates are found especially in eastern Europe and the continental land mass of Asia. The remaining three climates, Charleston (humid subtropical), Seattle (marine), and Santa Maria (Mediterranean), are well represented.

Table 4 is constructed in the same way as was Table 1 and the same general observations on cause and effect can be made with respect to it. Roughly speaking, a combination of high demand and simultaneous high solar radiation produces the least cost. An example of high radiation without high demand is New Delhi. An example of high demand without high radiation is Berlin. The former has high solar heat costs (as did Miami) but high percentage solar. The latter has moderate solar heat costs and very low percentage solar (like Boston, its climatological counterpart). The best locations strike a better balance between demand and solar availability. In no case was an overseas location as good as Santa Maria or Albuquerque. But the first five shown in Table 4 fall in the same range as the first three in Table 1. Judging by our two cases of steppe climate, Albuquerque and Madrid, it appears that this climate shows good prospects. On the other hand, the three principal climates covered in Table 4 show a range of cost variation

Table 5

COST OF SPACE HEAT (FOREIGN SITES)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Least Cost Solar Heat, $\$/10^6$ BTU (1961)				*Conventional, Energy Cost, Fuel Only, $\$/10^6$ BTU (U.S., 1961)*				
	15,000 BTU/DD House		*25,000 BTU/DD House*						
	Low	*High*	*Low*	*High*	*Firewood*	*Coal*	*Oil*	*Gas*	*Electricity*
Melbourne, Australia	2.40	3.12	1.90	2.68	3.18	—	3.12	2.16	5.39
Lisbon, Portugal	2.45	3.21	2.05	2.89	2.42	3.82	3.06	2.28[b]	20.40
Madrid, Spain	2.45	3.46	2.05	3.06	3.09	1.40	—	—	9.96
Buenos Aires, Argentina	2.55	3.34	2.10	2.96	—	2.60	1.56	—	30.60
Sydney, Australia	2.70	3.54	2.15	3.03	3.79	—	3.01	2.19	10.70
Paris, France	2.70	3.75	2.35	3.43	—	2.95	—	2.26	15.50
Belgrade, Yugoslavia	2.90	4.03	2.50	3.65	1.83	0.79[a]	—	—	9.00
Berlin, Germany	3.00	4.17	2.55	3.72	2.91	1.40	—	3.03	16.90
Rome, Italy	3.00	4.23	2.55	3.80	2.58	5.16	—	1.90[b]	18.50
Taranto, Italy	3.15	4.44	2.70	4.02	1.50	4.30	—	2.69[b]	18.30
Nagasaki, Japan	3.20	4.26	2.70	3.89	2.00	2.07	—	2.76	16.10
Athens, Greece	3.30	4.39	2.75	3.96	2.70	5.46	—	3.33[b]	17.50
Tokyo, Japan	3.25	4.58	2.85	4.25	4.40	2.60	—	2.99	8.93
Trieste, Italy	3.40	4.79	2.95	4.40	3.92	4.00	—	—	16.45
New Delhi, India	4.35	5.26	3.20	4.14	2.78	3.98	4.55	—	18.63

a Brown coal. Half the heat content of coal used.

b For cooking only.

Source: Solar heat costs are from optimal design systems for parameters given in Table 4. Conventional energy costs are derived from fuel prices given in International Labour Office, Yearbook of Labour Statistics (Geneva, annual). Indexes from Table 23 (1961) were combined with prices from Table 24 (1958) and international rates of exchange Table 31 (1961) to get 1961 U.S. prices. U.S. prices were converted to costs using the following heat (combustion) efficiencies: firewood, 32%; coal, 50%; oil, 75%; gas, 75%; electricity, 100%. The efficiency estimate for firewood is based on S. H. Schurr, et al., Energy in the American Economy 1850-1975, at 49-50 (John Hopkins Press, 1960); for coal, *id.* at 177; for oil and gas, American Society of Heating, Refrigerating and Air Conditioning Engineers, Guide and Data Book 692-694 (1963 edition).

that makes one skeptical of generalizations based on the Trewartha classification. Thus, Buenos Aires, with a solar heat cost of $2.10 per million BTU, and New Delhi, with $3.20, are both in the Charleston (Ca, humid sub-tropical) climate.

The ultimate test of solar suitability is, of course, in the comparison of the costs of solar heat with conventional fuels. Such a comparison appears in Table 5. Solar heat costs are calculated on the basis of equations (11a, b, c, d) with parameter values shown in Table 4, which are for the least cost combination with (11b). The results must be carefully interpreted. They are subject to the same qualifications as discussed earlier in connection with Table 2, and there is the additional important qualification that they are based on projected U.S. manufacturing costs applied to other countries. The various solar components might be more or less expensive at a foreign location, depending on comparative advantage, degree of industrialization and, in some cases, trade restrictions. These considerations make the comparison with local fuel prices more approximate than for U.S. sites, but do not create a consistent bias one way or the other.

Another aspect of the same problem appears in less serious form in the determination of fuel prices. International rates of exchange were used to convert local prices to their U.S. dollar equivalents. For industrialized nations, in most cases this is a satisfactory method of conversion. But where trade restrictions are important and prices are set by central government agencies, there is not necessarily a close relation between prices and true local costs. Thus, it will be noted in Table 5 that energy costs in Belgrade tend to be out of line with those in nearby countries. This may reflect on the official rate of exchange more than on special costs in Belgrade. Despite these problems, the comparisons shown in Table 5 have enough meaning to merit our attention, particularly when they can be supplemented by other studies.

Consider the case of the two Japanese cities, Nagasaki and Tokyo, which appear low on the list of solar prospects. A special study of household energy consumption in Japan attests to the importance of electric space heating. Electric power for space heat accounted in 1962 for 20% of electricity consumption in Japan; electricity was in turn found to account for 10.2% of all household energy consumed in urban areas and 3.4% in rural areas.[31] Hydroelectric power is important in Japan and probably accounts for the relatively low electric power costs shown for Tokyo. Moreover, the

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[31] Y. Tatsumi, Household Energy in Japan 2675-78 (1966) (Tokyo World Power Conference).

costs that appear in column (9) of Table 5 are averages for all electric power consumption. Bulk consumption, as for space heating, undoubtedly has lower rates. Nevertheless, even if one takes U.S. electric power costs, thermal or hydroelectric, and recognizes local variations in Japan (electricity in Nagasaki, for example, is not low cost), there would seem to be an opportunity for solar space heating where electric space heating is used.[32] Data on the extent of electric space heating are simply not available elsewhere, but the evidence for United States and Japan suggests that other locations can be found where it is in use and solar energy offers a lower cost alternative (or supplement) with the same "clean heat" properties.

At the primitive end of the scale, firewood continues to be an important fuel, particularly in rural areas. The above mentioned Japanese study attributed 82.5% of the calorific value of energy consumed in rural areas of Japan (1962) to firewood.[33] In rural India (1956), 42% of total energy came from firewood and 35% from the combustion of cow dung.[34] Vegetable fuels in rural areas are much cheaper than shown for the cities in Table 5. Moreover, standards of living are generally so low that one cannot recommend solar heat with its high capital investment.

The mineral fuels, coal, oil and gas account for the large volume of space heat in those locations having a high enough demand and a high enough standard of living to consider solar heating. A quick review of Table 5 will suggest the possibilities, though, as pointed out above, the figures are not to be interpreted with great precision.

In the two Australian cities, Melbourne and Sydney, the costs of solar house heating come close enough to the (rather high) costs of conventional fuels so that the calculation in individual cases could well depend on the specific installation at hand, including especially the size of house to be heated. Lisbon offers a good prospect for solar house heating. The gas price quoted there, as indicated in a footnote to Table 5, is for cooking only. Such prices have been included for what they imply of the possibility of subsequent expansion of gas facilities to include space heating. When account is taken of the "cooking-only" restriction on four of the figures for gas, it will be apparent that six of our fifteen overseas cities do not have prices quoted for either gas or oil, implying that usage of these fuels was not sufficiently widespread to include data applicable to

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[32] *Id.* at Table 7, shows an upward trend in electricity for space heat from 12.6% in 1958 to 20.1% in 1963 of all household electricity consumption.

[33] *Id.* at Table 4.

[34] National Council of Applied Economic Research, Domestic Fuels in India 15 (Asia Publishing House, New Delhi, 1959).

space heating. In only the two Australian cities are prices for both gas and oil listed. Coal is available everywhere else.

In Madrid and Buenos Aires, special access to conventional fuels undercuts what would otherwise be promising solar space heat locations. The relatively colder climates of Paris, Belgrade and Berlin likewise seem to have sufficient access to low cost conventional fuels, though the result in Belgrade is in doubt, as previously noted, due to possible inapplicability of the official exchange rate used in the calculations. The Mediterranean cities, Rome, Taranto, Athens and Trieste, all suffer from high cost coal, and despite relatively high cost solar space heat, appear to be good prospects for the latter, except and unless gas at cooking rates is made available for space heat. The two Japanese cities, Nagasaki and Tokyo, have more reasonable coal costs but relatively high gas costs. They do not have very good climates for solar space heat, but over the longer run may well find solar heat promising as a result of limited domestic supplies of conventional fuels.[35] New Delhi offers prospects for some solar space heating applications, more because of extremely high costs of conventional fuels than because of its suitability for solar energy. Even then, the New Delhi climate has an uncharacteristically high demand for space heat as compared with other parts of the solar-rich underdeveloped world. The importance of high demand in bringing down solar space heat costs has been demonstrated in this study.

CONCLUSIONS

The conclusions of the present study bear on two principal topics: (1) optimal design and (2) competitive status of solar space heat systems. Both are adequately described in the text. To summarize them here would be repetitious. Useful perspectives can be obtained, however, by comparing optimal design as developed herein with prior design practices. The contrast between prior design practice and that produced by the computer analysis explains in part the judgment of competitive status already presented in Tables 2, 5 and related discussion. The cost analysis, based on questionnaire results and other information, gives the remainder of the explanation.

Probably the most unexpected design result is the unimportance of collector heat capacity. This result makes possible the selection of materials and construction methods to minimize collector costs

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[35] M. Sapir & S. J. Van Hyning, The Outlook for Nuclear Power in Japan 13-15 (National Planning Association, Washington, 1956); and Japanese Economic Planning Agency, New Long Range Economic Plan of Japan 90-92 (Japan Times, Tokyo, 1961).

without great concern for total weight and bulk of the collector insofar as thermal inertia is involved. Heat losses due to capacity effects in the practical range of collector design are simply not important over the long term.

In the selection of collector tilt, prevailing practice has been to provide a tilt about 10° greater than the latitude. The optimum is closer to 15° above latitude, but fortunately for architectural considerations, collector efficiency is not sensitive to tilt within a range of ten degrees on either side of the optimum.

Most experimental space heat collectors have used double glass covers. The present optimization studies largely verify this practice, though in the warmest climates (Miami and Phoenix) single glazing yields lowest annual cost.

The proper size of the heat storage unit in relation to collector size has been found to be in the range anticipated by most, but not all, designers. The results herein show optimum storage capacity to vary from one to three typical winter days' solar supply, depending on the site. This is consistent with most previous practice, but not with the view, occasionally expressed, that it is economical to provide sufficient storage to meet demand throughout a cloudy period of a week or more.

With respect to collector size as related to house size or heating demand, the optimization results give a rational basis for design where virtually nothing had been available before. Speculations as to an "economical" collector size or an "optimum" fraction of heat to be solar supplied (for assumed prices of solar collector and auxiliary heat) have covered a wide range, commonly two-thirds to nine-tenths of the load being carried by solar if considerable collector cost reductions could be anticipated. The optimum designs found in this study are within or slightly below the expected range. For specific locations, a relatively exact optimal size can be found using marginal costs of conventional fuels expected to prevail over the heating system lifetime. Inspection of Figures 10 through 13 has shown that attempts to achieve very high capacity solar heat as a percent of total heat supplied can result in extraordinarily high solar heat costs, particularly in cold climates.

The optimization results have materially clarified the applicability of solar heating in various climates. The conclusions had been at least qualitatively foreseen by most investigators, but the much higher cost of solar heating in mild climates was not anticipated. The results verify the view that the most suitable areas for solar house heating are those with moderate to severe heating requirements, abundant sunshine, and ideally, heat needs throughout most of the year. Solar heating results, as computed for optimum designs, bear out the belief that in suitable climates, solar space heat could be available even today at costs below those of electric space heat in most places where the latter is in use. If further collector manufacturing improvements are achieved as hypothesized, and as seems probable in view of the history of productivity change, solar heat may well fall within the range of heat costs from higher priced mineral fuels.

The geographic distribution of economic advantage for solar house heat has been found from a juxtaposition of design, cost and demand. A general formulation, equation (15), has been derived for the prediction of solar heat output wherever temperature and solar radiation data are available. The demand for space heat can likewise be projected from temperature records and standard house heating characteristics, represented by heat demand per degree-day. The method involves relatively small standard errors and summarizes the effects of hour-by-hour interdependencies of climate and design.

A major problem in the application of derived results to locations outside the United States has been in the interpretation of cost data. Costs of producing solar heat equipment in the United States were derived from a component-by-component analysis, and are not likely to be the same in other countries. Secondly, conventional heat costs were estimated from prices that included other household uses of fuels.

More fundamentally, there is no assurance that conventional fuel prices correspond to true fuel costs where subsidies or taxes change those costs. The same observation is as valid in the United States as elsewhere, though one must recognize that distorting effects of public aids or taxes are a matter of degree and the energy sector is not the only part of the U.S. economy subject to them. As a practical matter it is well, therefore, to focus attention only on the special treatment given the fuel industries. The pros and cons of tax policies and import quotas become relevant, though they cannot be reviewed here.

A similar observation applies to conservation. The cost comparisons are based on market values, which reflect earnings from natural resource exploitation discounted at rates acceptable to present-day property owners, at least within the context of the previous paragraph and prevailing prorationing practices. If public control were extended seriously to restrain present consumption of liquid fuels on behalf of future consumption, prices would increase and solar space heat would be the more attractive. The authors neither advocate nor oppose such a policy but point out that costs as found are not immutable. They reflect scarcity values and also run the spectrum of public policies currently in force.

The introduction of a new technology such as solar space heat is beset by inertia in the spread of knowledge of its capabilities, plus inertia in the development of a supporting industrial infrastructure for supply, servicing and replacement. Mass production economies at the manufacturing level and "know-how" improvements at all levels can be expected—once markets begin to grow. Industrial take-off is slow and may be long postponed. But for solar space heat, a potential is there. This is the most important conclusion.

Table of Nomenclature

A Architectural index of space heating requirements in BTU per square foot of collector per degree-hour.

CC Cloud cover expressed in tenths of sky covered.

D Total heat demand, BTU per square foot of collector per hour.

DD Degree days.

DEF Heat deficit, BTU per square foot of collector per hour.

DH Degree hours.

E Heat transferred to storage after all heat losses, BTU per square foot of collector per hour.

HW Hot water demand, BTU per square foot of collector per hour.

h_s Heat transfer coefficient of insulation on storage, BTU per square foot of collector per hour per degree temperature difference between storage temperature and the room temperature of 70°, for given thickness of insulation and given K values.

I Ratio: Intensity of daily solar radiation on collector surface to intensity of daily solar radiation on a horizontal surface.

J Heat capacity of collector, BTU per degree Fahrenheit per square foot of collector.

K Heat capacity of storage, BTU per degree Fahrenheit per square foot of collector.

L Latitude, degrees.

L_o Optical heat loss, BTU per square foot of collector per hour.

L_s Thermal heat loss from storage, BTU per square foot of collector per hour.

L_T Thermal heat loss from collector, BTU per square foot of collector per hour.

N Number of glass plates on collector.

OVR Heat overflow, BTU per square foot of collector per hour.

Q Heat content of collector, BTU per square foot of collector in current hour.

QP Heat content of collector, BTU per square foot of collector in previous hour.

S Heat in storage, BTU per square foot of collector in current hour, measured above zero degrees Fahrenheit.

SP Heat in storage, BTU per square foot of collector in previous hour, measured above zero degrees Fahrenheit.

t_a Atmospheric temperature, degrees Fahrenheit.

t_c Collector temperature, degrees Fahrenheit.

t_s Storage temperature, degrees Fahrenheit.

U Modified climate index, degree hours plus one BTU of hot water demand per square foot of collector per hour.

X_c Solar radiation received on surface of collector, BTU per square foot of collector per hour.

X_h Solar radiation received on a horizontal surface, BTU per square foot of horizontal surface per hour.

Y Solar heat after optical loss, BTU per square foot of collector per hour.

Z Heat delivered from storage to living quarters and/or hot water, BTU per square foot of collector area per hour.

α Angle of solar radiation on a horizontal surface, degrees.

β Angle of collector tilt from the horizontal, degrees.

Υ Ratio: Current solar declination to maximum solar declination.