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SOLAR MODULATION OF GALACTIC COSMIC RAYS, 2

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The modulation of galactic cosmic rays in the interplanetary medium can be discussed in terms of a spherically symmetric model in which particles undergo convection, diffusion, and energy changes resulting from the expansion of the solar wind. In this model a Fokker-Planck equation determines, in principle, the particle number density when the solar wind speed, the diffusion coefficient, and the interstellar cosmic ray spectrum are specified (Parker, 1965, 1966; Gleeson and Axford, 1967, 1968a, b). This equation, however, is difficult to solve analytically, and, indeed, analytic solutions valid at all energies with realistic forms of the diffusion coefficient have not been obtained (see the first paper in this series, (Fisk and Axford, 1969)). It can be solved numerically, and in this Letter we will outline an appropriate numerical technique, and present as examples of the use of this technique some numerical solutions that provide reasonable fits to observed spectra of protons and helium

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ions with the assumption of quite simple forms for the diffusion coefficient and unmodulated spectra.

In the quasi-steady, spherically symmetric model of the interplanetary medium discussed by Parker (1965) and by Gleeson and Axford (1967), the cosmic ray number density U(r,T), per unit interval of kinetic energy T, satisfies a Fokker-Planck equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 VU) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha TU) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa \frac{\partial U}{\partial r}\right) \quad (1)$$

The corresponding streaming S(r,T) (radial current density), per unit interval of kinetic energy, is determined by (Gleeson and Axford, 1967):

$$S = VU - \kappa\frac{\partial U}{\partial r} - \frac{V}{3}\frac{\partial}{\partial T}(\alpha TU) \quad (2)$$

Here $\kappa(r,T)$ is the particle diffusion coefficient, V(r) is the solar wind speed, and $\alpha(T) = (T+2T_o)/(T + T_o)$ with T_o the rest energy of a particle.

Equation (1) is a parabolic partial differential equation, and with certain modifications can be solved using the numerical techniques which have been developed for dealing with simple space-time diffusion equations. Kinetic energy in equation (1) is the analogue of time in the simple diffusion equation. We will use here the Crank-Nicholson implicit finite difference technique (Diaz, 1958), which has the advantage over other techniques in that in general it determines solutions that are unconditionally stable. In this Letter we

will outline only the required modifications to equation (1). For details on the use of the Crank-Nicholson technique the reader is referred to the discussion by Diaz (1958).

The Crank-Nicholson technique can be used in determining the number density in the region $0 \leq r \leq R$, where R is the outer boundary of the modulating region, e.g. R could be some radial distance where the modulation becomes negligibly small. We assume that we know $\kappa(r,T)$, $V(r)$, and also the unmodulated spectrum, $U(r,T) = U_o(T)$ at $r = R$. Further, we must specify a boundary condition at the origin ($r = 0$), and also an "initial" condition, i.e., $U(r,T)$ at a given value of T for all r, $(0 \leq r < R)$. In specifying the former condition we could require that $r^2 S \to 0$ as $r \to 0$ (i.e., no sources or sinks at the origin) since we are concerned here only with galactic cosmic rays. However, this condition is difficult to treat numerically, and it is considerably simpler merely to note that for reasonable solutions for U, $r^2 U \to 0$ as $r \to 0$. We then rewrite equation (1) in terms of the variable $u(r,T) = r^2 U(r,T)$:

$$\frac{\partial}{\partial r}(Vu) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha T u) = \frac{\partial}{\partial r}\left(\kappa \frac{\partial u}{\partial r}\right) - 2\frac{\partial}{\partial r}\left(\frac{\kappa u}{r}\right) \qquad (3)$$

and solve this new equation subject to the boundary conditions $u(r,T) = 0$ at $r = 0$, and $u(r,T) = R^2 U_o(T)$ at $r = R$. $u(r,T)$ in turn determines $U(r,T)$, but note that we will be unable to obtain the number density accurately in the immediate vicinity of the origin. To specify the "initial" condition appropriate for equation (3) (i.e. $u(r,T)$ at a given value

of T for all r ($0 \leq r < R$), we note that at very large energies the effects of modulation can be neglected at all values of r. The appropriate initial condition is then $u(r,T) = r^2 U_o(T)$ at $T = T'$, ($0 \leq r < R$), where T' is a sufficiently large energy. Clearly, the choice for T' is dependent on the choice for the diffusion coefficient, but with realistic values of κ, $T' \sim$ 60 GeV/nucleon should be adequate. With these forms for the boundary and "initial" conditions, the Crank-Nicholson technique can be used in determining u(r,T) and hence U(r,T), beginning with u(r,T) at $T = T'$, ($0 \leq r \leq R$), and then calculating u(r,T) at lower energies in a step-by-step manner.

There is an alternative method for specifying the "initial" condition appropriate for equation (3). Gleeson and Axford (1968b) and Fisk and Axford (1969) have shown that the streaming S can be neglected in equation (3) when the particles undergo relatively little modulation, i.e., when the parameter $\mathcal{R} = \tilde{V}\tilde{r}/\tilde{\kappa} \leq 1$ (here the tilde denotes characteristic value). The resulting approximate equation,

$$VU - \kappa \frac{\partial U}{\partial r} - \frac{V}{3} \frac{\partial}{\partial T}(\alpha T U) \simeq 0 \qquad (4)$$

can often be easily solved (see Gleeson and Axford, 1968b), and its solutions, $U_a(r,T)$, used to specify the initial condition: $u(r,T) = r^2 U_a(r,T)$ at $T = T''$, ($0 \leq r < R$) where T'' is some energy at which $\mathcal{R} \leq 1$ for all r. In practice we anticipate that at earth $\mathcal{R} \leq 1$ at energies above a few hundred MeV/nucleon, and accordingly equation (4) can be used to determine an ac-

curate approximation to the number density at intermediate and high energies. Throughout the modulating region ($0 \leq r \leq R$), $R \leq 1$ at energies above, say, 1 GeV/nucleon. Clearly, the advantage in using this "initial" condition over the one described above is that here the computations are begun a much lower energy ($T'' \sim 1$ GeV/nucleon vs. $T' \sim 60$ GeV/nucleon), and hence considerable computational time can be saved in determining the modulation at energies $T \sim 0.01 - 1.0$ GeV/nucleon.

On assuming forms for U_o, κ, and V, the numerical technique outlined here can be used to compute the number densities and hence the intensities at earth for different species of particles. e.g., for protons and helium ions. (Note: intensity $J = vU/4\pi$, where v is particle speed). These predicted intensities can then be compared with observed spectra, thereby testing both the assumed forms for U_o, κ, and V, and also the theory itself. There is, however, considerable latitude available to us in the choice of the forms for U_o and κ to be used, and hence no definite conclusions will be possible without an exhaustive study of the consequences of all possible forms. We will present here only one set of possible numerical solutions for the intensities of protons and helium ions, which we have obtained using simple but realistic forms for U_o, κ, and V.

We assume that the unmodulated spectra of both protons and helium ions is described by a power law in total energy,

$U_o(T) = A\ (T+T_o)^{-2.65}$. The constant A is chosen so that the unmodulated spectra match the observed spectra at high energies where the effects of modulation can be neglected. In choosing the form for the diffusion coefficient to be used it is important to remember that at a given value of the radial distance acceptable diffusion coefficients must be expressible in the form particle velocity times a function of rigidity, where this function is the same for all species of particles. Hence, expressed in the form $\kappa = \beta\kappa_1(P,r)$, where $\beta = v/c$ (c is the speed of light) and P is particle rigidity, our choice for the diffusion coefficient must be the same for both protons and helium ions. We assume here that $\kappa = \kappa_o\beta P\exp(r/r_o)$, where κ_o is a constant that can be adjusted so that the predicted and observed intensities are in the best possible agreement. The assumption that $\kappa \propto \beta P$ has been predicted and to some extent confirmed (Jokipii, 1966; Gloeckler and Jokipii, 1966; O'Gallagher, 1967), and the form $\kappa \propto \exp(r/r_o)$ is a useful one since then the dependence of the modulation on radial distance is described by a single parameter, a fall-off distance r_o which we take to be 1 A.U. in agreement with the findings of Gleeson and Axford (1968b). We take the boundary of the modulating region to be at R = 25 A.U. (the modulation at this point is negligibly small), and we take V the solar wind speed to be constant at 400 km. sec. $^{-1}$.

In figure 1 we compare a numerical solution for the intensity, determined using the above forms for U_o, κ, and V, with the available observations of proton intensities during solar

minimum conditions in 1965. In figure 2 we show the corresponding helium data and numerical solution. κ_o is taken to be $\kappa_o = 8\times10^{17}$ cm.^{2}sec.$^{-1}$ MV^{-1}, e.g., at r = 1 A.U. and T = 6 GeV/nucleon, $\kappa = 1.38\times10^{22}$ cm.^{2}sec.$^{-1}$ for protons, and $\kappa = 2.75\times10^{22}$ cm.^{2}sec.$^{-1}$ for helium ions. As is evident in the figures, the predicted intensities agree reasonably well with the observations, except at low energies where neither of predicted curves is a particularly good fit.

In figures 1 and 2 we have also plotted the intensities determined by equation (4) for the forms for U_o, κ, and V considered here. As we indicated above, we anticipate that at earth equation (4) determines an accurate approximation to the number density and hence the intensity at energies above a few hundred MeV/nucleon. This equation is known as the "force-field" equation since particles behave in this approximation as if they were modulated by a heliocentric force field (see Gleeson and Axford, 1968b). Indeed, the force-field solutions shown in figures 1 and 2 are scarcely distinguishable from the corresponding numerical solutions down to energies of about 150 MeV/nucleon. In figures 1 and 2 we have also shown the intensities determined by the simple convection-diffusion equation,

$$VU = \kappa\frac{\partial U}{\partial r} , \qquad (5)$$

which was the basic equation used in earlier theories that neglected the effects of particle energy changes (Parker, 1963).

The differences between the simple convection-diffusion solutions and the corresponding numerical solutions at energies below ~ 400 MeV/nucleon clearly indicate the inadequacies of the earlier simple theory.

In summary, we have outlined in the Letter a numerical technique which can be used to solve the Fokker-Planck equation, equation (1), and we have presented some numerical solutions that provided reasonable fits to observed spectra of protons and helium ions with the assumption of quite simple forms of the unmodulated spectra (U_o) and the diffusion coefficient (κ). We should not conclude from this, however, that we have in fact found the best or even the most likely forms for U_o and κ since other combinations of these quantities will probably yield spectra in better agreement with the observations. Rather, the numerical solutions presented here should be considered only as examples illustrating the use of the numerical technique.

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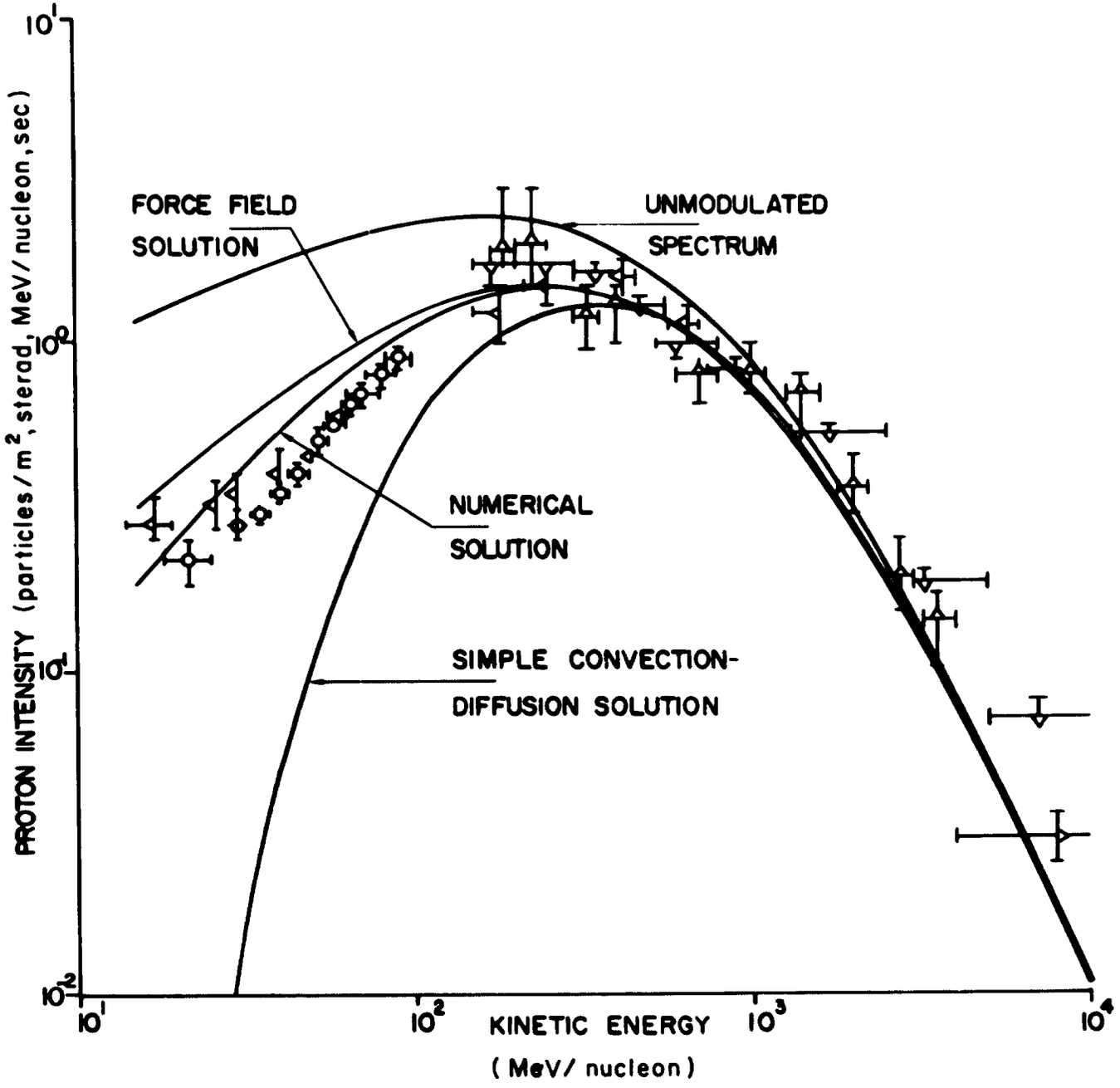
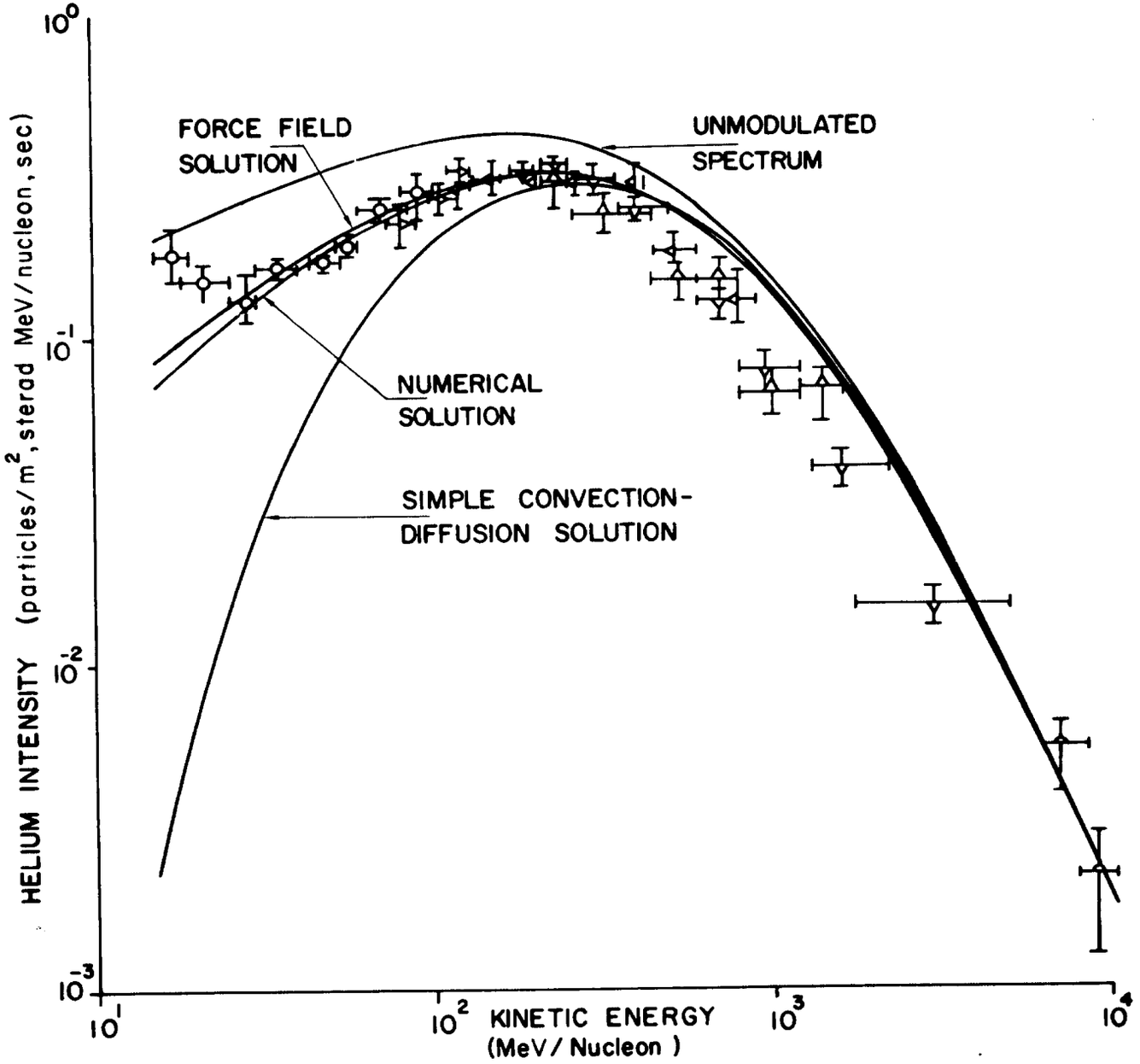
FIGURE 1

A comparison between a numerical solution for the proton intensity, and the observations of proton intensities during solar minimum conditions in 1965. The intensities determined by the force-field equation (equation (4)) and the simple convection-diffusion equation (equation (5)), using the same forms of U_o, κ and V as in the numerical solution, are also shown. The unmodulated spectrum shown in this figure is a plot of the intensity corresponding to the unmodulated number density; $J = vU_o/4\pi$, where U_o is given by a power law in total energy. The data points are taken from a paper by Gloeckler and Jokipii (1967); the symbol O is used to represent the observations of Fan, et al. (1966); the symbol $\triangleleft$ represents the observations of Balasubrahmanyan, et al. (1966a, b); the symbol Δ represents the observations of Waddington and Freier (1966); the symbol ∇ represents the observations of Ormes and Webber (1966); and the symbol $\triangleright$, the observations of McDonald (1958).

FIGURE 2

A comparison between a numerical solution for the helium ion intensity, and the observations of helium ion intensities during solar minimum conditions in 1965. The intensities determined by the force-field equation (equation (4)) and the simple convection-diffusion equation (equation (5)), using the same forms of U_o, κ, and V as in the numerical solution, are also shown. The unmodulated spectrum shown in this figure is a plot of the intensity corresponding to the unmodulated number density; $J = vU_o/4\pi$, where U_o is given by a power law in total energy. The data points are taken from a paper by Gloeckler and Jokipii (1967); the symbol O is used to represent the observations of Fan, et al. (1966); the symbol $\triangleleft$ represents the observations of Balasubrahmanyan, et al. (1966a, b); the symbol Δ represents the observations of Freier and Waddington (1965); the symbol ∇ represents the observations of Ormes and Webber (1966); the symbol $\triangleright$ represents the observations of Hofmann and Winckler (1966); and the symbol $\frown$, the observations of Anand, et al. (1966).


FORCE FIELD
SOLUTION
UNMODULATED
SPECTRUM
NUMERICAL
SOLUTION
SIMPLE CONVECTION-
DIFFUSION SOLUTION
PROTON INTENSITY (particles/m², sterad, MeV/nucleon, sec)
KINETIC ENERGY
(MeV/nucleon)


FORCE FIELD SOLUTION
UNMODULATED SPECTRUM
NUMERICAL SOLUTION
SIMPLE CONVECTION-DIFFUSION SOLUTION
HELIUM INTENSITY (particles/m², sterad MeV/nucleon, sec)
KINETIC ENERGY (MeV/Nucleon)
10⁰
10⁻¹
10⁻²
10⁻³
10¹
10²
10³
10⁴