Solid-state traveling-wave amplification in the collisionless regime*

Avraham Gover, Keith H. Burrell, and Amnon Yariv

California Institute of Technology, Pasadena, California 91109
(Received 20 May 1974; in final form 8 August 1974)

Traveling-wave interaction between flowing plasma and electromagnetic wave supported by external slow wave structure is analyzed in the collisionless regime. The plasma is described with the collisionless Boltzmann equation and the interaction is analyzed using the coupled-mode approach. The analysis is applied to a plasma consisting of drifiting charge carriers in a solid. Illustrative examples demonstrate interaction leading to wave growth for frequencies in the far-infrared regime. It is suggested that experimental measurement of wave growth or damping due to this effect can provide information about the velocity distribution of drifting carriers in the solid.

I. INTRODUCTION

As Landau[1] first demonstrated, plasma waves in a finite temperature plasma have features due to collective particle behavior which cannot be obtained from the macroscopic kinetic equations, but which must be derived from the Boltzmann equation. Landau showed that the interaction of a plasma wave with particles traveling with velocities near its phase velocity could lead to damping of the wave even if the plasma is collisionless (relaxation time $\tau \to \infty$).

Using Landau's results, it is possible to prove[2] that no plasma wave is unstable in a plasma whose velocity distribution function has only a single maximum. However, one can show that, if the plasma wave interacts with an electromagnetic wave supported by an external slow wave structure, the instability conditions can be met for a plasma carrying a steady current. The goal of this paper is to analyze a simplified model and demonstrate this effect.

Traveling-wave amplification of an electromagnetic wave through interaction with charged particles in motion is a familiar phenomenon. It has been successfully analyzed and implemented in the microwave traveling-wave tube[3] and also in the solid-state traveling-wave amplifier.[4-8] In the first case, the charged particles are in a form of a monoenergetic beam of electrons moving in a vacuum, while in the second case, they are the drifting charge carriers in a semiconductor.

In both cases, the charged particles are usually described by the moment equations. This approach is valid for different reasons in the two systems. The TWT electron beam is nearly monoenergetic; for such a beam, the Boltzmann equation reduces immediately to the moment equation. Even though the carriers in the solid-state amplifier have a finite temperature, collisions are frequent enough that the effects discussed by Landau for a collisionless plasma are unimportant and the moment equations are again a good approximation.

The condition for the onset of electromagnetic-wave amplification is similar in both cases. Growth occurs when a characteristic particle velocity v_0 exceeds the phase velocity of the electromagnetic slow wave component. For the TWT, v_0 is the common velocity of the particles in the beam, while it is the average drift velocity of the carriers for the solid-state amplifier. For the latter, if $v_0 \ll c$ the condition for amplification is $v_0 > v_{p1} \approx \omega L/2\pi$ (for first harmonic operation), where v_{p1} is the phase velocity of the first-order electromagnetic space harmonic, ω is the radian frequency of the electromagnetic wave, and L is the period of the waveguide corrugations (slow wave circuit).

If ω is high enough, the drifting carriers in the solid-state amplifier are a finite-temperature collisionless plasma ($\omega\tau \gg 1$). The existing analysis of traveling-wave interactions is not applicable to this case, since the moment equations used in it cannot correctly describe the Landau waves that propagate in a collisionless plasma. We intend, therefore, in this paper, to extend the analysis of traveling-wave interaction to this regime, employing the Boltzmann equation to describe the drifting plasma.

This work is motivated by recent work on periodically perturbed waveguides[7,9] and the implementation of solid-state traveling-wave amplifiers,[8] which has indicated that devices operating in the collisionless regime may be feasible and even more efficient than those in the collision-dominated regime. Corrugated semiconductor surfaces with periods of the order of 1000 Å have been produced using ion machining techniques.[9] The development of uv, x-ray, and electron resist lithography could allow the production of surface corrugation periods of the order of 100 Å. With carrier drift velocities as high as 2×10^7 cm/sec available in semiconductors, the gain condition $v_0 > \omega L/2\pi$ would predict amplifying and oscillating devices in the far infrared ($10^{13} \lesssim \omega \lesssim 10^{14}$ rad/sec). Since relaxation times can be of the order of 10^{-12} sec, such devices would thus operate in the collisionless regime.

II. THE DISPERSION RELATION

To analyze the interaction between the plasma waves and the elctromagnetic slow wave components, we use a one-dimensional model and a coupled-mode approach.[3,4,8] First, we calculate the plasma disturbance caused by an electromagnetic-wave field $E_c \propto \exp[i(\omega t - \beta z)]$, which is supported by an external slow wave structure. Second, the field induced in the slow wave structure by a plasma wave is found. Self-consistent substitution of the results of the two derivations then yields the dispersion relation.

The plasma response to the external field is obtained from the linearized Boltzmann equation. We employ the electrostatic approximation; hence, Maxwell's equations reduce to Poisson's equation. If we assume negative charge carriers, we have

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)f_1(z,u,t)=\frac{e}{m}E(z,t)\frac{\partial}{\partial u}f_0(u), \tag{1}$$

$$\frac{\partial}{\partial z}E_p(z,t)=-\frac{e}{\epsilon}\int_{\infty}^{+\infty}du f_1(z,u,t). \tag{2}$$

Here, z is the coordinate in the direction of wave propagation, u is the z component of the particle velocity, m is the charge-carrier effective mass, and ϵ is the dielectric constant, excluding the plasma contribution. The zero- and first-order velocity distribution functions, f_0 and f_1, respectively, are calculated from the three-dimensional distribution functions by integrating over the transverse velocities. $E(z,t)$ is the total electric field, while $E_p(z,t)$ is the field due to space charge; thus,

$$E(z,t)=E_p(z,t)+E_c(z,t), \tag{3}$$

where E_c is the external field induced by the slow wave circuit.

We may solve for f_1 and $E(z,t)$ in terms of $E_c(z,t)$ by using Fourier transforms in z. To avoid the difficulties pointed out by Landau,[1] we chose to add a small negative imaginary part to the frequency, i.e., $\omega\to\omega-i\eta$. This corresponds to turning on the disturbance in the far past, and will allow us to define uniquely an integral that is otherwise ambiguous. At the end of the analysis, we will set $\eta=0$.

The Fourier transform of a function $h(z)$ will be written

$$h(\beta)=\int_{-\infty}^{+\infty}dz\,h(z)\exp(i\beta z).$$

In what follows, we will suppress the common exponential time dependence.

Using these definitions, the transforms of the solutions to Eqs. (1)–(3) can be written

$$E(\beta)=\epsilon_p(\beta,\omega)E_c(\beta) \tag{4}$$

$$f_1(\beta,u)=\frac{ie}{m\beta}\frac{d}{du}f_0(u)\left(u-\frac{\omega}{\beta}+\frac{i\eta}{\beta}\right)^{-1}\epsilon_p(\omega,\beta)E_c(\beta), \tag{5}$$

where

$$\epsilon_p(\beta,\omega)\equiv\left[1-\frac{\omega_p^2}{n_0\beta}\int_{-\infty}^{+\infty}du\frac{df_0}{du}\left(u-\frac{\omega}{\beta}+\frac{i\eta}{\beta}\right)^{-1}\right]^{-1}, \tag{6}$$

and where n_0 is the unperturbed carrier density and $\omega_p=(n_0e^2/m\epsilon)^{1/2}$ is the plasma frequency. The integral in Eq. (6) would have been undefined if $\eta=0$.

For later use, we will also need the z-directed current density. Since $J_z(\beta)=-e\int_{-\infty}^{+\infty}du\,uf_1(\beta,u)$,

$$J_z(\beta)=-i\frac{e^2\omega}{m\beta^2}\epsilon_p(\beta,\omega)E_c(\beta)\int_{-\infty}^{+\infty}du\frac{df_0}{du}\left(u-\frac{\omega}{\beta}+\frac{i\eta}{\beta}\right)^{-1}. \tag{7}$$

We have used the identities $u/(u+\alpha)=1-\alpha/(u+\alpha)$ and $\int_{-\infty}^{+\infty}du\,(df_0/du)=0$ to put the integral in Eq. (7) in its present form. That $f_0(u)$ should vanish as $|u|\to\infty$ is necessary for any reasonable distribution function.

To cast our results in a slightly more universal form, we define the following moments of the unperturbed distribution function: the carrier number density $n_0=\int_{-\infty}^{+\infty}f_0(u)\,du$, the drift velocity $v_0=n_0^{-1}\int_{-\infty}^{+\infty}uf_0(u)\,du$, and the temperature T and thermal velocity $v_{\rm th}$, $\kappa T=\frac{1}{2}mv_{\rm th}^2=mn_0^{-1}\int_{-\infty}^{+\infty}(u-u_0)^2f_0(u)\,du$, where κ is the Boltzmann constant. Also, we define a normalized distribution function

$$f_0(u)=\frac{n_0}{v_{\rm th}}g\left(\frac{u-v_0}{v_{\rm th}}\right). \tag{8}$$

Using these definitions, we proceed to define the plasma dispersion function

$$G(\zeta)=\int_{-\infty}^{+\infty}\frac{g(x)\,dx}{x-\zeta},\quad \operatorname{Im}\zeta<0. \tag{9}$$

This definition disagrees with the usual convention in plasma physics, where $\operatorname{Im}\zeta>0$ is usually taken.

Combining these definitions with Eqs. (6) and (7), we find

$$J_z(\beta)=-i\frac{\epsilon\omega}{\beta^2}\frac{\frac{1}{2}k_{de}^2G'(\zeta)}{1-\frac{1}{2}(k_{de}^2/\beta^2)G'(\zeta)}E_c(\beta), \tag{10}$$

where $k_{de}=(n_0e^2/\epsilon kT)^{1/2}$ is the Debye wave number, $\zeta=-[v_0-(\omega-i\eta)/\beta]/v_{\rm th}$, and where primes denote differentiation with respect to the argument.

Since we now have the current induced by the slow electromagnetic component $E_c(\beta)$, the circle can be closed by combining this with the expression for the field $E_c(\beta)$ induced in the slow wave circuit by $J_z(\beta)$, which is given by the Pierce equation.[3] Using this,

$$E_c(\beta)=i\frac{\beta^2\beta_1K_1S}{\beta_1^2-\beta^2}J_z, \tag{11}$$

where S is the cross-sectional area of the interaction region, K_1 is the interaction impedance between the slow electromagnetic wave and the plasma wave, and $\beta_1=2\pi/L$. L is the period of the slow wave structure.

The dispersion relation is obtained by requiring Eqs. (10) and (11) to be self-consistent. This yields

$$\frac{\epsilon\omega\beta_1K_1S}{\beta_1^2-\beta^2}\frac{\frac{1}{2}k_{de}^2G'(\zeta)}{1-\frac{1}{2}(k_{de}^2/\beta^2)G'(\zeta)}=1. \tag{12}$$

III. APPROXIMATE SOLUTION OF THE DISPERSION RELATION

In general, one needs both a knowledge of the zero-order distribution function and a reasonable amount of computer calculation to solve Eq. (12) for β. However, if K_1 is small, it is reasonable to assume that the solution to the coupled system will not differ substantially from that for $K_1=0$, when there is no interaction. The $K_1=0$ modes are just the electromagnetic modes $\beta=\pm\beta_1$, and the hot plasma modes that satisfy $D(\beta)\equiv1-(k_{de}^2/2\beta^2)G'(\zeta)=0$.

Since we are interested in the electromagnetic wave, for small K_1 we can expand Eq. (12) about $\beta=\beta_1$, keeping only first-order terms. Setting $\beta=\beta_1+\Delta\beta$, we find

$$\Delta\beta=-\frac{\alpha G'(\zeta_1)}{D(\beta_1)}, \tag{13}$$

where $\zeta_1 = -(v_0 - \omega/\beta_1)v_{\mathrm{th}}^{-1}$, $\alpha = \frac{1}{4}\epsilon\omega k_{de}^2 K_1 S$, and where we have finally taken the limit $\eta \to 0+$.

Because β_1 is real, we may write the real and imaginary parts of β as

$$\mathrm{Im}\beta = -\frac{\alpha\,\mathrm{Im}G'(\zeta_1)}{|D(\beta_1)|^2} \qquad (14)$$

$$\mathrm{Re}\beta = \beta_1 - \frac{\alpha}{|D(\beta_1)|^2}\left(\mathrm{Re}G'(\zeta_1) - \frac{1}{2}\frac{k_{de}^2}{\beta_1^2}|G'(\zeta_1)|^2\right). \qquad (15)$$

As a consequence of taking $\eta \to 0+$,

$$G'(\zeta_1) = P\int_{-\infty}^{+\infty}\frac{g'(x)}{x-\zeta_1}\,dx - i\pi g'(\zeta_1).$$

Hence, if the zero-order distribution function is such that $g'(\zeta_1) > 0$, the system can support growing modes ($\mathrm{Im}\beta > 0$) and amplification is possible. If it is also true that $g'(0) = 0$, then the gain criterion becomes $v_0 > \omega/\beta_1$, which is the condition obtained from the moment equations. Consequently, the latter condition is seen to be a special case, valid only for distributions $f_0(v)$ whose maximum occurs at the drift velocity v_0.

Two limits of Eq. (14) are of interest. If $k_{de}^2/\beta_1^2 \ll 1$, we have

$$\mathrm{Im}\beta \approx \pi\alpha g'(\zeta_1). \qquad (16)$$

On the other hand, if $k_{de}^2/\beta^2 \gtrsim 1$, it is possible to have $\mathrm{Re}D(\beta_1) \approx 0$, which yields

$$\mathrm{Im}\beta \approx \frac{\alpha}{\pi}\frac{\beta_1^2}{k_{de}^2}\frac{1}{g'(\zeta_1)}. \qquad (17)$$

Physically, Eq. (17) means that maximum gain results when β_1 comes as close as possible to satisfying $D(\beta_1) = 0$, which implies a good phase match between electromagnetic and plasma waves. As Landau showed,[1] no nonzero real β can satisfy the plasma-wave dispersion relation exactly, but the gain increases the closer β_1 comes to satisfying $D(\beta) = 0$.

If the denominator in Eq. (17) is too small, one wonders whether our first-order approximation to the dispersion relation, Eq. (12), is valid. To check it, let us extend the solution for $\Delta\beta$ to second order by expanding both $\beta_1^2 - \beta^2$ and $1 - (k_{de}^2/2\beta^2)G'(\zeta)$ to second order about $\beta = \beta_1$. We obtain

$$A\left(\frac{\Delta\beta}{\beta_1}\right)^2 + B\left(\frac{\Delta\beta}{\beta_1}\right) + C = 0, \qquad (18)$$

where $A = 2 + (k_{de}^2\omega/2\beta_1^2 v_{\mathrm{th}})G''(\zeta_1)$, $B = D(\beta_1)$, and $C = (\alpha/\beta_1)G'(\zeta_1)$.

The solution to Eq. (18) is

$$\frac{\Delta\beta}{\beta_1} = \frac{-B \pm (B^2 - 4AC)^{1/2}}{2A}. \qquad (19)$$

If $4|AC| \ll |B|^2$, the smaller of the solutions in Eq. (19) reduces to Eq. (13). Accordingly, this is the condition for the validity of our previous approximation.

If the matching between the electromagnetic wave and the plasma wave is good, then $4|AC| \gg |B|^2$; thus, the amplifying solution to Eq. (19) is

$$\frac{\Delta\beta}{\beta_1} = i\left(\frac{C}{A}\right)^{1/2} \qquad (20)$$

IV. DISCUSSION AND EXAMPLES

The wave-particle interaction discussed here is of interest for its application to broad-band high-frequency solid-state amplifiers and oscillators. However, the formalism developed here could also provide a basis for a method of measuring the distribution function of a drifting finite-temperature plasma, e.g., a solid-state plasma.

If one can measure $\Delta\beta$ experimentally, then the derivative of the normalized distribution function $g'(\zeta_1)$ can be found from Eq. (13). However, since $\mathrm{Re}\Delta\beta$ may be difficult to measure, especially in a solid, a series of measurements of $\mathrm{Im}\Delta\beta$ for various ω, β, and k_{de} could furnish enough information to obtain $g'(\zeta_1)$ from Eq. (14) alone. If it is possible to work in parameter ranges where Eqs. (16) or (17) hold, then evaluation of $g'(\zeta_1)$ is straightforward when $\mathrm{Im}\Delta\beta$ is known. It follows that it may be possible to measure directly the velocity distribution function of drifting carriers in some regimes. Such a method will measure the distribution of velocities in the drift direction rather than the distribution of electron energies. It may be complementary to other existent methods to measure velocity distribution.[10]

In order to estimate the amount of gain available from the solid-state traveling-wave amplifier we will use a drifting Maxwellian as a preliminary crude model of the carriers' velocity distribution function. Consequently, we take $g(x) = \pi^{-1/2}\exp(-x^2)$. The dispersion function is then

$$G(\zeta) = \pi^{-1/2}\int_{-\infty}^{+\infty} dx\,[\exp(-x^2)/(x-\zeta)]; \quad \mathrm{Im}\zeta < 0.$$

The tabulated plasma dispersion function $Z(\zeta)$, which is defined by Fried and Conte,[11] has $\mathrm{Im}\zeta > 0$ in its definition. For real ζ, we have $G(\zeta) = Z^*(\zeta)$. For convenience, $G(\zeta)$ is plotted in Fig. 1 for real ζ.

Figure 1 shows that the present $G'(\zeta_1)$ can support gain ($\mathrm{Im}\beta > 0$) whenever $\zeta_1 < 0$. The drifting Maxwellian, of course, satisfies $g'(0) = 0$; thus, the gain condition $\zeta_1 < 0$ and the previously derived $v_0 > \omega/\beta_1$ are equivalent statements.

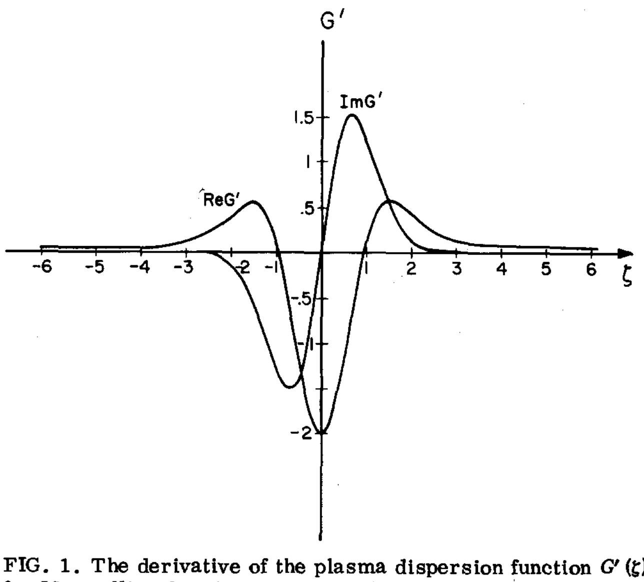

FIG. 1. The derivative of the plasma dispersion function $G'(\zeta)$ for Maxwellian distribution and real argument ζ.

As a concrete example, we chose the solid-state traveling-wave amplifier proposed and analyzed[7,8] previously. This consists of a dielectric thin-film waveguide with a surface corrugation of period L. A conducting layer of thickness $L/2\pi$ (the penetration depth of the first-order space harmonic) adjacent to the corrugated surface carries the current which interacts with the wave. If the index of refraction of the medium is $n=3.5$, the interaction impedance is given by[7]

$$K_1 = 0.09\frac{a^2}{Wc}\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2}\omega, \tag{21}$$

where $(\mu_0/\epsilon_0)^{1/2}$ is the impedance of free space, c the speed of light, a the corrugation depth, and W the device width. (The cross-sectional area is given by $S = WL/2\pi$.)

By choosing $\beta_1^2=4k_{de}^2$, $a=\frac{1}{2}L=\pi/\beta_1$, $v_0/v_{\text{th}}=1.85$, and $\omega/\beta_1 v_{\text{th}}=1.2$, we obtain $\zeta_1=-0.65$, which is near the minimum of $\text{Im}G'(\zeta_1)$ (see Fig. 1). Consequently, Eq. (14) yields a gain $g\equiv 2\,\text{Im}\beta=1.5\times10^{-21}\omega v_0$, where g, ω, and v_0 are in cm^{-1}, rad/sec, and cm/sec, respectively. The calculated gain is given in Table I for two proposed dielectrics, along with some of the physical conditions necessary to attain that gain. For this calculation, we assumed a frequency ω corresponding to a free-space wavelength of 100 μm.

As mentioned previously, higher gain should be available in the regime where Eq. (17) is valid. Here, we have $\text{Re}G'(\zeta_1)\approx 2\beta_1^2/k_{de}^2$, which requires $\zeta_1<-0.9$ to ensure that $\text{Re}G'(\zeta_1)>0$. For maximum gain, one should go to large $|\zeta_1|$, but since this would require very large drift velocities, one cannot increase $|\zeta_1|$ indefinitely.

Unfortunately, in this high-gain region, a necessary condition to satisfy $\text{Re}G'(\zeta_1)\approx 2\beta_1^2/k_{de}^2$ is $\beta_1^2 < k_{de}^2$. Under these conditions, it can be argued that screening by the charge carriers will limit the penetration of the first-order space harmonic into the plasma, thus invalidating Eq. (21). However, to get a crude idea of the gains available, we will retain Eq. (21), but assume that the penetration depth is k_{de}^{-1} instead of β_1^{-1}.

For the parameter values $a=\frac{1}{2}L$, $v_0/v_{\text{th}}=3.5$, $\omega/\beta_1 v_{\text{th}}=0.5$, and $\beta_1^2/k_{de}^2=0.07$ (i.e., $\zeta_1=-3$), Eq. (17) yields $g=4.8\times10^{-20}\omega v_0$. (Again, g, ω, and v_0 are in cm^{-1}, rad/sec, and cm/sec, respectively.) Results analogous to Table I are given in Table II. The frequency ω again corresponds to a free-space wavelength of 100 μm. If higher values of v_0/v_{th} are possible, the gain rises drastically, since it is proportional to $\exp[+(v_0-\omega/\beta_1)^2 v_{\text{th}}^2]$.

For measurements of the carrier's velocity distribution function, operation in the attenuation region is also of interest. Appreciable effects on the propagating waves are possible here under physical conditions that are more easily achievable. If $\omega/\beta_1 v_{\text{th}}=3$ and $v_0/v_{\text{th}}=0$ we obtain from Eq. (17) $g=-3.3\times10^{-20}\omega v_{\text{th}}$. Using an ω corresponding to 100 μm free-space wavelength, we find the results given in Table III. Note that the attenuation grows with frequency here, contrary to the behavior of free-carrier loss in the collision-dominated regime. This effect is different from the usual Landau damping because of the resonant effect of the slow wave structure.

TABLE I. Example of gain.

	Ge	GaAs
g	$0.34\ \text{cm}^{-1}$	$0.56\ \text{cm}^{-1}$
v_0	1.2×10^7 cm/sec	2×10^7 cm/sec
L	260 Å	435 Å
T	77 °K	31 °K
n_0	$6.6\times10^{16}\ \text{cm}^{-2}$	$9.5\times10^{15}\ \text{cm}^{-3}$

TABLE II. Example of gain in the regime $k_{de}^2/\beta_1^2 \gtrsim 1$.

	Ge	GaAs
g	$11\ \text{cm}^{-1}$	$18.2\ \text{cm}^{-1}$
v_0	1.2×10^7 cm/sec	2×10^7 cm/sec
L	57.3 Å	95.5 Å
T	21.5 °K	8.6 °K
n_0	$10^{19}\ \text{cm}^{-3}$	$3\times10^{18}\ \text{cm}^{-3}$

TABLE III. Example of attenuation.

	Ge	GaAs
g	$-8.1\ \text{cm}^{-1}$	$-21\ \text{cm}^{-1}$
v_0	0	0
L	870 Å	2300 Å
T	330 °K	300 °K
n_0	$6.5\times10^{16}\ \text{cm}^{-3}$	$4.5\times10^{17}\ \text{cm}^{-3}$

V. CONCLUSION

In this paper, we have analyzed the interaction between a slow electromagnetic wave and a collisionless flowing finite-temperature plasma by using a one-dimensional model. The primary purpose was to explore the feasibility of solid-state amplifiers and oscillators operating in the infrared; however, application of this wave-plasma interaction to measurements of particle distribution functions were also discussed. The analysis in Secs. II and III is of quite general validity; the assumptions made in specializing to traveling-wave interaction with a solid-state plasma are, however, much more restrictive.

The basic coupled-mode analysis is a standard technique, but our detailed predictions involved two approximations that could affect their numerical accuracy. The one-dimensional model, although analytically convenient, ignores all tranverse variation in the fields. The other questionable assumption is our use of a drifting Maxwellian to model the distribution function of the flowing carriers. At present, these assumptions cannot be rigorously justified, and we consequently suggest that the gains calculated in Sec. IV should be regarded as a rough indication of the magnitude of the effect. Further experimental and theoretical work will be needed for accurate quantitative predictions.

The gains found in Sec. IV are low to moderate, and, if attainable, would be suitable for an oscillator or low-gain amplifier. The proposed operating conditions would be difficult to achieve at present. However, developments in semiconductor technology should make fabrication easier, and further optimization of the device by varying the operating conditions and dielectric materials should be possible.

Measurement of the velocity distribution function of the drifting carriers through their interaction with the electromagnetic wave is an application which requires experimental conditions that are much easier to attain. Such measurements could yield useful information for conditions with lower drift velocity, higher temperatures, or longer perturbation periods.

*Research supported by the Office of Naval Research and by the Army research Office.

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