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Soliton interaction as a possible model for extreme waves in shallow water

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Abstract. Interaction of two long-crested shallow water waves is analysed in the framework of the two-soliton solution of the Kadomtsev-Petviashvili equation. The wave system is decomposed into the incoming waves and the interaction soliton that represents the particularly high wave hump in the crossing area of the waves. Shown is that extreme surface elevations up to four times exceeding the amplitude of the incoming waves typically cover a very small area but in the near-resonance case they may have considerable extension. An application of the proposed mechanism to fast ferries wash is discussed.

1 Introduction

The phenomenon of particularly high and steep (freak or rogue) waves is one of the most dangerous events that a traveller at sea may encounter. At the early stage of freak wave studies, their existence and appearance were explained by basically linear models like interaction of waves with opposing currents (see Peregrine, 1976; Jonsson, 1990 and bibliography therein) or with uncharted seamounts (see White and Fornberg, 1998 and bibliography therein), blockage of short waves by longer waves and currents (Shyu and Phillips, 1990), interaction of surface waves with internal waves (Donato et al., 1999) or linear superposition of long waves with preceding shorter waves (e.g. Sand et al., 1990; Stansberg, 1990). Later on, a number of deeply interesting results were obtained based on the wave ray theory (e.g. Shyu and Tung, 1999; White and Fornberg, 1998) and on the assumption of the presence of either specific bathymetry or structure of currents.

Freak waves became a subject of significant interest for scientists in 1990s when it was established that they occur much more frequently than predicted by surface wave statistics. They are too high, too asymmetric and too steep, and not necessarily related with other dynamical processes in the ocean (Sand et al, 1990). During the last decade, many efforts were concentrated in order to find an appropriate mechanism that can cause considerable changes in the wave amplitudes exclusively resulting from (possibly nonlinear) superposition of long-crested waves and leading to spatially localised extreme surface elevations. For water waves, probably the first mechanism of this type has been first used to explain the origin of freak waves in (Dean, 1990). Modulation instability of wave trains with respect to longitudinal and transversal perturbations (Trulsen and Dysthe, 1997, Osborne et al., 2000) was one of the possibilities in order to understand the nature of freak waves or to establish the area where they are expected.

The most promising has been the approach of nonlinear wave focusing occurring in situations resembling rough seas in natural conditions and claiming that a large number of waves with different frequencies and propagation directions may produce a large long-living transient wave group at a particular point. This phenomenon has been theoretically validated in several simple but realistic models, e.g. in the framework of the simplest one-dimensional (1-D) Korteweg-de Vries equation (Pelinovsky et al., 2000) and for the two-dimensional wave field in the framework of the Davey-Stewartson equation (Slunyaev et al., 2002). Numerically, it has been demonstrated to work in irregular or random sea even for extreme wave design purposes (Smith and Swan, 2002) and also in the framework of generalized 2-D Schrödinger equation (Onorato et al., 2002). There exist brilliant experiments showing that this effect indeed occurs in laboratory conditions (e.g. Baldock and Swan, 1996; Johannessen and Swan, 2001).

A detailed description of another source for considerable changes in the wave amplitudes that can be related to the nonlinear superposition of long-crested waves and that also leads to spatially localised extreme surface elevations is the aim of the present paper. As different from the effect of wave focusing that generally presumes the existence of a number of different wave components that partially may be short-crested, the proposed mechanism is effective for a small number but

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definitely long-crested and soliton-like waves. This situation apparently is uncommon for storm waves or swell but may frequently occur in relatively shallow areas with heavy fast ferry traffic (Soomere et al., 2003, Parnell and Kofoed-Hansen, 2001). A simple way to describe the interaction of soliton-like waves in water of a finite depth is to consider two long-crested solitary waves travelling in different directions on a surface of water under gravity (Hammack et al., 1989, 1995). This is the simplest setup for multi-directional wave phenomena that can occur on sea. In this paper the extreme wave hump is described resulting from the crossing of these waves. This wave hump resembles Mach stem that arises when waves propagate in a semi-bounded area (also called Mach reflection, see, e.g. Funakoshi, 1980; Peregrine, 1983; Tanaka, 1993). Contrary to the case of a linear superposition when the resulting wave height does not exceed the sum of the heights of the counterparts, this hump can be up to four times higher than the incoming wave(s).

A suitable mathematical model for the description of nonlinear shallow water gravity waves is the Kadomtsev-Petviashvili (KP) equation (Kadomtsev and Petviashvili, 1970). Exact solutions for the KP equation can be found by using Hirota bilinear formalism (Hirota, 1971). The 2-D two-soliton interaction has been analysed, among others, in (Satsuma, 1976; Miles, 1977a, 1977b; Freeman, 1980), with main attention to the phase shifts and resonance phenomena. The resonance in this context is related to two interacting solitons (Miles, 1977a, b) resulting in a single soliton that was resonant with both interacting waves. In this framework, many authors have demonstrated that the amplitude of the water elevation at the intersection point of two solitons may exceed that of the sum of incoming solitons (see, for example, Segur and Finkel, 1985; Haragus-Courcelle and Pego, 2000; Tsuji and Oikawa, 2001; Chow, 2002) but neither the limits of the elevation nor the spatial occupancy of the high elevation have been analysed in detail. This model allows to consider the solitons with arbitrary amplitudes whereas in the case of the Mach reflection the amplitudes of the counterparts generally are related to each other.

Recently, Peterson and van Groesen have explicitly studied the two-soliton interaction (Peterson and van Groesen, 2000, 2001) with a special decomposition distinguishing the single solitons and the interaction soliton that connects the single solitons and mostly is concentrated in the intersection area of the single solitons. The decomposition permits to study the properties of an interaction soliton in detail and clarify the mechanism of its emergence. Shown is that under certain conditions the amplitude of the interaction soliton may exceed considerably the amplitudes of the interacting single solitons.

The general idea elaborated below is the following. As in (Peterson and van Groesen, 2001), we associate waves with solitons by assuming the KP equation to be a valid model for such waves in shallow water, that is, waves are relatively long and small but finite when comparing to the water depth. Oblique interaction of two solitons (below called incoming solitons) propagating in slightly different directions involves the so-called interaction soliton that connects two solitons with their shifted counterparts (Peterson and van Groesen, 2001) and can be associated with the wave hump resulting from their crossing (cf. experimental evidence in Hammack et al., 1989, 1995).

We aim to clarify the regions in the parameter space of a two-soliton solution where the height of the interaction soliton exceeds the value predicted by the linear theory and to evaluate its maximum height and length and the conditions when they are achieved. When two waves of amplitudes a_1 and a_2 meet, the maximum amplitude M of their (linear or nonlinear) superposition can be written as $M = m(a_1 + a_2)$, where "amplification factor" m may depend on both a_1 and a_2 and their intersection angle γ_{12}. Linear theory gives $m = 1$. We show that, at least, for specific combinations of a_1, a_2 and γ_{12}, the maximum "amplification factor" is $m_{\max} = 2$ as in the case of the Mach reflection.

The paper is organized as follows. In Sect. 2 the solution of the KP equation is given decomposed into a sum of two soliton terms and interaction soliton term. The analysis of the spatial occupancy and the height of the interaction soliton is presented in Sect. 3. Section 4 includes the estimates of the the critical angle between incoming solitons, leading to extreme elevations, and occurrence and spatial structure of extreme elevations. In Sect. 5 physical evidence of extreme waves, possible modelling of such entities using the notion of interaction solitons and further prospects of modelling are discussed.

2 Two-soliton solution

The KP equation in normalized variables reads (Segur and Finkel, 1985)

$$(\eta_t + 6\eta\eta_x + \eta_{xxx})_x + 3\eta_{yy} = 0\,, \qquad (1)$$

where normalised variables (x, y, t, η) are related to physical variables $(\tilde{x}, \tilde{y}, \tilde{t}, \tilde{\eta})$ in a coordinate frame moving in the x-direction as follows:

$$x = \frac{\varepsilon^{1/2}}{h}\left(\tilde{x} - \tilde{t}\sqrt{gh}\right)\,, \qquad y = \frac{\varepsilon}{h}\tilde{y}\,,$$
$$t = \frac{\varepsilon^{3/2}\sqrt{gh}}{6h}\tilde{t}\,, \qquad \eta = \frac{3}{2\varepsilon h}\tilde{\eta} + O(\varepsilon)\,, \qquad (2)$$

where η is the elevation of the water surface ($\eta = 0$ corresponds to the undisturbed surface), $\varepsilon = |\tilde{\eta}|_{\max}/h \ll 1$ is maximal wave amplitude normalized against the depth of the fluid h and g is gravity acceleration. In this framework, the phase speed of a disturbance always exceeds the maximum linear wave phase speed $\sqrt{gh}$. Since the interaction pattern of a two-soliton solution is stationary in the moving coordinate frame, in the following we take $t = 0$ without loss of generality. Below we mean that a gravity wave with amplitude $\tilde{\eta}_0$ and wave length L propagating in water of depth h, is a shallow-water wave if its Ursell number is $U = \tilde{\eta}_0 L^2 h^{-3} \approx 1$. The KP equation itself is valid provided $\tilde{l}/\tilde{k} = O(\tilde{\kappa}h) = O(\varepsilon)$, where $(\tilde{k}, \tilde{l})$ is a wave vector

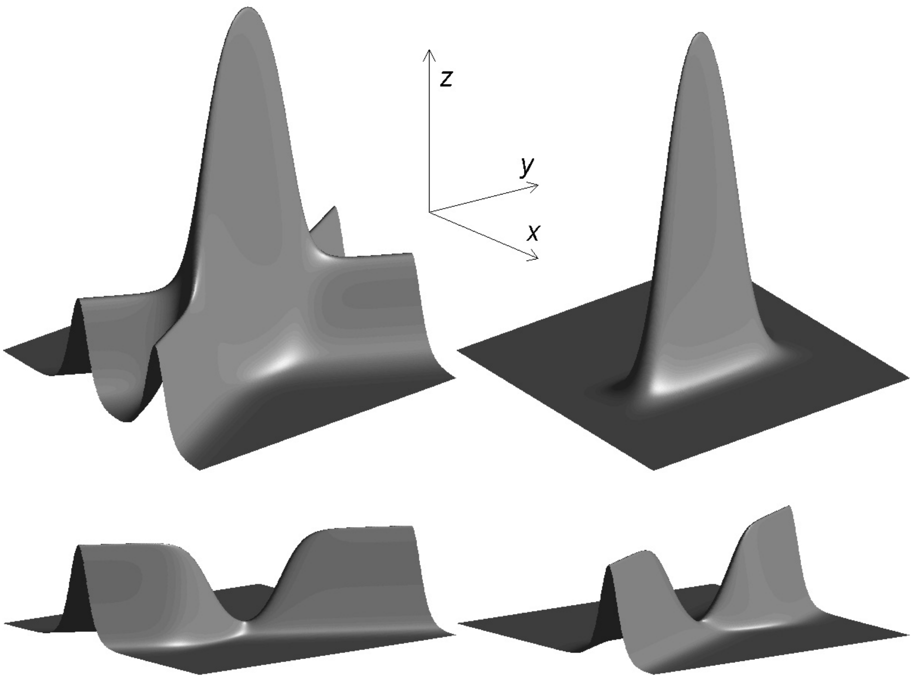

Fig. 1. The shape of surface in the vicinity of the intersection of solitons s_1, s_2 (upper left panel) and surface elevation caused by the interaction soliton s_{12} (upper right) and incoming solitons (lower panels) in normalised coordinates (x, y), cf. Fig. 3. Area $|x| \le 30,\ |y| \le 30,\ 0 \le z \le 4a_1$ is shown at each panel.

for the waves in question, $\tilde{\kappa} = |(\tilde{k}, \tilde{l})|$, and x-direction is the principal direction of wave propagation.

A two-soliton solution of the KP Eq. (1) reads (Segur and Finkel, 1985)

$$\eta(x, y, t) = 2\frac{\partial^2}{\partial x^2} \ln \theta ,$$

where

$$\theta = 1 + e^{\varphi_1} + e^{\varphi_2} + A_{12}\, e^{\varphi_1+\varphi_2} ,$$

$\varphi_{1,2} = k_{1,2}\, x + l_{1,2}\, y + \omega_{1,2}\, t$ are phase variables, $\boldsymbol{\kappa}_1 = (k_1, l_1)$, $\boldsymbol{\kappa}_2 = (k_2, l_2)$ are the wave vectors of the incoming solitons, the "frequencies" $\omega_{1,2}$ can be found from the dispersion relation of the linearized KP equation $P(k_{1,2}, l_{1,2}, \omega_{1,2}) = k_{1,2}\, \omega_{1,2} + k_{1,2}^4 + 3l_{1,2}^2 = 0$ and A_{12} is the phase shift parameter

$$A_{12} = -\frac{P(k_1-k_2, l_1-l_2, \omega_1-\omega_2)}{P(k_1+k_2, l_1+l_2, \omega_1+\omega_2)} = \frac{\lambda^2-(k_1-k_2)^2}{\lambda^2-(k_1+k_2)^2} , \qquad (3)$$

where $\lambda = l_1/k_1 - l_2/k_2$ (Peterson and van Groesen, 2001).

The two-soliton solution can be decomposed into a sum of two incoming solitons s_1, s_2 and the interaction soliton s_{12} (Peterson and van Groesen, 2000)

$$\eta = s_1 + s_2 + s_{12} , \qquad (4)$$

where the counterparts s_1, s_2 and s_{12} (Fig. 1) are defined as:

$$s_{1,2} = \frac{A_{12}^{1/2}\, k_{1,2}^2 \cosh\left(\varphi_{2,1} + \ln A_{12}^{1/2}\right)}{\Theta^2} ,$$

$$s_{12} = \frac{\left(k_1-k_2\right)^2 + A_{12}\left(k_1+k_2\right)^2}{2\Theta^2} ,$$

$$\Theta = \cosh\left[\left(\varphi_1-\varphi_2\right)/2\right] + A_{12}^{1/2} \cosh\left[\left(\varphi_1+\varphi_2+\ln A_{12}\right)/2\right] . \qquad (5)$$

The heigth and orientation of the interaction soliton only depend on the amplitudes of the incoming solitons and the angle (interaction angle) between their crests. Within restrictions of the KP model, interaction may result in either the positive or the negative phase shift $\Delta_{12} = -\ln A_{12}$ of the counterparts. The positive phase shift (when the interaction soliton generally does not exceed the larger incoming one) typically occurs for interactions between solitons with fairly different amplitudes. The negative phase shift is typical in interactions of solitons with comparable amplitudes and results generally in an interaction soliton which height exceeds that of the sum of the two incoming solitons. This observation agrees with the fact that the total mass and energy of all three solitons (as well as other integrals of motion) in a specific direction (called the principal propagation direction) are constant in the transversal direction.

3 The height and length of the interaction soliton

Since we are basically interested in extreme amplitudes, in what follows we concentrate on the region in the parameter space that corresponds to the negative phase shift case, equivalently, to the region where $A_{12} > 1$ (Fig. 2). In this region, the maximum height of the interaction soliton exceeds that of the counterparts (except in the limiting case $A_{12} \to 1$ with no phase shift when the height of the interaction soliton tends to $\left(k_1^2 + k_2^2\right)/4$, that is the mean height of the incoming

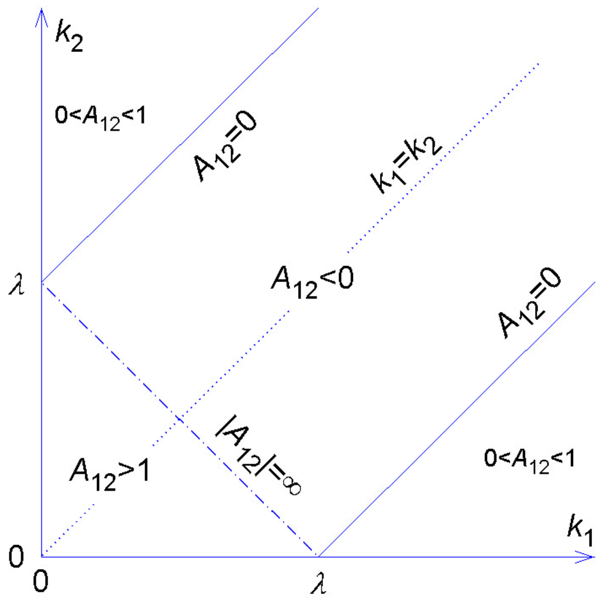

Fig. 2. "Map" of negative and positive phase shift areas for the two-soliton solution of the KP equation for fixed $l_{1,2}$. The phase shift parameter has a discontinuity along the line $k_1 + k_2 = \lambda$ where $|A_{12}| = \infty$.

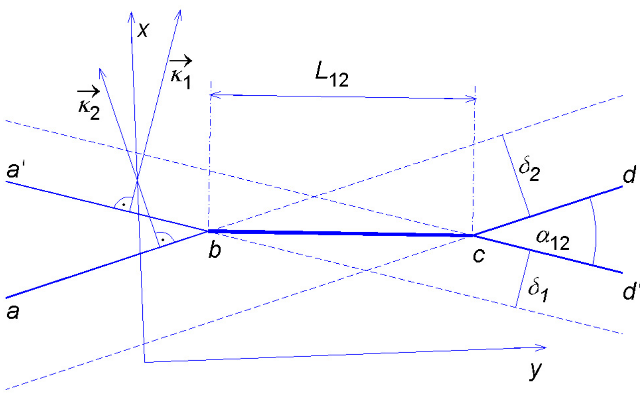

Fig. 3. Idealised patterns of crests of incoming solitons (solid lines) and the interaction soliton (bold line) corresponding to the negative phase shift case. Notice that the crest of the interaction soliton is not necessarily parallel to the y-axis except in the case of equal heights (equivalently, equal lengths of the wave vectors) of incoming solitons.

solitons). Further, in the case of considerably different amplitudes of incoming solitons ($A_{12} < 1$), the amplitude of the interaction soliton is smaller than the mean height of incoming solitons. However, in the case of equal amplitude incoming solitons, the surface elevation in the interaction region may up to four times exceed the amplitude $a_{1,2} = k_{1,2}^2/2$ of the incoming solitons (see below; cf. Tsuji and Oikawa, 2001).

It is convenient to take the x-axis to bisect the interaction angle α_{12} between the wave vectors $\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2$ (Fig. 3) that yields $\lambda = 2l_1/k_1 = 2\tan\left(\frac{1}{2}\alpha_{12}\right),\ l_1/k_1 = -l_2/k_2$ and

$$A_{12} = \frac{4l_1^2 - k_1^2(k_1 - k_2)^2}{4l_1^2 - k_1^2(k_1 + k_2)^2} . \qquad (6)$$

In (Peterson and van Groesen, 2000) it is shown that the phase shifts δ_1, δ_2 of the incoming solitons satisfy the relation $|\delta_{1,2}\kappa_{1,2}| = |\Delta_{12}|$. It is natural to interpret the interval $[b, c]$ on Fig. 3 (where the crests of the incoming solitons practically coincide) as the crest of the interaction soliton. From Fig. 3 it follows that the (geometrical) length of the interaction soliton L_{12} equals to the longer diagonal of the parallelogram with the sides $\delta_{1,2}/\sin\alpha_{12}$ (that are perpendicular to wave vectors $\boldsymbol{\kappa}_{2,1}$, respectively) and the acute angle α_{12}. This consideration immediately gives

$$L_{12} = \frac{|\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2|}{|\boldsymbol{\kappa}_1 \times \boldsymbol{\kappa}_2|}|\Delta_{12}| = \frac{|\Delta_{12}| \cdot |\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2|}{\kappa_1\kappa_2 \sin\alpha_{12}} . \qquad (7)$$

From Eq. (7) it follows that, formally, L_{12} may vary from 0 to infinity. Since the first factor in Eq. (7) has a minimum value $\sqrt{\kappa_1^{-2} + \kappa_2^{-2}} > 0$ if $\boldsymbol{\kappa}_1 \perp \boldsymbol{\kappa}_2$ (this value indeed cannot be achieved, because in this case the KP equation becomes invalid) and since in physically meaningful cases at least one wave vector $\boldsymbol{\kappa}_{1,2}$ has finite length, L_{12} may approach zero if and only if $A_{12} \to 1$. This is possible only if $k_1 k_2 = 0$, i.e. in the trivial case when one of the incoming solitons has zero amplitude. Further, L_{12} approaches infinity either if $A_{12} \to \infty$ or in the trivial subcase $\boldsymbol{\kappa}_1 \parallel \boldsymbol{\kappa}_2$. The first case is equivalent to the limiting process $l_1 \to \frac{1}{2}k_1(k_1 + k_2)$ which gives a nontrivial solution.

For a given nonzero angle α_{12} between the incoming solitons and fixed $k_1 + k_2$, the interaction soliton length L_{12} has a maximum value if the vectors $\boldsymbol{\kappa}_1,\ \boldsymbol{\kappa}_2$ have an equal length. In the particular case of equal height solitons $k_1 = k_2 = k$, $l_1 = -l_2 = l,\ A_{12} = l^2/\left(l^2 - k^4\right)$ and we have

$$L_{12} = \frac{\sqrt{(k_1 + k_2)^2 + (l_1 + l_2)^2}}{|k_1 l_2 - k_2 l_1|} \ln A_{12} = \frac{\ln A_{12}}{l} , \qquad A_{12} > 1 . \qquad (8)$$

In the case of equal amplitude solitons, the direction of x-axis (the principal propagation direction) coincides with that of vector $\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2$. With the use of Eqs. (3) and (5) it can be shown that the maximum heights of the the counterparts $s_1,\ s_2$, and s_{12} read (Peterson and van Groesen, 2001):

$$a_{1,2} = \frac{1}{2}k_{1,2}^2,\ a_{12} = \frac{(k_1 - k_2)^2 + A_{12}(k_1 + k_2)^2}{2\left(1 + A_{12}^{1/2}\right)^2} . \qquad (9)$$

Notice that the maximum heights of the incoming solitons are achieved at $\varphi_1 = 0,\ \varphi_2 \to -\infty$ or $\varphi_2 = 0,\ \varphi_1 \to -\infty$. From the definition of the interaction soliton in Eq. (5) it follows that its height is always nonzero and has a maximum value at the center of the interaction region defined by $(\varphi_1, \varphi_2) = \frac{1}{2}(\Delta_{12}, \Delta_{12})$. Figure 4 shows dependence of

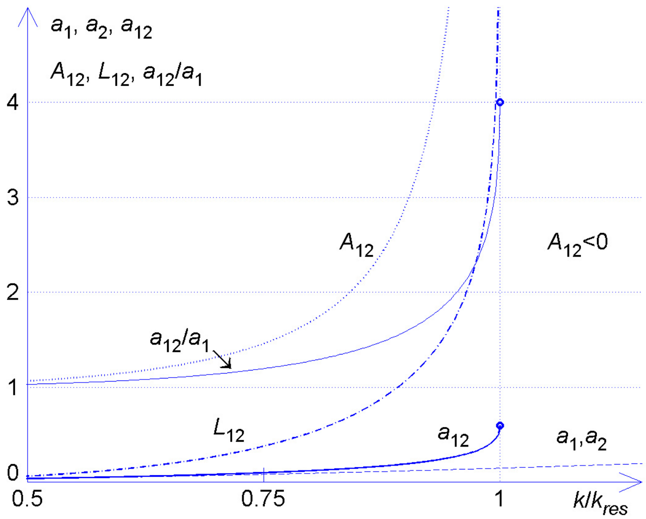

Fig. 4. The dependence of the phase shift parameter A_{12}, length of the interaction soliton L_{12} and amplitudes of the incoming a_1 and the interaction soliton a_{12} on the wave vector component k along the line $k_1 = k_2 = k$ in Fig. 2 for $l_1 = -l_2 = 0.3$. The case $k = k_{\text{res}} = \lambda/\sqrt{2}$ corresponds to the intersection of the line $k_1 = k_2$ with the border of the area $A_{12}>1$, i.e. with the line where $|A_{12}| = \infty$. Notice that for small values of k/k_{res} the assumption $|l| \ll |k|$ is violated.

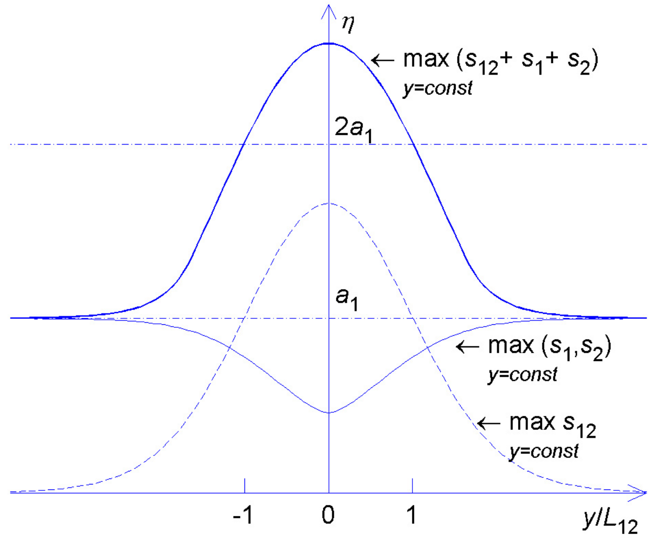

Fig. 5. The maximum surface elevation $\max_{y=const}(s_1+s_2+s_{12})$ and the height of the incoming solitons $\max_{y=const}(s_1, s_2)$ and the interaction soliton $\max_{y=const}(s_{12})$ along their crests for $k_1 = k_2 = 0.5$; $l_1 = -l_2 = 0.3$. The origin coincides with the centre of the interaction soliton.

quantities A_{12}, L_{12}, a_1, a_2, a_{12} on the wave vector length in the case of equal amplitude incoming solitons. For fixed $l_{1,2}$ and equal height solitons, from Eqs. (6), (8) and (9) it follows that A_{12} monotonously increases from 1 to infinity when parameter k of incoming solitions increases from 0 to $\hat{k} = \lambda/\sqrt{2}$. Further, L_{12} increases from 0 to infinity and the amplitude of an interaction soliton monotonously increases from 0 to $2\hat{k}^2 = \lambda^2$.

In terms of the decomposition (Eq. 4), the components s_1, s_2 and s_{12} are shown in Fig. 5, where $\zeta = y/L_{12}$ and values correspond to crests approximately following the line $abcd$ (s_2), $a'bcd'$ (s_1) or bc (s_{12}) in Fig. 3. The origin of ζ is taken in the centre of the interaction soliton. It is seen that although the visible part of interaction crest has the length L_{12} (Fig. 3), the changes in components occur beyond this limit. Namely, the length of the area where the height of the interaction soliton exceeds the double amplitude of the incoming solitons (or, equivalently, where nonlinear effects create higher elevation than simple superposition of heights of two waves) may considerably exceed L_{12}. The reason is that the above definition of the interaction soliton length L_{12} in Eq. (7) is based on the geometrical image of crest patterns of the interacting solitons.

4 The critical angle, occurrence and spatial structure of extreme elevations

Equation (2) shows that the normalized co-ordinate axes have different scaling factors $x \sim \varepsilon^{1/2}\tilde{x}$, $y \sim \varepsilon\tilde{y}$ that must be taken into account when interpreting the results in physical co-ordinates. Let $\tilde{\alpha}_{12}$ be the angle between soliton crests in the physical space. If we define $\tilde{\lambda} = 2\tan\left(\frac{1}{2}\tilde{\alpha}_{12}\right)$, then it is easy to show that $\tilde{\lambda} = \lambda\varepsilon^{1/2}$. The physical length $\tilde{L}_{12}$ of the interaction soliton crest is

$$\tilde{L}_{12} = \frac{h}{\varepsilon}L_{12} = \frac{2h|\tilde{\Delta}_{12}|}{\varepsilon^{1/2}k\tilde{\lambda}} ,$$

where

$$\tilde{\Delta}_{12} = -\ln\frac{\tilde{\lambda}^2}{\tilde{\lambda}^2 - 4\varepsilon k_1^2} .$$

The above has shown that the highest surface elevation occurs in the center of the interaction area when two solitons of comparable height interact. In the particular case of equal in height solitons $k_1 = k_2 = k$, $l_1 = -l_2 = l$ and Eqs. (3) and (9) are simplified as follows:

$$A_{12} = \frac{l^2}{l^2 - k^4} , \quad a_{12} = \frac{2A_{12}k^2}{\left(1 + A_{12}^{1/2}\right)^2} .$$

The interaction soliton has maximum amplitude $2k^2$ if $A_{12} = \infty$. In this case we have $l = k^2$ that is consistent with the assumption of the KP equation that $|l| \ll |k|$ provided $|k| < 1$.

For given wave depth h and incoming soliton amplitudes $\tilde{\eta}$, the resonance takes place and the extent of extreme water

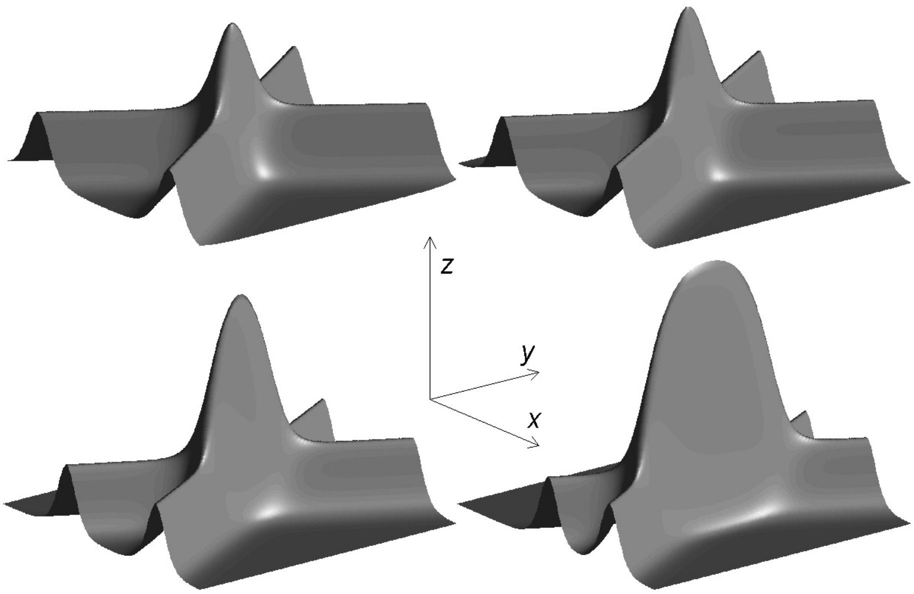

Fig. 6. Surface elevation in the vicinity of the interaction area, corresponding to incoming solitons with equal amplitudes $a_1 = a_2$, $l = -l_1 = 0.3$, $k_{res} = \sqrt{0.3}$ and $k = 0.8k_{\rm res}$ (upper left panel), $k = 0.9k_{\rm res}$ (upper right), $k = 0.99k_{\rm res}$ (lower left), $k = 0.9999k_{\rm res}$ (lower right) in normalised coordinates (x, y), cf. Fig. 3. Area $|x| \le 30$, $|y| \le 30$, $0 \le z \le 4a_1$ is shown at each panel.

level elevation described by $\tilde{L}_{12}$ may increase dramatically if the solitons intersect under a physical angle

$$\tilde{\alpha}_{12} = 2 \arctan \sqrt{3\tilde{\eta}/h} \ .$$

The critical angle (at which the interaction soliton height and length are extreme) for two solitary waves in realistic conditions is reasonable. For example, for waves with heights $\tilde{\eta} = 1.8$ m meeting each other in an area with typical depth $h = 50$ m, the critical angle is about 36°.

From Fig. 4 it follows that the maximum relative amplitude of the interaction soliton mostly remains modest although it always exceeds that of incoming solitons. Elevations exceeding two times the incoming soliton height occur relatively seldom, only if $\left(k_{\rm res} - k\right)/k_{\rm res} \approx< 0.05$.

The analysis until now has concerned only the occurrence of extreme elevations. We studied numerically the appearance of the interaction soliton depending on the wave vector length along the line $k_1 = k_2$ for a fixed pair of wavenumbers $l_1 = -l_2$ (Fig. 6). If the wavenumber k_1 differs from $k_{\rm res} = \sqrt{l_1}$ more than 10%, the length of the interaction soliton (the extent of the extreme elevation) remains modest (cf. Fig. 4). When $k_1 \to k_{\rm res}$ (equivalently, the wave vectors of the incoming solitons approach the border of the area $A_{12}>1$ in Fig. 2), the area of extreme elevation widens. However, the length of the interaction soliton considerably exceeds k_1^{-1} only if the difference between k_1 and $k_{\rm res}$ (equivalently, the difference between the interaction angle and the critical angle) is less than 1% (Fig. 6).

5 Discussion

The fact that the total energy density of all three solitons, integrated in the principal propagation direction, is constant in the transversal direction confirms that the described amplitude amplification is not accompanied by energy pumping into the interaction region. Further on, the central part of the interaction region with extreme water surface elevation, if "cut" out from the interaction picture and let to evolve on its own in a channel of suitable width, apparently will not have the properties of a KP (KdV) soliton. The reason is that generally (except in the limiting case $|A_{12}| \to \infty$) it does not correspond to a one-soliton solution of the KP equation, because its profile is wider compared to that of the one-soliton solution with the same fixed height.

Since there occurs no energy concentration in the extreme surface elevation region, one might say that the described effect is interesting only theoretically. This is true to some extent. But in natural conditions the sea bottom is never perfect, and the moving interaction soliton, generally, after a while meets conditions where KP equation itself might be invalid or the negative phase shift is impossible. The transition from the region $k_1 < k_{\rm res}$ to the case $k_1 > k_{\rm res}$ needs a further analysis. In this situation, the interaction soliton may break as any other water wave does. This particular moment contains acute danger, because, as it is generally believed, "no non-breaking wave is dangerous", e.g. for offshore yachts (Kirkman and McCurdy, 1987).

The discussed model of extreme water elevations or freak waves assumes the existence of more or less regular long-crested 2-D wave trains. This assumption suggests that a freak wave caused by the soliton interaction mechanism is a rare phenomenon at open sea in natural conditions. Indeed, the influence of the most important source of surface waves – wind fields during storms – generally results in an extremely irregular sea surface and the appearance of long-living solitary waves is unlikely. The presented mechanism may be far more important in relatively shallow coastal areas with high ship traffic density. It is well known that long-crested soliton-like surface waves are frequently excited by contemporary high speed ships if they sail with critical or supercritical speeds (e.g. Chen and Sharma, 1997; navigational speeds are distinguished according to the depth Froude number $F_d = v_{ship}/\sqrt{gh}$, that is the ratio of the ship speed and the maximum phase speed of gravity waves. Operating at speeds resulting $F_d = 1$ is defined as critical and $F_d > 1$ as supercritical).

Groups of soliton-like ship waves intersecting at a small angle may appear if two high speed ships meet each other or if a ship changes its course. The first opportunity frequently happens in areas with heavy high speed traffic like the Tallinn Bay, Baltic Sea, where up to 70 bay crossings take place daily whereas at specific times three fast ferries depart simultaneously (Soomere et al., 2003).

The length of interactions solitons (equivalently, the extent of the area where considerable amplification of wave heights may take place) generally is moderate, thus these structures are short-crested three-dimensional (3-D) stationary waves. This property should be particularly emphasized, because the incoming solitons are virtually one-dimensional and their interaction pattern is a two-dimensional structure. In terms of dimensions, this situation is principally different from that arising in studies of focusing of transient and directionally spread waves. In these experiments (both numerical and laboratory) a number waves with different frequencies and propagation directions are focused at one point at a specific time instant to produce a time-varying transient wave group (e.g. Johannessen and Swan, 2001). In this context, the forming of an interaction soliton resembles the 3-D freak wave appearance in numerical simulations based on the generalized 2-D Schrödinger equation (Onorato et al., 2002).

Substantial areas of extreme surface elevation may occur only if the heights of the incoming waves, their intersection angle and the local water depth are specifically balanced. Thus, the fraction of sea surface occupied by the interaction solitons corresponding to extreme elevations apparently is very small as compared with the area covered by intense wash. By that reason, detecting of an interaction soliton in one-point in situ measurements is unlikely. Yet the nonlinear amplification of wave heights in the crossing points of wave crests (that happens always when negative phase shift takes place) may be one of the reasons why superposition of critical wash generated before and after turns is mentioned as particularly dangerous (Kirk McClure Morton, 1998).

In populated or industrial areas, a possible hit of the central part of a near-resonant long-crested interaction soliton on an entrance of a channel (harbour entrance, river mouth etc.) may cause serious consequences. The reason is that this structure is basically different from a superposition of two linear wave trains at the same place. Linear waves continue to move in their original directions, and result in a system of interfering waves in the channel. If a near-resonant interaction soliton enters a channel, it concentrates energy of both incoming solitons in one structure, the further behaviour and stability of which is yet unclear. In a highly idealised case of interactions of five solitons surface elevations may exceed the amplitudes of the incoming solitons more than an order (Peterson, 2001).

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