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Chin. Phys. B Vol. 22, No. 6 (2013) 060306

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(Received 28 June 2012; revised manuscript received 4 October 2012)

The relativistic Duffin–Kemmer–Petiau equation in the presence of Hulthén potential in (1+2) dimensions for spin-one particles is studied. Hence, the asymptotic iteration method is used for obtaining energy eigenvalues and eigenfunctions.

Keywords: Duffin–Kemmer–Petiau equation, Hulthén potential, asymptotic iteration method

PACS: 03.65.Pm, 03.65.Ca, 98.80.cq. **DOI:** 10.1088/1674-1056/22/6/060306

1. Introduction

The first order Duffin–Kemmer–Petiau (DKP) equation describes spin-zero and spin-one particles.[1–3] The DKP formalism has been studied to analyze relativistic interactions of spin-one and spin-zero hadrons with nuclei. In analogy with Dirac phenomenology for proton–nucleus scattering, the DKP formalism is in better agreement with the experimental data for a phenomenological treatment of the elastic meson–nucleus scattering than Proca and KG theory.[4,5] There is a renewed interest in considering the DKP equation in the presence of interactions. Recently, many articles have been devoted to investigating the DKP theory under different types of potentials, hence we may cite Refs. [6]–[17], also this equation explains the quark confinement problem of quantum chromodynamics theory[18] and we may cite the papers on the meson–nuclear interaction.[19] Moreover, the relativistic model of α–nucleus elastic scattering has been treated by the formalism of the DKP theory[20] and the covariant Hamiltonian[21] in the causal approach.[22,23] In addition, there has been an increasing interest in the DKP oscillator.[24–29] Since the wave function includes all the necessary information about considering systems, the energy eigenvalues and corresponding eigenfunctions between interacting systems in relativistic quantum mechanics and in non-relativistic quantum mechanics have been studied more efficiently in recent years. The main purpose of the present work is to study the DKP equation in the presence of the Hulthén potential in (1+2)-dimensional space-time for spin-one particles by asymptotic iteration method (AIM) which is based on solving the second-order differential equations, where it has been used for solving the Schrödinger, Dirac, DKP, and Klein–Gordon wave equations in the presence of different types of potentials. The Hulthén potential is a short range potential and its form is[30]

$$V(r) = -ze^2\delta\frac{\mathrm{e}^{-\delta r}}{1-\mathrm{e}^{-\delta r}},$$

where δ is a screening parameter that is used for determining the range of the Hulthén potential and z is an atomic number. Therefore, the Hulthén potential is a particular case of Eckart potential which explains the interaction between two atoms or molecular structure and nuclear interaction.[31–35] Moreover, the Hulthén potential and its different forms are used in non-relativistic and relativistic spaces.[36–42]

The organization of the rest of this paper is as follows. In Section 2 , we explain the DKP equation and in Section 3 we discuss the DKP equation in the presence of interaction in (1+2) dimensions. Then, in Section 4, we introduce the basic equations of the AIM, and in Section 5 we derive the solutions of the DKP equation in the presence of Hulthén potential by the AIM. Finally, concluding remarks and discussion are given in Section 6.

2. Duffin–Kemmer–Petiau equation

The one-dimensional DKP equation for non-interacting bosons of spin-zero and spin-one is ($\hbar = c = 1$)[1–3]

$$(\mathrm{i}\beta^\mu\partial_\mu - m)\Psi = 0, \tag{1}$$

where β^μ are the DKP matrices that satisfy in this algebra

$$\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\lambda\nu}\beta^\mu. \tag{2}$$

Moreover

$$g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1), \quad (g^{\mu\nu})^2 = 1, \tag{3}$$

where we can use the following relations,[43]

$$\beta_0\beta_k\beta_0 = 0,\ k = 1,2,3, \tag{4}$$

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$$\beta_0^3 = \beta_0, \tag{5}$$

$$\partial_\mu b^\mu \beta_\nu \partial_\mu b^\mu = \partial_\mu b^\mu b_\nu, \; b_\mu = (b_0, \boldsymbol{b}), \tag{6}$$

where the b_μ is a generic four-vector.

$$(\boldsymbol{\beta}\cdot\boldsymbol{b})\beta_0(\boldsymbol{\beta}\cdot\boldsymbol{b}) = 0. \tag{7}$$

Multiplying Eq. (1) by $\beta_\mu \partial^\mu$ and using Eq. (6) we have

$$(\mathrm{i}\beta_\mu \partial^\mu \beta_\mu \beta_\mu \partial^\mu - m\beta_\mu \partial^\mu \beta_\mu)\Psi = 0, \tag{8}$$

so,

$$(\mathrm{i}\partial_\mu \beta_\mu \partial^\mu - m\beta_\mu \partial^\mu \beta_\mu)\Psi = 0, \tag{9}$$

hence, equation (1) becomes

$$(m\partial_\mu - m\beta_\mu \partial^\mu \beta_\mu)\Psi = 0, \tag{10}$$

and we can obtain

$$\partial_\mu \Psi = \beta_\mu \partial^\mu \beta_\mu \Psi. \tag{11}$$

Multiplying Eq. (1) by β_0, getting the zero component of Eq. (11), multiplying the resulting equation by the imaginary unity, one determines, upon adding the results,[43]

$$\{\mathrm{i}\left[\partial_0 + \partial^k(\beta_0\beta_k - \beta_k\beta_0)\right] - m\beta_0\}\Psi = 0. \tag{12}$$

On the other hand, $\mathrm{i}\partial_0\Psi = H\Psi$, where H is the DKP Hamiltonian,

$$H = -i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta_0 m = \boldsymbol{\alpha}\cdot\boldsymbol{P} + \beta_0 m,$$
$$\alpha_k \equiv \beta_0\beta_k - \beta_k\beta_0, \; k = 1,2,3, \tag{13}$$

within the ten-dimensional representation of spin-one sector

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \bar{0}^{\mathrm{T}} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{0}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix},$$
$$\beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & -\mathrm{i}S_i \\ -e_i^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \bar{0}^{\mathrm{T}} & -\mathrm{i}S_i & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}, \tag{14}$$

with S matrices being 3×3 ones, $(S_i)_{jk} = -\mathrm{i}\varepsilon_{ijk}$ where ε_{ijk} takes the values $1, -1, 0$ for an even permutation, an odd permutation, and repeated indices, respectively; e_i matrices are 1×3 ones with $(e_i)_{1j} = \delta_{ij}$, i.e.,

$$e_1 = (1,0,0) \quad e_2 = (0,1,0) \quad \text{and} \quad e_3 = (0,0,1),$$

where $\boldsymbol{I}$ and $\boldsymbol{0}$ respectively represent unit and null 3×3 matrices and $\bar{0}$s are 1×3 ones.

On the other hand, the DKP equation in the presence of the interaction is

$$(\mathrm{i}\beta^\mu\partial_\mu - m - U)\Psi = 0, \tag{15}$$

where the general form of the interaction is

$$U = S(r) + PS_\mu(r) + \beta^\mu V_\mu(r) + \beta^\mu PV_{p\mu}(r), \tag{16}$$

furthermore, for an elastic scattering, U is[44]

$$U = S(r) + PS_\mu(r) + \beta^0 V(r) + \beta^0 PV_P(r), \tag{17}$$

where each term has a specific Lorentz character. Two Lorentz vectors may be written as β^μ and $P\beta^\mu$ by assuming rotational invariance and parity conservation hence the projection operator is

$$P = \frac{1}{3}(\beta^\mu\beta_\mu - 2) = \mathrm{diag}(1,1,1,1,0,0,0,0,0,0).$$

The DKP matrix has three irreducible representations: one-dimensional representation that is trivial, five-dimensional representation for spin-zero particles, and ten-dimensional representation for spin-one particles.[1–3]

3. The DKP equation in the presence of interaction in (1+2)-dimensional space

The DKP equation in the presence of an interaction in (1+2) dimensions is

$$(E\beta^0 - \beta^0 PV(\rho) + \mathrm{i}\beta^1\partial_1 + \mathrm{i}\beta^2\partial_2 - m)\Psi = 0, \tag{18}$$

of which a solution in the following form is obtained as

$$\Psi(x,y,t) = \exp(-\mathrm{i}E_{n,\lambda}t)\psi_{n,\lambda}(x,y) \tag{19}$$

by removing the time dependence. We pick up two quantum numbers and display our ten-component stationary spinor as

$$\psi^{\mathrm{T}}_{n,\lambda}(x,y) = (\psi^{(1)}_{n,\lambda}, \psi^{(2)}_{n,\lambda}, \psi^{(3)}_{n,\lambda}, \psi^{(4)}_{n,\lambda}, \psi^{(5)}_{n,\lambda}, \psi^{(6)}_{n,\lambda}, \psi^{(7)}_{n,\lambda}, \psi^{(8)}_{n,\lambda}, \psi^{(9)}_{n,\lambda}, \psi^{(10)}_{n,\lambda})^{\mathrm{T}}. \tag{20}$$

Substitution of the latter spinor into Eq. (19) yields the coupled equations

$$\mathrm{i}\partial_1\psi^{(5)}_{n,\lambda}(x,y) + \mathrm{i}\partial_2\psi^{(6)}_{n,\lambda}(x,y) - m\psi^{(1)}_{n,\lambda}(x,y) = 0, \tag{21}$$

$$E_{n,\lambda}\psi^{(5)}_{n,\lambda}(x,y) + \mathrm{i}\partial_2\psi^{(10)}_{n,\lambda}(x,y) - m\psi^{(2)}_{n,\lambda}(x,y) = 0, \tag{22}$$

$$E_{n,\lambda}\psi^{(6)}_{n,\lambda}(x,y) - \mathrm{i}\partial_1\psi^{(10)}_{n,\lambda}(x,y) - m\psi^{(3)}_{n,\lambda}(x,y) = 0, \tag{23}$$

$$E_{n,\lambda}\psi^{(7)}_{n,\lambda}(x,y) + \mathrm{i}\partial_1\psi^{(9)}_{n,\lambda}(x,y) - \mathrm{i}\partial_2\psi^{(8)}_{n,\lambda}(x,y) - m\psi^{(4)}_{n,\lambda}(x,y) = 0, \tag{24}$$

$$E_{n,\lambda}\psi^{(2)}_{n,\lambda}(x,y) - V(\rho)\psi^{(2)}_{n,\lambda}(x,y) - \mathrm{i}\partial_1\psi^{(1)}_{n,\lambda}(x,y) - m\psi^{(5)}_{n,\lambda}(x,y) = 0, \tag{25}$$

$$E_{n,\lambda}\psi^{(3)}_{n,\lambda}(x,y) - V(\rho)\psi^{(3)}_{n,\lambda}(x,y) - \mathrm{i}\partial_2\psi^{(1)}_{n,\lambda}(x,y) - m\psi^{(6)}_{n,\lambda}(x,y) = 0, \tag{26}$$

$$E_{n,\lambda}\psi^{(4)}_{n,\lambda}(x,y) - V(\rho)\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(7)}_{n,\lambda}(x,y) = 0, \tag{27}$$

$$\mathrm{i}\partial_2\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(8)}_{n,\lambda}(x,y) = 0, \tag{28}$$

$$-\mathrm{i}\partial_1\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(9)}_{n,\lambda}(x,y) = 0, \tag{29}$$

$$\mathrm{i}\partial_1\psi^{(3)}_{n,\lambda}(x,y) - \mathrm{i}\partial_2\psi^{(2)}_{n,\lambda}(x,y) - m\psi^{(10)}_{n,\lambda}(x,y) = 0. \tag{30}$$

If we suppose $\psi^{(2)}_{n,\lambda}(x,y) = 0$, it is simply seen that the other components of $\psi^{(3)}_{n,\lambda}(x,y)$ can be obtained from coupled equations as

$$\mathrm{i}\partial_1\psi^{(5)}_{n,\lambda}(x,y) + \mathrm{i}\partial_2\psi^{(6)}_{n,\lambda}(x,y) - m\psi^{(1)}_{n,\lambda}(x,y) = 0, \tag{31a}$$

$$E_{n,\lambda}\psi^{(5)}_{n,\lambda}(x,y) + \mathrm{i}\partial_2\psi^{(10)}_{n,\lambda}(x,y) = 0, \tag{31b}$$

$$E_{n,\lambda}\psi^{(6)}_{n,\lambda}(x,y) - \mathrm{i}\partial_1\psi^{(10)}_{n,\lambda}(x,y) - m\psi^{(3)}_{n,\lambda}(x,y) = 0, \tag{31c}$$

$$-\mathrm{i}\partial_1\psi^{(1)}_{n,\lambda}(x,y) - m\psi^{(5)}_{n,\lambda}(x,y) = 0, \tag{31d}$$

$$E_{n,\lambda}\psi^{(3)}_{n,\lambda}(x,y) - V(r)\psi^{(3)}_{n,\lambda}(x,y) - \mathrm{i}\partial_2\psi^{(1)}_{n,\lambda}(x,y)$$

$$-m\psi^{(6)}_{n,\lambda}(x,y)=0, \tag{31e}$$

$$\mathrm{i}\partial_1\psi^{(3)}_{n,\lambda}(x,y)-m\psi^{(10)}_{n,\lambda}(x,y)=0. \tag{31f}$$

Similarly we can obtain the $\psi^{(4)}_{n,\lambda}(x,y)$ from four coupled equations as follows:

$$E_{n,\lambda}\psi^{(7)}_{n,\lambda}(x,y)+\mathrm{i}\partial_1\psi^{(9)}_{n,\lambda}(x,y)-\mathrm{i}\partial_2\psi^{(8)}_{n,\lambda}(x,y) -m\psi^{(4)}_{n,\lambda}(x,y)=0, \tag{32a}$$

$$E_{n,\lambda}\psi^{(4)}_{n,\lambda}(x,y)-V(\rho)\psi^{(4)}_{n,\lambda}(x,y)-m\psi^{(7)}_{n,\lambda}(x,y)=0, \tag{32b}$$

$$\mathrm{i}\partial_2\psi^{(4)}_{n,\lambda}(x,y)-m\psi^{(8)}_{n,\lambda}(x,y)=0, \tag{32c}$$

$$-\mathrm{i}\partial_1\psi^{(4)}_{n,\lambda}(x,y)-m\psi^{(9)}_{n,\lambda}(x,y)=0. \tag{32d}$$

Elimination of other components in favor of the fourth one yields

$$(E_{n,\lambda}(E_{n,\lambda}-V(\rho))-m^2)\psi^{(4)}_{n,\lambda}(x,y)+\partial_1^2\psi^{(4)}_{n,\lambda}(x,y) +\partial_2^2\psi^{(4)}_{n,\lambda}(x,y)=0. \tag{33}$$

Now, we rewrite Eq. (33) as

$$(\partial_1^2+\partial_2^2)\psi^{(4)}_{n,\lambda}(x,y) = -(E_{n,\lambda}(E_{n,\lambda}-V(\rho))-m^2) \times\psi^{(4)}_{n,\lambda}(x,y). \tag{34}$$

The first and second terms on the right-hand side in Eq. (34) lead to

$$(\partial_1^2+\partial_2^2)\psi^{(4)}_{n,\lambda}(x,y)=\nabla^2\psi^{(4)}_{n,\lambda}(x,y), \tag{35}$$

where

$$\nabla^2=\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}. \tag{36}$$

By choosing the wave function as

$$\psi^{(4)}_{n,\lambda}=\psi^{(4)}_{n,\lambda}(\rho,\varphi)=R^{(4)}_n(\rho)Q_\lambda(\varphi), \tag{37}$$

we can easily obtain

$$Q_\lambda(\varphi)=\mathrm{e}^{\mathrm{i}\lambda\varphi}, \tag{38}$$

$$\nabla^2\psi^{(4)}_{n,\lambda}=\Big(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\lambda^2}{\rho^2}\Big)R^{(4)}_n(\rho)\,\mathrm{e}^{\mathrm{i}\lambda\varphi}. \tag{39}$$

Equations (36)–(39) lead to the following first order differential equation

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\lambda^2}{\rho^2}\Big)R^{(4)}_n(\rho) +(E^2_{n,\lambda}-E_{n,\lambda}V(\rho)-m^2)R^{(4)}_n(\rho)=0. \tag{40}$$

To solve Eq. (40), we will introduce the asymptotic iteration method.

4. Basic equations of the AIM

We briefly outline the asymptotic iteration method here; the details can be found in Refs. [45]–[47]. The AIM was proposed to solve second-order differential equations of the form

$$y''=\lambda_0(x)y'+s_0(x)y, \tag{41}$$

where $\lambda_0(x)\neq 0$; $s_0(x)$ and $\lambda_0(x)$ are in $C_\infty(a,b)$. The variables, $s_0(x)$ and $\lambda_0(x)$, are sufficiently differentiable. The differential equation (41) has a general solution

$$y(x)=\exp\Big(-\int^x\alpha\,\mathrm{d}x'\Big) \times\Big[C_2+C_1\int^x\exp\Big(\int^{x'}[\lambda_0(x'')+2\alpha(x'')]\,\mathrm{d}x''\Big)\,\mathrm{d}x'\Big], \tag{42}$$

if, for sufficiently large n,

$$\frac{s_n}{\lambda_n}=\frac{s_{n-1}}{\lambda_{n-1}}=\alpha, \tag{43}$$

where

$$\lambda_n(x)=\lambda'_{n-1}(x)+s_{n-1}(x)+\lambda_0(x)\lambda_{n-1}(x),$$
$$s_n(x)=s'_{n-1}(x)+s_0(x)\lambda_{n-1}(x),\ \ n=1,2,3,\ldots. \tag{44}$$

The termination condition of the method together with Eq. (44) can also be written as

$$\delta(x)=\lambda_{n-1}(x)s_n(x)-\lambda_n(x)s_{n-1}(x)=0 \tag{45}$$

for a given potential, the idea is to convert the relativistic wave equation into the form of Eq. (41). Then, s_0 and λ_0 are determined and s_n and λ_n parameters are calculated. The energy eigenvalues are obtained by the termination condition given by Eq. (45). However, the exact eigenfunctions can be derived from the following wave function generator

$$y_n(x)=C_2\exp\Big(-\int^x\alpha_k\,\mathrm{d}x'\Big). \tag{46}$$

5. Solutions of the DKP equation in the presence of Hulthén potential

We investigate the Hulthén potential in the following equation

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\lambda^2}{\rho^2}\Big)R^{(4)}_n(\rho) +(E^2_{n,\lambda}-E_{n,\lambda}V(\rho)-m^2)R^{(4)}_n(\rho)=0, \tag{47}$$

where the Hulthén potential

$$V(\rho)=\frac{-V_0}{\mathrm{e}^{\alpha\rho}-1}, \tag{48}$$

with V and α being real parameters.

Hence for removing the first derivation in Eq. (47) we need to choose the the following wave function

$$R^{(4)}_n(\rho)=\frac{u^{(4)}_n(\rho)}{\sqrt{\rho}}. \tag{49}$$

So, we have

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1}{4\rho^2}-\frac{\lambda^2}{\rho^2}\Big)u^{(4)}_n(\rho) +\Big(E^2_{n,\lambda}+\frac{E_{n,\lambda}V_0}{\mathrm{e}^{\alpha\rho}-1}-m^2\Big)u^{(4)}_n(\rho)=0. \tag{50}$$

We use the following approximation

$$\frac{\alpha^2}{(\mathrm{e}^{\alpha\rho}-1)^2}\approx\frac{1}{\rho^2}, \tag{51}$$

which is valid for small α. Substituting this approximation into Eq. (50), we have

$$\Big(\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1}{4}\frac{\alpha^2}{(\mathrm{e}^{\alpha\rho}-1)^2}-\frac{\lambda^2\alpha^2}{(\mathrm{e}^{\alpha\rho}-1)^2}\Big)u^{(4)}_n(\rho)$$

$$+\left(E_{n,\lambda}^2+\frac{E_{n,\lambda}V_0}{\mathrm{e}^{\alpha\rho}-1}-m^2\right)u_n^{(4)}(\rho)=0. \quad (52)$$

So, by using the change of variable $z=\mathrm{e}^{-\alpha\rho}$ we can rewrite Eq. (52) as

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\frac{1}{z}\frac{\mathrm{d}}{\mathrm{d}z}+\frac{\xi_0}{(1-z)^2}-\frac{\xi_1}{z^2}+\frac{\xi_2}{z(1-z)}\right)u_n^{(4)}(z)=0, \quad (53)$$

where

$$\xi_0=\frac{1-4\lambda^2}{4},\quad \xi_1=\frac{m^2-E_{n,\lambda}^2}{\alpha^2}\quad \text{and}\quad \xi_2=\frac{E_{n,\lambda}V_0}{\alpha^2}.$$

In order to solve Eq. (53) with the aid of AIM, we should convert Eq. (53) into the form of Eq. (41). The wave function should satisfy the boundary conditions, i.e., $u_n^{(4)}(0)=0$ at $z=0$ for $\rho\to\infty$ and $u_n^{(4)}(1)=0$ at $z=1$ for $\rho\to 0$. Therefore, the reasonable physical wave function is proposed as follows:

$$u_n^{(4)}(z)=z^{\sqrt{\xi_1}}(1-z)^{(1+\sqrt{1-4\xi_0})/2}g(z). \quad (54)$$

If Eq. (54) is inserted into Eq. (53), the second-order homogeneous linear differential equation is obtained in the following form

$$g_n''(z)=-\frac{-1+z(2+\sqrt{1-4\xi_0}+2\sqrt{\xi_1})-2\sqrt{\xi_1}}{z(-1+z)}g_n'(z)-\frac{1+\sqrt{1-4\xi_0}+2(1+\sqrt{1-4\xi_0})\sqrt{\xi_1}-2\xi_2}{2z(-1+z)}g_n(z), \quad (55)$$

where

$$\lambda_0(z)=\frac{1-z(2+\sqrt{1-4\xi_0}+2\sqrt{\xi_1})+2\sqrt{\xi_1}}{z(-1+z)}, \quad (56a)$$

$$s_0(s)=-\frac{1+\sqrt{1-4\xi_0}+2(1+\sqrt{1-4\xi_0})\sqrt{\xi_1}-2\xi_2}{2z(-1+z)}. \quad (56b)$$

By means of Eq. (44), we may calculate $\lambda_n(s)$ and $s_n(s)$ as follows:

$$\lambda_1(z)=\frac{4+12\sqrt{\xi_1}+8\xi_1}{2z^2(-1+z)^2}+\frac{11+2(1-4\xi_0)+\sqrt{1-4\xi_0}(9+6\sqrt{\xi_1})+18\sqrt{\xi_1}+8\xi_1+2\xi_2}{2(-1+z)^2}-\frac{11+\sqrt{1-4\xi_0}(3+6\sqrt{\xi_1})+30\sqrt{\xi_1}+16\xi_1+2\xi_2}{2z(-1+z)^2}, \quad (57a)$$

$$s_1(z)=\frac{-2(1+\sqrt{\xi_1})+z(4+\sqrt{1-4\xi_0}+2\sqrt{\xi_1})}{2z^2(-1+z)^2}\cdot\frac{(1+\sqrt{1-4\xi_0}+2\sqrt{\xi_1}+2\sqrt{1-4\xi_0}\sqrt{\xi_1}-2\xi_2}{2z^2(-1+z)^2}. \quad (57b)$$

Combining these results obtained by the AIM with quantization condition given by Eq. (45) yields:

$$s_0\lambda_1-s_1\lambda_0=0$$
$$\Rightarrow \xi_{2,0,\lambda}=\frac{1}{2}+\sqrt{1-4\xi_0}\left(\frac{1}{2}+\sqrt{\xi_1}\right)+\sqrt{\xi_1}, \quad (58a)$$
$$s_1\lambda_2-s_2\lambda_1=0$$
$$\Rightarrow \xi_{2,1,\lambda}=\frac{5}{2}+\sqrt{1-4\xi_0}\left(\frac{3}{2}+\sqrt{\xi_1}\right)+3\sqrt{\xi_1}, \quad (58b)$$
$$s_2\lambda_3-s_3\lambda_2=0$$
$$\Rightarrow \xi_{2,2,\lambda}=\frac{13}{2}+\sqrt{1-4\xi_0}\left(\frac{5}{2}+\sqrt{\xi_1}\right)+5\sqrt{\xi_1}. \quad (58c)$$

When the above expressions are generalized, ξ_0,ξ_1,ξ_2 statements containing the energy eigenvalues turn into

$$\xi_{2n}=\frac{1}{2}+n+n^2+\sqrt{1-4\xi_0}\left(\frac{1}{2}+n+\sqrt{\xi_1}\right)+\sqrt{\xi_1}+2n\sqrt{\xi_1}. \quad (59)$$

If the above ξ_0, ξ_1, ξ_2 expressions are inserted, the energy eigenvalues statement of the DKP equation for the Hulthén potential is as follows:

$$-E_{n,\lambda}V_0+\alpha^2\left[\frac{1}{2}+n+n^2+\lambda\left(1+2n+2\sqrt{\frac{m^2-E_{n,\lambda}^2}{\alpha^2}}\right)+(1+2n)\sqrt{\frac{m^2-E_{n,\lambda}^2}{\alpha^2}}\right]=0. \quad (60)$$

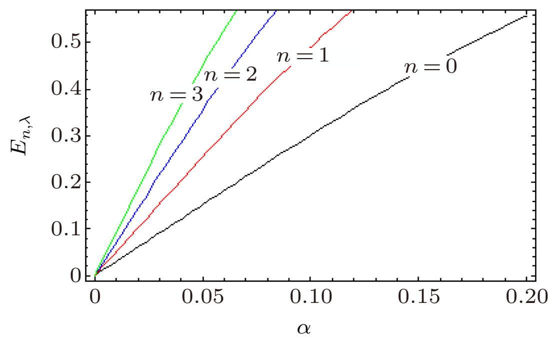

Fig. 1. (color online) Variations of energy eigenvalue with the α screening parameter. The parameters are in atomic units ($\hbar=c=m=\lambda=V_0=1$).

In Fig. 1 we can deduce that with the decrease of parameter α the values of energy become close together, and when $\alpha=0$ the energy eigenvalues have a same value, so a degeneracy can be reached. A simple glance at the energies reveals that the energy difference between the levels decreases with principal quantum number increasing. For example,

$E_{8,3}-E_{8,2}=0.032923\ \mathrm{fm}^{-1}$, $E_{7,3}-E_{7,2}=0.034133\ \mathrm{fm}^{-1}$,
$E_{6,3}-E_{6,2}=0.035264\ \mathrm{fm}^{-1}$, $E_{5,3}-E_{5,2}=0.036297\ \mathrm{fm}^{-1}$,
$E_{4,3}-E_{4,2}=0.037213\ \mathrm{fm}^{-1}$, $E_{3,3}-E_{3,2}=0.037990\ \mathrm{fm}^{-1}$,
$E_{2,3}-E_{2,2}=0.038613\ \mathrm{fm}^{-1}$, $E_{1,3}-E_{1,2}=0.039065\ \mathrm{fm}^{-1}$,
$E_{0,3}-E_{0,2}=0.039335\ \mathrm{fm}^{-1}$.

the unit 1 fm $=10^{-15}$ m. Moreover, numerical solutions of the DKP equation without any approximation are obtained by using the finite difference method[48] to check the exactness of our energy eigenvalue equation (60). These results are given in Table 1.

Table 1. Energy eigenvalues for $m = 1$, $V_0 = 0.5$, $\alpha = 0.01$.

$\vert n,\lambda\rangle$	Our calculation	Numerical results
$\vert 0,0\rangle$	0.020096	0.027401
$\vert 0,1\rangle$	0.060191	0.060898
$\vert 0,2\rangle$	0.099999	0.098268
$\vert 0,3\rangle$	0.139334	0.13946
$\vert 1,0\rangle$	0.060390	0.070536
$\vert 1,1\rangle$	0.100593	0.10801

Now, the corresponding eigenfunctions can be obtained for the DKP equation with Hulthén potential by using the wave function generator given by Eq. (46). The first lowest states are given as follows:

$$g_0(z)=1, \tag{61a}$$

$$g_1(z)=-C_2(1+2\sqrt{\xi_1})\Big(1-\frac{2+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}}{1+2\sqrt{\xi_1}}z\Big), \tag{61b}$$

$$\begin{aligned} g_2(z)=&\,C_2(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})\\ &\times\Big[1-\frac{2(3+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})}z\\ &+\frac{(3+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})(4+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})}z^2\Big], \end{aligned} \tag{61c}$$

and

$$\begin{aligned} f_3(z) =& -C_2(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})(3+2\sqrt{\xi_1})\Big[1-\frac{3(4+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})}z\\ &+\frac{3(4+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})(5+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})}z^2+\frac{3(4+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})(5+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})}z^2\\ &-\frac{(4+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})(5+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})(6+2\sqrt{\xi_1}+\sqrt{1-4\xi_0})}{(1+2\sqrt{\xi_1})(2+2\sqrt{\xi_1})(3+2\sqrt{\xi_1})}z^3\Big]. \end{aligned} \tag{62}$$

Thus, the wave function $g_n(z)$ can be written as

$$g_n(z)=(-1)^n C_2\frac{\Gamma(1+2\sqrt{\xi_1}+n)}{\Gamma(1+2\sqrt{\xi_1})}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};z). \tag{63}$$

Hence, we can write the total radial wavefunctions as follows:

$$u_n^{(4)}(z)=Nz^{\sqrt{\xi_1}}(1-z)^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};z), \tag{64}$$

$$\psi_{n,\lambda}^{(4)}(\rho,\phi)=\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}. \tag{65}$$

Since $\psi_{n,\lambda}^{(4)}(\rho,\phi)$ is found, the ten-component stationary spinor is also obtained

$$\psi_{n,\lambda}^{(3)}(\rho,\phi)=\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}, \tag{66}$$

$$\begin{aligned} \psi_{n,\lambda}^{(1)}(\rho,\phi)=&\frac{\mathrm{i}}{E_{n,\lambda}}\Big(\sin\phi\frac{\mathrm{d}}{\mathrm{d}\rho}+\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\\ &\times\Big[\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big], \end{aligned} \tag{67}$$

$$\begin{aligned} \psi_{n,\lambda}^{(5)}(\rho,\phi)=&\frac{1}{\mathrm{i}m}\Big(\cos\phi\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\Big\{\frac{\mathrm{i}}{E_{n,\lambda}}\Big(\sin\phi\frac{\mathrm{d}}{\mathrm{d}\rho}+\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\\ &\times\Big(\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big)\Big\}, \end{aligned} \tag{68}$$

$$\begin{aligned} \psi_{n,\lambda}^{(6)}(\rho,\phi)=&\frac{1}{m}\Big\{(E_{n,m}-V(\rho))\\ &\times\Big[\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big]\\ &+\frac{1}{E_{n,\lambda}}\Big(\sin\phi\frac{\mathrm{d}}{\mathrm{d}\rho}+\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\Big(\cos\phi\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\\ &\times\Big[\frac{N\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big]\Big\}, \end{aligned} \tag{69}$$

$$\psi_{n,\lambda}^{(7)}(\rho,\phi)=\frac{N(E_{n,\lambda}-V(\rho))\,\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{m\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi},\quad(70)$$

$$\psi_{n,\lambda}^{(8)}(\rho,\phi)=\frac{\mathrm{i}}{m}\Big(\sin\phi\frac{\mathrm{d}}{\mathrm{d}\rho}+\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\times\Big[\frac{N\,\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big],\quad(71)$$

$$\psi_{n,\lambda}^{(9)}(\rho,\phi)=\frac{1}{\mathrm{i}m}\Big(\cos\phi\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\times\Big[\frac{N\,\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big],\quad(72)$$

$$\psi_{n,\lambda}^{(10)}(\rho,\phi)=\frac{\mathrm{i}}{m}\Big(\cos\phi\frac{\mathrm{d}}{\mathrm{d}\rho}-\frac{\sin\phi}{\rho}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big)\times\Big[\frac{N\,\mathrm{e}^{-\alpha\rho\sqrt{\xi_1}}(1-\mathrm{e}^{-\alpha\rho})^{(1+\sqrt{1-4\xi_0})/2}{}_2\mathrm{F}_1(-n,1+2\sqrt{\xi_1}+\sqrt{1-4\xi_0}+n;1+2\sqrt{\xi_1};\mathrm{e}^{-\alpha\rho})}{\sqrt{\rho}}\,\mathrm{e}^{\mathrm{i}\lambda\phi}\Big].\quad(73)$$

Now we check the energy relation for J/ψ. Choosing $\alpha = 0.2\ \mathrm{fm}^{-1}$, $V_0 = 4.75466\ \mathrm{fm}^{-1}$, and $m_c = 7\ \mathrm{fm}^{-1}$, we determine $M_{\mathrm{theor}} = 1.485\ \mathrm{fm}^{-1}$ which is in acceptable agreement with its experimental value $M_{\mathrm{exp}} = 15.485\ \mathrm{fm}^{-1}$.

6. Concluding remarks

We obtained energy eigenvalues and corresponding eigenfunctions by the asymptotic iteration method. On the other hand, we investigated spin-one particles by using the Duffin–Kemmer–Petiau equation with the Hulthén potential in (1+2) dimensions. This study presents a different approach, the AIM, to the calculation of the bound state solutions of the DKP equation with the Hulthén potential in (1+2) dimensions for unity spin particles. For an arbitrary quantum number n state, we obtained the energy eigenvalues and corresponding eigenfunctions in the $1/\rho^2$ approximation by AIM. The advantage of the AIM is that it gives the eigenvalues directly by converting the second-order differential equation into the form of $g'' = \lambda_0(x)g' + s_0(x)g$. The non-normalized wavefunctions are easily constructed by iterating the values of s_0 and λ_0. The method presented in this study is general and worth extending to the solution of other interactions. Also we elucidated the energy spectra for various quantum numbers and the behaviors of the energy spectrum versus some parameters were also investigated. The results of this study are definitely useful for investigating a wide range of physical problems, from meson spectroscopy to cosmology.

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