

## SOLVABILITY OF NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER IN REFLEXIVE BANACH SPACE

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**Abstract.** In this work, we are concerned with weak and pseudo solutions for some initial value problems of fractional order and their corresponding functional integral equation of fractional order. These initial value problems includes many initial value problems that arise in nonlinear analysis and its applications.

**Key Words and Phrases:** Fractional Pettis integral, weakly relatively compact, functional differential equation, O'Regan fixed point theorem.

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