Solver-Based Mixed Integer Linear Programming (MILP) Based Novel Approach for Hydroelectric Power Generation Optimization

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ABSTRACT In Pakistan, hydroelectric power is one of the reliable sources of electricity with a capacity of 8,713 MW, which is 29% of the total energy mix. Hence, with such a vast resource capacity of hydroelectric power, its optimization and dispatch planning will have a great significance. This research work discusses the importance of hydroelectric power generation planning for both storage and run-of-river (ROR) hydropower plants, as well a; a solver-based optimization technique is proposed for the first time to resolve the intricate job of generation planning for hydroelectric power plants in MATLAB. A mathematical-optimization model is also developed, which uses a Mixed-Integer Linear-Programming (MILP) algorithm, based on the objective function of profit maximization, which considers a random varying revenue plan as model input. Three hydropower generators of different capacities and efficiencies are considered for the optimization problem. MILP based solution is proposed for both storage and ROR hydropower plants with two dispatch schedules, i.e., Normal dispatch schedule and optimum dispatch schedule. The objective functions are solved, and the profit (in dollars) from each dispatch schedule is calculated and compared. The preliminary optimization results show an increase of $22,000 and $29,130 in the profit for storage and ROR hydropower plants, respectively, which is 19% more than average income. Hence, ensuring the credibility of the proposed algorithm for maximizing the revenue ($), is aimed to facilitate and assist better planning for electric power producers.

INDEX TERMS Power generation planning, hydroelectric power system, MATLAB, mixed integer linear programming, mathematical optimization model, water storage dams, run-of-river.

I. INTRODUCTION

Electricity is the basic need of today's life in almost every field of life. It is traded in the same way as other commodities in the deregulated energy market. The big challenge in selling electricity as a commodity is its reliability. Any disruption in its supply will have a substantial impact upon a simple residential up to large industrial and metropolitan loads. One of its major causes is the input power supply uncertainties to the generating stations or an unexpected increase in demand. For the electric power systems like hydroelectric power systems, the variability in the input power is mainly due to the

The associate editor coordinating the review of this manuscript and approving it for publication was Siqi Bu.

uncertainty of available water heads in the dam project and the uncertainty due to the changing water discharge in (Run of river) ROR projects. Such variabilities and uncertainties of supplies must be mitigated before integrating the system to the national grid; otherwise, the resulting unbalance between the generation and demand can cause serious reliability problems, which lead to the full system shutdown or blackouts. That requires efficient transmission and distribution planning to balance out the fluctuations and variations in the output of hydroelectric power plants in the real time.

The kinetic and potential energy of the water flow in hydroelectric power plants is used to produced Electrical energy, utilizing turbine generator set [1]. Hydroelectric power plants (HPP) exist in the shape of ROR hydroelectric projects, which

generates electricity by tapping the water instantly from the rivers and supplying it to the turbine generator set, and also in the shape of water storage dam projects, which generates electricity through supplying the stored water to the turbine generator set with help of penstocks. The overall cost of dams projects is higher than the ROR projects. But due to the variable discharge rate of ROR, they are less reliable; making the dams projects on the upper hand based on the scale of reliability.

Integrated power systems (IPS) were proposed as a solution to the water storage capacity issues. IPS is very much complex system to analyze. In an IPS, the generating stations, transmission lines, distribution lines, substations and demand locations are grouped as a single component. Based on the capacity of these components, weather conditions and location, their operational characteristics also varies e.g. by varying the diameter of the conductor and the length of the transmission lines, the power losses vary in it. Therefore, the long transmission lines and thin conductors will be having more power losses as compared to the short transmission line with thick conductors. Power system operators (PSO) and electric utilities have a major role in integrated power systems. The monitoring and controlling of power systems and the supply of reliable and uninterrupted electricity are the responsibilities of PSOs. With the increase in the components of the power system, its complexity increases, so monitoring and controlling for the PSO becomes difficult. Hence, the hydroelectric power generation planning and optimization need a detailed mathematical model to analyze and assess the complexities of the integrated power systems. The hydro-electric power stations are located very far from the demand sites in Pakistan. Therefore, an optimal and efficient power transmission and distribution planning is required to supply reliable power to the demand customers. There is an immense need to optimize and plan the dispatch of hydroelectric power plants by developing a mathematical optimization model. First time, MILP based solution is proposed for the PSOs to monitor and dispatch the hydroelectric generators optimally and maximize the objective function of profit for the proposed storage and ROR setups. Establishing an objective function for the scenario of the poly generation system is a novel approach od this paper. Reasons for selecting and preferring MILP optimization over others is explained in detail in the related work portion of this article.

The structure of this paper is as: Part-I consists of reasons and importance of transmission and distribution planning and as well as need of mathematical model for optimization. Part-II contains structure and components of hydroelectric plants, whereas Part-III consist of proposed methodology. While Part-IV contains results, which is followed by discussion and conclusion.

II. HYDRO ELECTRIC POWER

Hydropower is the rate of extracting hydraulic energy from the flow of water as result of its position or velocity or both [1]. The small-scale hydropower plants are mostly
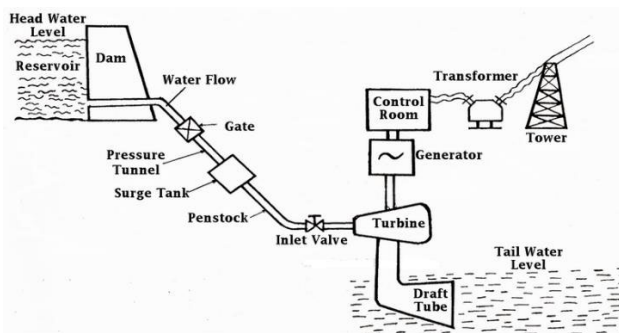
FIGURE 1. Hydroelectric Power plant and its basic components adapted from [2].

utilized for mechanical application as well as for the generation of electric power, whereas, the large-scale hydropower plants also known as hydroelectric power plants, are typically used for power generation only. A typical hydroelectric power is shown in Fig. 1. Hydroelectric power generation is considered as one of most economical in terms of cost/kwh [3]. For a well-planned and operated hydroelectric power generation the normalized cost of electricity generation ranges from 0.02 US\$/kwh to 0.19 US\$/kwh [4]. Due this reason hydroelectric power is the first choice for investment by the power utility companies as a base load provider. In 2015, hydropower generated 16.6% of the world's total electricity and 70% of all renewable electricity and was expected to increase by about 3.1% each year for the next 25 years [5]. In the current era, electricity is one of the most important aspect and the basic need of human lives. The overall economy of a country depends upon the business growth of that country.

Hence the electricity reliability issues have a great impact upon the economy of the country which may result in bankruptcies. In this scenario, hydroelectric power plants are the most reliable source of electricity as they are able to respond quickly to any fluctuations in the demand of electric power as compared to other electric power sources [6], [7]. Hydroelectric power plants are the most efficient source of electricity as they covert the mechanical energy from turbines directly to the electrical energy. The efficiency of HPP is mostly near to 85% [8].

A. HYDROELECTRIC POWER PLANTS
Hydro-mechanical energy is converted to electrical energy through the principle of Faraday's Law of electromagnetic induction. In ROR project the kinetic energy of water is utilized, whereas in storage power plants the potential energy of the stored water is first converted into kinetic energy by means of penstock and applied to the mechanical turbine tied with shaft of electric generator which produces the respective electrical energy. The amount of electric power produced in hydroelectric power plants is given by [9]:
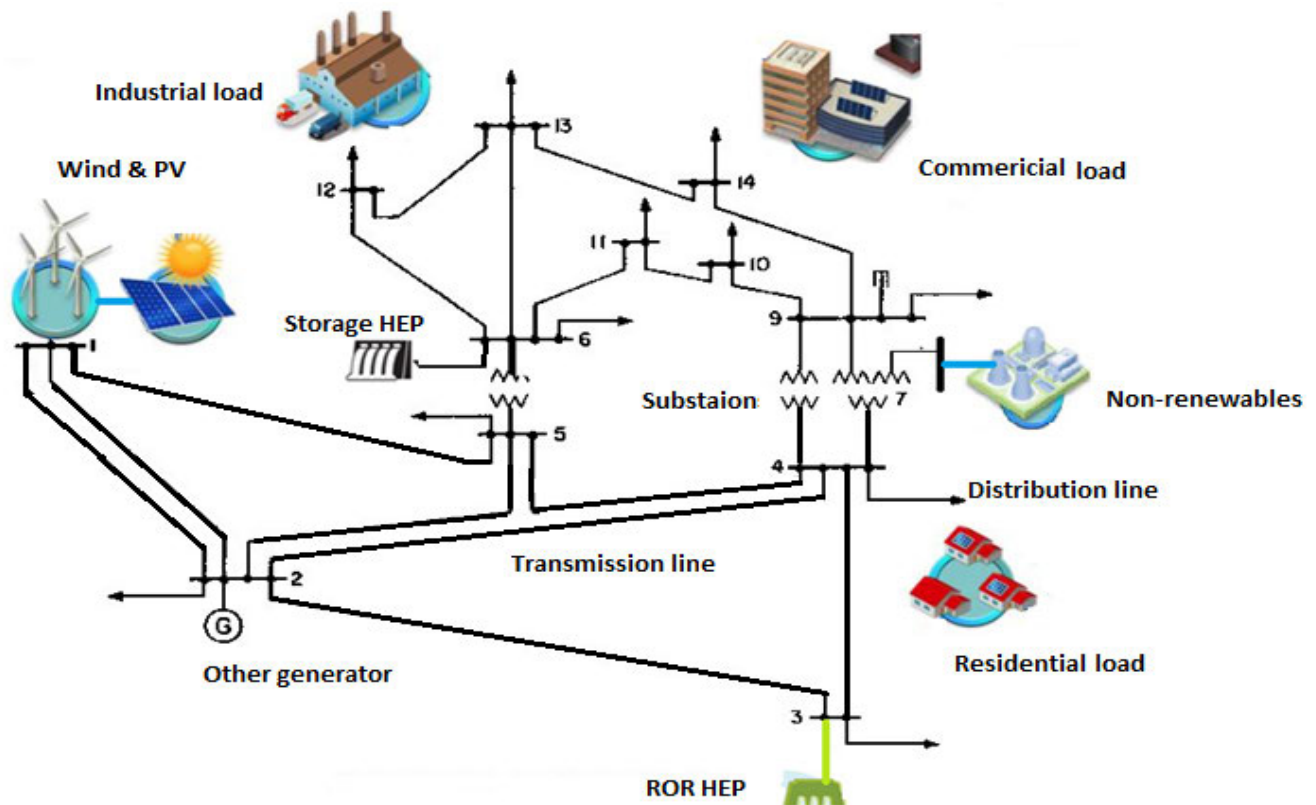
$$P = \eta \rho g h Q \tag{1}$$

FIGURE 2. A typical IPS adapted from [10].

where,

ρ density of water (kg/m^3)
g gravitational acceleration (m^2/sec)
η overall efficiency
Q water-discharge rate (m3/sec)
h head in the reservoir (m)

Different hydro power plants (HPPs) are interconnected to form an integrated power system (IPS). A typical integrated power system is shown in Fig. 2. The power in an IPS is generated from different energy sources both renewable (solar, wind etc.) and non-renewable (fossil fuels etc.) and transmitted to different types of electrical loads (industrial, commercial, residential etc.). These power users are connected to the generators though transmission lines, distribution lines and substations.

Hydroelectric power plants (HEP) have tendency to respond very quickly to a change in the load as compared to other power plants. Therefore, this characteristic (high ramp rate) of hydroelectric power plants enables the power system operators (PSO) to provide the frequency control and the peaking power quickly to the grid by dispatching from hydroelectric generators. Moreover, hydroelectric power is an environmentally friendly and the cheapest source of energy. The run of river hydroelectric power plant (ROR HEP) generators can be dispatched quickly for the generation of electric power due to their high ramp rate. Therefore, hydroelectric power

is the best source of energy as compared to other renewable energy resources like wind and solar [11]. The water storing capability and scheduling of storage HPP make them able to optimize the utilization of electricity as compared to other energy sources [12].

B. RELATED WORK

The electricity generation from hydroelectric power plants is greatly affected by the climate and environmental changes, due to which the supply uncertainties take place [13], [14]. If these uncertainties in the supply are not taken care of, and the PSOs do not well plan the transmission and distribution systems, may result in socio-economic problems. Different planning techniques are used in the field of power system optimization. The electrical energy is transported in two stages, i.e., first from the generating station to the transmission station substations and second to the distribution stations from the transmission substation. Due to the changing demands, transmission losses, and the limited capacity of the transmission and distribution network, the problem arises, which are all grouped into the transportation problem (TP). The transportation problem handles the minimization of the shipping cost by determining the shipping schedule while satisfying the supply and demand limits [15], [16]. The shortcoming of the above technique is that it only addresses transportation and optimizes cost by suggesting different transportation routes. In [17], Lou *et al.*

proposed methodology for mathematical modeling and developed a mathematical model for the sustainability of small-scale hydropower plants. In [18] an improved and updated Particle Swarm Optimization (PSO) technique is used for optimization, which is resilient particle swarm optimization (RPSO). In RPSO, velocity of particle is not dependent on the size of the distance between the individual and the optimal particle but only dependent on its direction. The shortcoming of RPSO is that it has very fewer parameters and does not address generation capacity optimization. Moreover, in [19]–[22], different deterministic techniques are explained for the scheduling of hydroelectric power plants. The deterministic model gives an outstanding result for uncertainty in the field of demand and supply. The only shortcoming of this technique is that it does not optimize the final cost. To consider many types of uncertainties at the same time and get an optimum solution, a stochastic optimization approach is used in [16], [22]–[26]. In [27] some of the "Artificial Neural Network (ANN)" based optimization methods are discussed for optimal scheduling of hydroelectric generations. Based on computational time, complexity, and output results of the models, every technique has its own advantages and disadvantages. The models discussed in the literature, some consider generation and demand variations for implementing the variability in the power system while some consider certainty for a small period in the future conditions. These articles proposed techniques for planning the generation or demand. In [30] author proposes virtual energy hub (VEH) which optimizes its revenue from participating in the electrical and thermal energy markets by examining both local markets.Uncertainties of the proposed system are modelled by using a pure stochastic optimization method. The price uncertainty of this system is modelled by an information gap decision theory (IGDT) method. It cannot be applied to poly generation system. Whereas in [31] case study has been taken, in which only two home energy hub (HEH) and two Conventional building (CBs) exist in which all elements of this structure exchange information with Central energy management system (CEMS). HEHs send the information including the amount of excess power/shortage, energy tariff, and the CBs only send their amount of shortage power at each hour to CEMS unit. photovoltaic system is the only renewable energy generation resource in the HEHs. In this manner electricity cost has been reduced so that when it is required only then harnessed and utilized. While in [32] the author proposes a multi-objective two-stage stochastic unit commitment scheme for integrated gas and electricity networks taking into account novel flexible energy sources such as power to gas P2G technology and demand response (DR) programs as well as high penetration of wind turbines. In this paper, P2G technology is introduced as a promising option for increasing the wind power dispatch in power systems. This technique increases generation capacity only. It has nothing to do with cost optimization. In [33] cost optimization is done by demand shifting potential, which can be tapped through shifting certain amounts of energy demand from some time periods to others with lower expected demand, typically to match price values and to ensure that existing generation will be economically sufficient. Authors claim that it is also possible to obtain the maximum profit with the coalition formation. The impact of the consumption shifting in the multiple home microgrid (H-MG) schedule has been considered while conducting both individual and coalition operations. In [34] author proposes, the concept of multi-objective-techno-economic environmental optimization for scheduling electric vehicle charging/discharging. End user energy cost, battery degradation, grid interaction and CO_2 emissions in the home micro-grid context are modeled and concurrently optimized for the first time while providing frequency regulation. The multi-objective techno-economic-environmental optimization (MOTEEO) approach is proposed and applied in three case studies and for different charging strategies in order to find the synergy of four objectives: energy cost, EV battery degradation, grid net exchange and CO_2 emissions along with ancillary services. In [35] three solutions have been proposed to cover the challenges of gas system constraints and the uncertainty of wind power: 1) using information-gap decision theory (IGDT) based robust approach to address the uncertainty caused by the intrinsic nature of wind power, 2) Integration of compressed air energy storage (CAES), and demand response (DR) in day-ahead scheduling and 3) considering flexible ramping products in order to ensure reliable operations, there must be enough ramp to eliminate the variability of wind power in real-time dispatch stage.

Generation cost optimization for multi generator set, operating in parallel, has not been addressed in the literature. However, in this research work, a first-time solver-based mixed-integer linear programming (MILP) technique is proposed considering the variations in all the components. In this article, we proposed MILP for the first time based on the real-time data of Neelam Jhelum Power Plant. There are multiple reasons for selecting MILP optimization technique; some of them are:

Mixed-integer Linear programming is a state-of-the-art technique, widely used for the optimization of poly-generation systems [36]. Poly-generation systems are those systems that produce electricity, heat, and cooling at the same time, i.e., thermal power plants. The non-linear constraints (load, temperature, and size) concerning the performance curve of the generating unit mostly results in non-linearity, non-convexity, and many variables that are extremely challenging to solve with the cutting-edge techniques. For this reason, MILP is the best possible technique to solve the optimization problem. The main advantages of MILP over the other proposed algorithms utilized for optimization problem are.

1) MILP algorithm simplifies the objective function by converting all non-linear variables to linear variables in order to mitigate the non-convexity of the problem.
2) MILP represents the inclusion and exclusion of a generating unit as a binary variable to minimize the operation constraint of the units in ON and OFF modes.

3) Due to the convexity of MILP, it provides globally optimum and accurate solution.

4) It provides no optimum solution only when there is a contradiction between the constraints and the problem is unbounded in the direction of the objective function [37].

5) The most substantial benefit of MILP algorithm is the availability of extremely fast and effective commercially available solvers i.e. CPLEX [38], Gurobi [36], Xpress [39], MATLAB [40].

In [37], a MILP model is developed for a tri-generation plant for a hospital complex. Bischi *et al.* [38] proposed a MILP optimization model for minimizing the O&M cost of a co-generation system. Ren *et al.* [39] utilized the MILP approach to determine the optimum schedule of the generating units by minimizing the overall energy cost for a grid-connected distributed energy system. Moreover, in [40] Ameri *et al.* proposed the MILP solution for the optimization of a tri-generation systems connected to the cooling and heating systems of the district. However, in this research work the MILP approach is used for the generation planning and cost optimization for hydroelectric power plants for the first time. A two-stage solver-based MILP solution is proposed to optimally dispatch the hydroelectric power generators during the high revenue intervals to maximize the overall energy profit form the plant.

III. THE PROPOSED METHODOLOGY

The research problem presented in this article is formulated based on a solver-based approach for the optimal dispatch of power generators. The energy supply sources are dam and ROR power projects. The generation system efficiently generates the electric power and then this power is distributed in large amount to consumer ends through transmission and distribution system. In order to comprehensively formulate the research problem, three different generators are considered having different design parameters, operational and maintenance costs. Therefore, their generation efficiencies vary from one generator to another generator. In the electrical market the electricity pricing is different for different phases of the day. By taking the advantage of this variable pricing, the revenue/MWh could be increased, and the cost could be reduced by dispatching the generators during high pricing hours.

In this research work, a solution for optimal power generation is proposed by incorporating the variability in the generation. For this purpose, a solver-based mathematical model is developed for storage and ROR hydroelectric power plants in MATLAB. The model in the first-stage consists of the hydroelectric power planning and optimization for storage HPP, whereas in the second stage it is done for ROR hydroelectric power plants. An optimal power generation solution is proposed by considering three generators for a time frame of 48 hours. Three power capacity levels are defined for each generator i.e. OFF, HIGH and LOW as shown in Table 1.

TABLE 1. Power level for the hydroelectric generators.

Generator	Power Level	Fuel (water) Consumption
Generator 1	*OFF*	*Zero*
	HIGH	*Maximum*
	LOW	*Minimum*
Generator 2	*OFF*	*Zero*
	HIGH	*Maximum*
	LOW	*Minimum*
Generator 3	*OFF*	*Zero*
	HIGH	*Maximum*
	LOW	*Minimum*

Depending upon the power levels, each generator has a rate of fuel (water) consumption and power production. The rate of fuel (water) consumption is minimum for LOW power level, maximum for HIGH power level and zero for OFF power level for each generator. In this model the maximum and minimum generation capacity parameters along with the generation cost (starting cost + running cost), revenue per megawatt hour (MWh) and generation efficiency are considered for the problem formulation. Moreover, if the past generation data, the capacity parameters and the cost data are known, then the mathematical model is applicable to other sources of energy i.e. thermal, PV, wind, nuclear etc.

TABLE 2. Definition of indexes.

Indexes	Description
i	Time period index i.e. $1 \le i \le 48$ for this research work.
j	Generator index i.e. $1 \le j \le 3$ for this research work.
k	Power level index i.e. • $k = 0$ for OFF power level • $k = 1$ for LOW power level • $k = 2$ for HIGH power level

A. MATHEMATICAL OPTIMIZATION MODEL

1) PROBLEM NOTATION AND DECISION VARIABLE

The scheduling problem in this research work is formulated as a binary inter programing problem. The indexes i, j and k are defined in Table 2. Moreover, the problem notations and parameters used in this research work as defined in Table 3, whereas the decision variables are defined in Table 4.

2) OBJECTIVE FUNCTION

The objective-function of the mathematical optimization model is to maximize the total profit from total electric power generation by minimizing the total generation cost over a certain period. The objective function consists of running cost (operational/ maintenance) of the generators ($/MWh), the revenue from generators ($/MWh) and the cost for-starting the-generators.

The per-unit energy-cost ($/Mwh) is considered constant for the selected interval of the problem formulation. For each interval, the power generation cost of a generator is given by the product of the generated energy

TABLE 3. Definition of problem notation and parameters.

Notation	Description
$nPeriods$	The number of time periods i.e. 48 in this work.
$poolPrice(i)$	The total revenue (dollars) per MWh at interval i.
$ngen$	Total number of generators.
$gen(j,k)$	Total power (MW) generated by generator j at power level k.
$fuel(j,k)$	Total fuel (water) utilized by generator j at power level k.
$totalHead$	Total head available for 1 day.
$\eta(j,k)$	Efficiency of generator j at power level k.
$Rg(i,j)$	Revenue of generation for generator j at interval i.
$Cg(i,j)$	Cost of generation for generator j at interval i.
$Cs(i,j)$	Cost of startup for generator j at interval i.

TABLE 4. Decision variables.

Decision Variables	Description
$y(i,j,k)$	"Binary scheduling vector".
$z(i,j)$	Generator startup vector. • $z(i,j) = 1$, when generator j is OFF at interval i and ON at interval $i + 1$. • $z(i,j) = 0$, when generator j is ON continuously.
$s(i,j,k)$	The generated energy by generator j in MWh at interval i at power level k for storage plants.
$r(i,j,k)$	The generated energy by generator j in MWh at interval i at power level k for ROR plants.
$D(i,j,k)$	Total designed capacity generator j at interval i at power level k.
$x(i,j,k)$	The solution vector
m	Numbers of generators, which, in our case, are 3.

by the generator (in Mwh) to the per-unit energy cost (in \$/Mwh). Similarly, in the same way the generation cost of all the generators is computed and then added together to find the total generation cost. The same process is applied to compute the total incoming generation revenue and startup cost form all the generators. Finally, the total generation cost, startup generator cost and the incoming revenue are summed together to calculate the objective function for a certain selected period.

Mathematically,

$$Maximize\ Pr = Time * \left[\left[\sum_{i=1}^{i=n} \sum_{j=1}^{j=m} Rg\,(i,j) \right] - \left[\sum_{i=1}^{i=n} \sum_{j=1}^{j=m} Cg\,(i,j) \right] - \left[\sum_{i=1}^{i=n} \sum_{j=1}^{j=m} Cs\,(i,j) \right] \right] \quad (2)$$

where, Pr is the total profit from the generation of electric power, $n = 48$ and $m = 3(No : of\ Generators)$, for this research work.

3) CONSTRAINTS

The constraints considered for mathematical optimized model are given as;

1. The total amount power produced by each generator at interval i should be less than or equal to the total capacity. i.e.

$$r\,(i,j,k)\,, s\,(i,j,k) \leq D\,(i,j,k) \quad \forall i,j,k \quad (3)$$

2. The generated power by each generator cannot be negative. i.e.

$$r\,(i,j,k)\,, s\,(i,j,k) \geq 0 \quad \forall i,j,k \quad (4)$$

3. "Binary scheduling vector" constraint:
   • When generator j is operating on power level k at interval i, then

$$y\,(i,j,k) = 1 \quad \forall i,j,k \quad (5)$$

   • When generator j is OFF, then

$$y\,(i,j,k) = 0 \quad \forall i,j,k \quad (6)$$

4. Generator startup indicator variable constraint:
   • When generator j is OFF at interval i and ON at interval i+1.

$$z\,(i,j) = 1 \quad \forall i,j,k \quad (7)$$

   • When generator j is ON continuously.

$$z\,(i,j) = 0 \quad \forall i,j,k \quad (8)$$

Whereas, equation (7) and (8) shows generator startup vector, when it is equal to 1 or 0 for any specific generator j, which means generator is off/on at interval i and on/off at interval $i+1$. Where is k is the collective power level of all three generators. So, for single generator startup vector k exists.

B. SIMULATION SETUP

To tackle the problem of hydroelectric power generation planning and optimization under different level of input supplies, a random set of data is taken for a period of 48 hours for both storage and ROR power projects. The data set is uniformly distributed for each interval. The sample size of the data can be increased, so as the accuracy of the results can be increased but the total computation will also be increased accordingly. A total of three generator sets are considered with different power generation capacities, fuel (water) usage and efficiencies to address the uncertainties of power generation in the optimum dispatch model. The optimum dispatch model utilizes an optimization algorithm known as solver-based optimization problem setup [29].

For the problem formulation, the optimization toolbox in MATLAB R2020a is used to solve the model. The optimization toolbox is an optimization-solver for system of non-linear equations, linear programming, least-squares, quadratic programming, problem-based and solver-based optimization of a mathematical model.
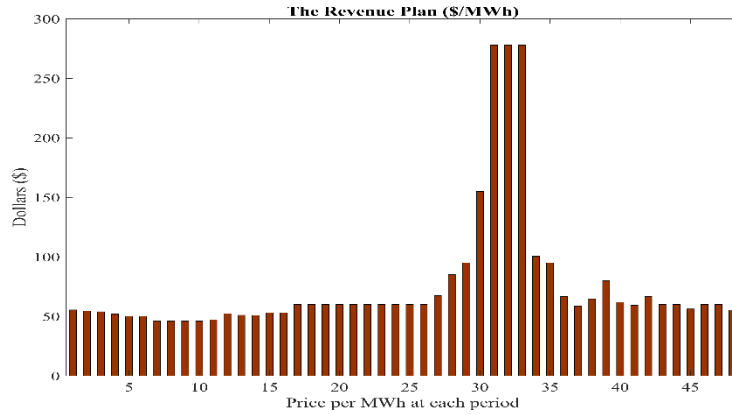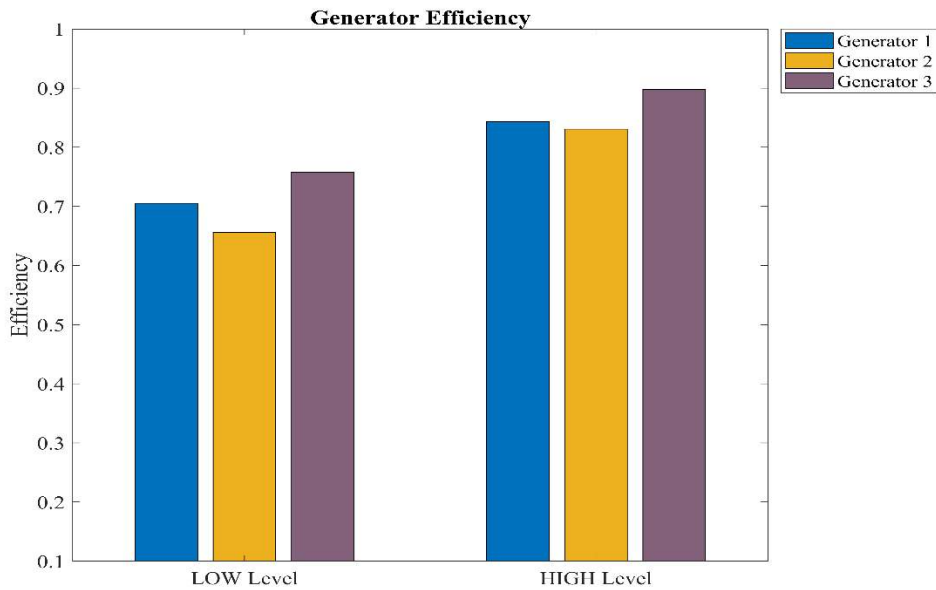
FIGURE 3. Revenue plan for mathematical model.

FIGURE 4. Generator Efficiency at LOW and High-power levels.

1) POOL PRICE

Pool price is the revenue plan (in dollars) per megawatt hour (MWh) for the power plant which is decided by the electric regulatory market. To address the problem in this research work, a variable revenue ($/MWh) is selected as input to the model as shown in Figure 3. By taking the advantage of different revenue prices at each interval, we can schedule the power plant generators in those intervals in which the revenue ($/MWh) is the highest in order maximize the total revenue and alternatively maximizing the objective function i.e. Total profit.

2) RESOURCE CAPACITY PARAMETERS

As stated earlier, the efficiencies of the hydroelectric generators are assumed to be different from one another. Therefore, the power capacity and the fuel (water) input are different for each generator. The list of resource capacity parameters which are used as input to the model are shown in Table 5.

C. GENERATOR EFFICIENCY

All the generators used in this research work are set to have different generating efficiency from one another at HIGH and LOW operating levels. The efficiency of the generators at LOW and HIGH power levels are depicted in Figure 4. It can be noticed that the efficiency of generator-3 is a bit more than generator-1 and generator-2 at its corresponding operating-points i.e. LOW and HIGH. Moreover, the efficiency of generator-1 and generator-2 at its HIGH operating level is higher than the efficiency of generator-3 at its LOW operating level.

IV. ANALYSIS OF SIMULATION RESULTS

In this research work numerical experimentation is done on both storage and ROR hydroelectric power plants. A solver-based optimization solution approach is implemented which utilizes a mixed integer linear programing (MILP) solution technique in MATLAB. After fitting the data and parameters
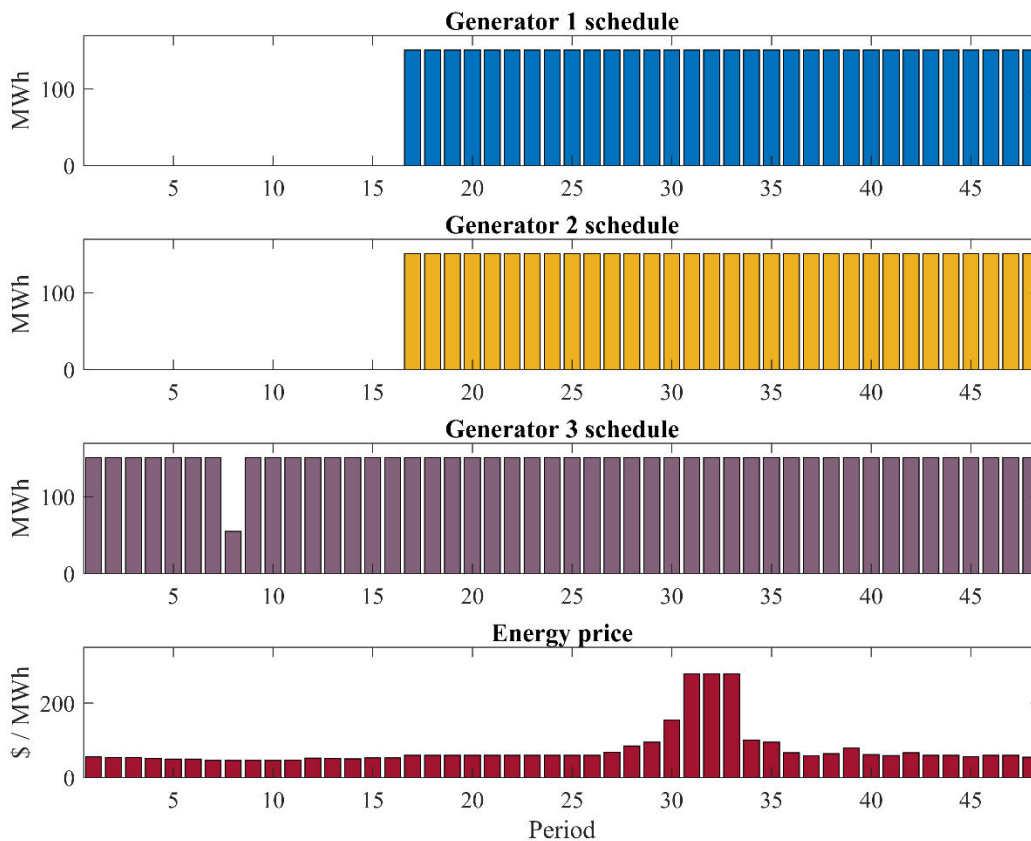
FIGURE 5. Normal dispatch schedule of generators for storage power plant.

TABLE 5. Resource capacity parameters.

Generator	Capacity (MW)		Fuel (Water turbine) Input (MW)		Efficiency* (%)	
	H[1]	L[2]	H	L	H	L
1	151	55	179	78	84.4	70.5
2	152	61	183	93	83.1	65.6
3	150	50	167	66	89.8	75.8

[1]HIGH power level, [2]LOW power level, *Shown in figure 4.

presented in the above sections into the mathematical model, the following solutions for problem are obtained.

A. MIXED INTEGER LINEAR PROGRAMMING (MILP) SOLUTION FOR STORAGE POWER PLANT

In the real-world problems, there is an uncertainty in the decision variables like power-generation (in this case), power transmission and distribution. A mixed-integer linear programming (MILP) is a solver-based solution approach to a problem in which some of the decision variables are constrained to be an integer, whereas other variables can be non-integer at the optimal solution. The scope of the optimization problems that one can define and solve can be greatly broadened using integer variables. Based on the energy revenue price ($/MWh) form the electrical market and the resource

capacity parameters and the efficiency of the generators, the optimum solution is found by solving the objective function through MILP approach. Two power generation plans are discussed and analyzed for the storage hydroelectric power plant.

1) NORMAL DISPATCH SCHEDULE

First, the normal scheduled generation plan is formulated, and the solution is plotted as-function of-time in-Figure-5. The below generation planning results for storage hydropower plants shows the normal dispatch schedule for all the three generators based on the energy price ($/MWh) from the electrical market. It can be noted that generator which is more efficient is normally scheduled for most of the time to avoid losses and maximize the overall profit from the power generation. Generator-1 and generator-2 runs at their HIGH power-level whenever they are on. Whereas, the generator-3 runs mostly at its High-power level except for one-time unit for which it dips down to its LOW power level. The results from the normal schedule of generators is shown in Table 6.

2) OPTIMUM DISPATCH SCHEDULE

After the normal scheduled generation plan, the optimum dispatch schedule for the generators is formulated and the solution is plotted as function of time in Figure 6. The generation
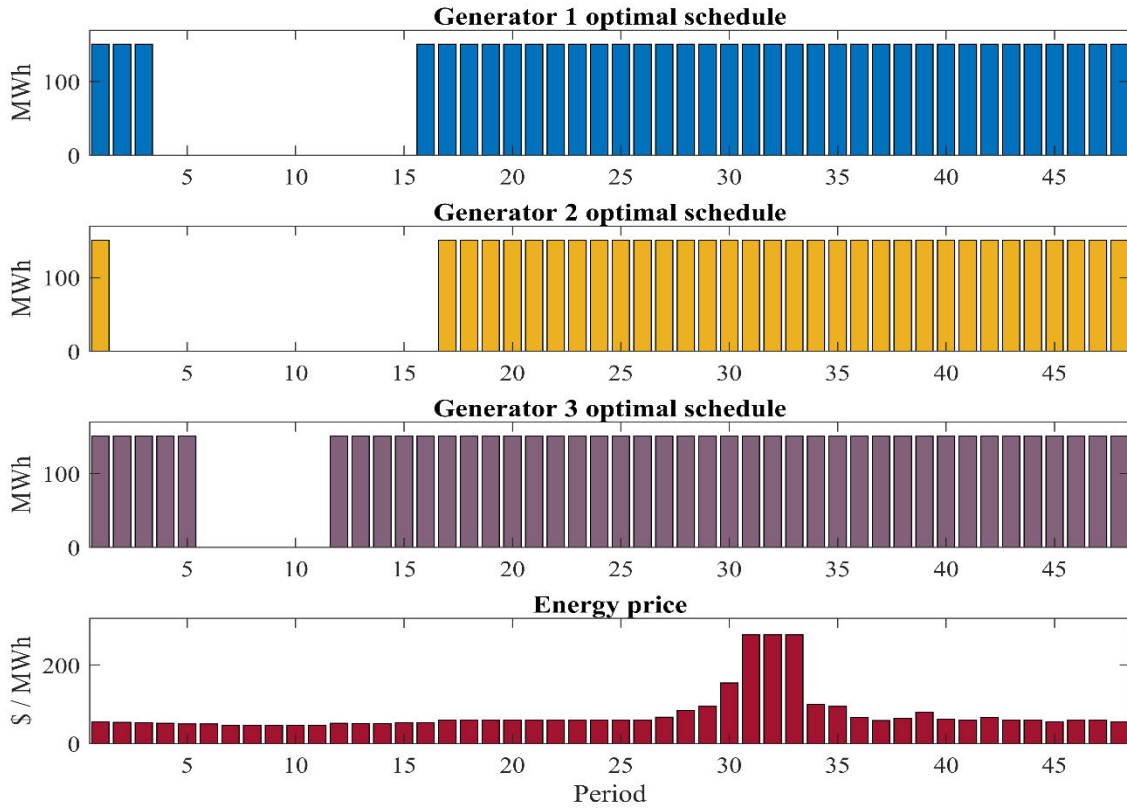
FIGURE 6. Optimum dispatch schedule of generators for storage power plant.

TABLE 6. Results for normal schedule of generators for storage plant.

Generator	Dispatch Period	Power Level	Efficiency	Objective Profit
1	32	HIGH	84.4%	$
2	32	HIGH	83.1%	**1,324,800**
3	whole interval	HIGH except LOW at period = 8	89.8% at HIGH 75.8% at LOW	

TABLE 7. Results for optimum schedule of generators for storage plant.

Generator	Dispatch Period	Power Level	Efficiency	Objective Profit
1	36	HIGH	84.4%	$
2	33	HIGH	83.1%	**1,346,800**
3	42	HIGH	89.8%	

planning results for storage hydropower plant shows the optimum dispatch schedule for all the three generators. It can be noted that all the generators are dispatched according to their efficiency and the energy price ($/Mwh) from the electrical market. The generator 3 runs longer than generator 1 and 2, which is obvious as it is the most efficient generator among all. By comparing the optimum dispatch schedule to the normal dispatch scheduled, it can be noted that the generators are now optimally dispatch for only those time units at which the energy revenue price ($/Mwh) is the highest. Also, all the generators are now dispatched at their High-Power level.

Hence increasing the overall incoming revenue ($/Mwh) and alternatively maximizing the overall object function of profit ($) from the plant. The results from the optimum schedule of generators is shown in Table 7.

Objective profit has been calculated based on above mentioned equations, but for understanding we are calculating objective profit for one scenario. Using equation (2) for maximum profit, putting values, we get:

First term of profit equation is,

$$Time * \left[\left[\sum_{i=1}^{i=n}\sum_{j=1}^{j=m} Rg\,(i,j)\right]\right]$$

So, for each generator, the calculation is as:

$G^1 = $32h x 151Mw = 4832Mwh
$G^2 = $32h x 152Mw = 4864Mwh
$G^3 = $47h x 150Mw + 1h *50Mw=7050 + 50= 7100Mwh
Total $=(G^1+G^2+G^3)* \$280$
$= (4832+4864+7100)$ Mwh $*\$280$
$= 16796$Mwh $*\$280$
$= \$4702880$
Then for the second term of the calculation is,

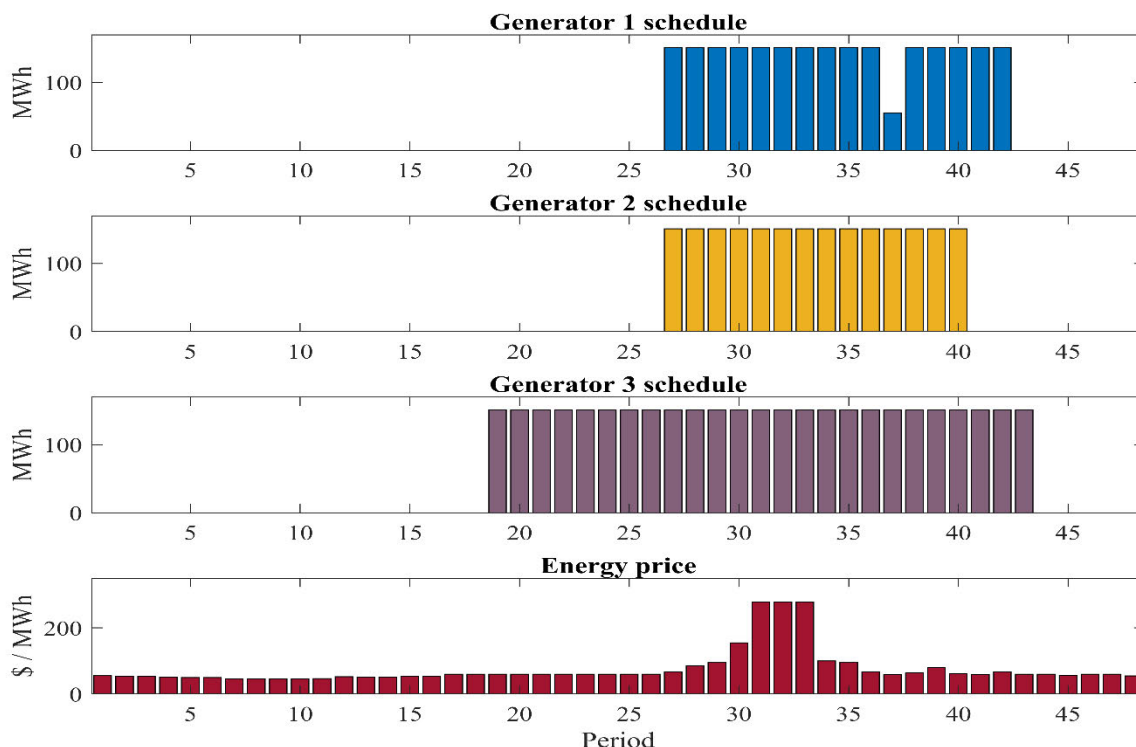$$\left[\sum_{i=1}^{i=n}\sum_{j=1}^{j=m} Cg\,(i,j)\right]$$

FIGURE 7. Normal dispatch schedule of generators for ROR power plant.

TABLE 8. Total Savings for Storage hydropower plant.

Normal Schedule Profit (NSP)	Optimal Schedule Profit (OSP)	Total savings (TS = OSP - NSP)
$ 1,324,800	$ 1,346,800	$ 22,000

where $Cg(i, j)$ is cost of generation at any time instance, which in our case is, as per two year record of Neelam Jhelum power plant, is $2533560 for all three generators. and in the same manner the third component of equation (2) is,

$$\left[\sum_{i=1}^{i=n} \sum_{j=1}^{j=m} Cs\,(i, j) \right]$$

Which is starting cost of generation, as per record of Neelam Jhelum, it is $844520 for all three generators.

So, putting values of all three component in profit equation gives us net objective profit of $1324800, Which is shown in table 6. Similar is the case for all the scenarios of each table.

3) OBJECTIVE FUNCTION
The objective function in this research work is to maximize the overall profit from the plant. It can be clearly seen that the overall profit from the storage plant is maximized through the optimal dispatch schedule of the generators. The respective objective profits for normal and optimum dispatch schedules are shown in Table 6 and Table 7 respectively. The overall savings for the assumed period are shown in Table 8.

Hence a total of $ 22,000 have been saved by optimizing the storage hydroelectric power generation schedule for the assumed time period in this research work.

B. MIXED INTEGER INEAR PROGRAMMING (MILP) SOLUTION FOR ROR POWER PLANT
Based on the energy revenue price ($/Mwh) form the electrical market, the resource capacity parameters and the efficiency of the generators, the optimum solution is found by solving the objective function through MILP approach. Two power generation plans are discussed and analyzed for the ROR hydroelectric power plant.

1) NORMAL DISPATCH SCHEDULE
First, the normal scheduled generation plan for ROR hydroelectric plant is formulated and the solution is plotted as function of time in Figure 7. The generation planning results for ROR hydropower plants shows the normal dispatch schedule for all the three generators based on the energy price ($/MWh) from the electrical market. It can be noted that all the generators are operated only for the time intervals having the highest incoming revenue ($/Mwh) from the electricity market. Moreover, the generator which is more efficient is normally scheduled for most of the time to avoid losses and maximize the overall profit from the power generation. Generator-2 and generator-3 runs at their HIGH power-level whenever they are on. Whereas, the generator-1 runs mostly at its HIGH power level except for one-time unit for which
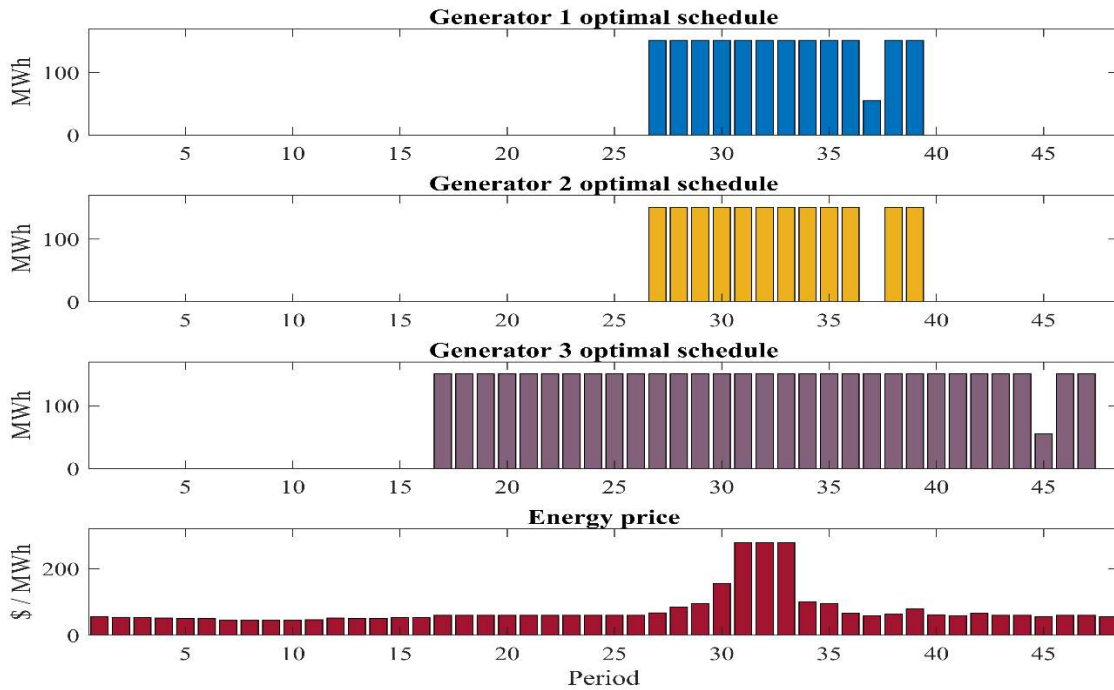
FIGURE 8. Optimum dispatch schedule of generators for ROR power plant.

TABLE 9. Results for normal schedule of generators for ROR plant.

Generator	Dispatch Period	Power Level	Efficiency	Objective Profit
1	16	HIGH except LOW at period = 37	84.4% at HIGH 70.5% at LOW	$ 8,53,650
2	14	HIGH	83.1%	
3	25	HIGH	89.8%	

TABLE 10. Results for optimum schedule of generators for ROR plant.

Generator	Dispatch Period	Power Level	Efficiency	Objective Profit
1	13	HIGH except LOW at period = 37	84.4% at HIGH 70.5% at LOW	$ 8,82,780
2	12	HIGH	83.1%	
3	31	HIGH except LOW at period = 45	89.8% at HIGH 75.8% at LOW	

it dips down to its LOW power level. The results from the normal schedule of generators is shown in Table 9.

2) OPTIMUM DISPATCH SCHEDULE

After the normal scheduled generation plan, the optimum dispatch schedule for the generators is formulated and the solution is plotted as function of time in Figure 8. The generation planning results for ROR hydropower plants shows the optimum dispatch schedule for all the three generators. It can be noted that all the generators are dispatched according to their efficiency and the energy price ($/Mwh) from the electrical market. The generator 3 runs longer than generator 1 and 2, which is obvious as it is the most efficient generator among all. By comparing the optimum dispatch schedule to the normal dispatch scheduled, it can be noted that the generators are now optimally dispatch for only those time units at which the energy revenue price ($/Mwh) is the highest. Due to higher efficiency of generator 3, its total operating time has been optimally maximized. Now generator-1 runs mostly at its High-power level except a single dip in its power

level to LOW for a single unit of time. Similarly, generator-2 runs at its HIGH-power level for it whole time of dispatch. Also, generator-3 is now dispatched at its HIGH power level for most of the time except a single dip in its power level to LOW for a single unit of time. Hence increasing the overall incoming revenue ($/Mwh) and alternatively maximizing the overall object function of profit ($) from the ROR plant. The results from the optimum schedule of generators is shown in Table 10.

3) OBJECTIVE FUNCTION

The objective function in this research work is to maximize the overall profit from the plant. It can be clearly seen that the overall profit from the ROR plant is maximized through the optimal dispatch schedule of the generators. The respective objective profits for normal and optimum dispatch schedules

are shown in Table 9 and Table 10 respectively. The overall savings for the assumed period are shown in Table 11.

TABLE 11. Total savings for ROR hydropower plant.

Normal Schedule Profit (NSP)	Optimal Schedule Profit (OSP)	Total savings (TS = OSP - NSP)
$ 8,53,650	$ 8,82,780	$ 29,130

Hence a total of $ 29,130 have been saved by optimizing the ROR hydroelectric power generation schedule for the assumed time period in this research work.

V. CONCLUSION

A solver-based mathematical optimization model is developed for the first time for a hydroelectric power system, i.e., for both storage and ROR plants. For the same capacity, storage hydroelectric power plants are more expensive than ROR hydroelectric power plant. Due to the simple water diversion channel, ROR power plants are more environmentally friendly as compared to storage plants. However, ROR projects are less reliable than the storage power projects due to variability in their output energy. A mathematical optimization model that could optimally predict the power generation plan would be useful for the power system operators to schedule the power plants optimally. The numerical formulation of the problem gives a solution that would maximize the total profit by ensuring the optimized power generation dispatch all the time.

The optimization approach presented in this article considers the variability in different capacity parameters. It solves the model through mixed-integer linear programming (MILP) technique to give the most optimal and economic dispatch of the hydroelectric generators by taking the system constraints into account. A two-stage solver-based mathematical model is developed by utilizing the optimization toolbox in MATLAB. The first stage was used to optimize the power generation of the storage plant, whereas the second stage was used to optimize the power generation of ROR project. Three set of generators having different operating efficiencies and power capacities are used to propose an optimal solution for power generation by utilizing a variable energy price ($/MWh) input to the model. The objective function was solved, and the total profit was calculated for two scenarios i.e., regular dispatch schedule and optimum dispatch schedule. Comparison has been made between the two dispatch scenarios, and the profit difference was calculated (in dollars) between the two dispatch scenarios. It concludes that the profit ($) from the optimum dispatch was far more than the expected dispatch, hence ensuring the credibility of the proposed model. The proposed technique suggests an optimal on-off schedule of poly generating stations considering profits and running costs and results in increased profit, which can be implemented in any practical scenario. The model presented in this research work also applies to other types of power plants like thermal, solar, wind, and nuclear energy.

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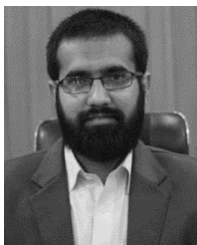
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