Solving Fuzzy Linear Programming Problem as Multi Objective Linear Programming Problem

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***Abstract*— This paper proposes the method to the solution of fuzzy linear programming problem with the help of multi objective constrained linear programming problem when the constraint matrix and the cost coefficients are fuzzy in nature and it is also explained with an illustrative example.**

***Index Terms*— Fuzzy Linear Programming Problem, Multi Objective Fuzzy Linear Programming Problem, Fuzzy Sets.**

I. INTRODUCTION

Linear programming is a one of the most important operational research (OR) techniques. It has been applied to solve many real world problems but it fails to deals with imprecise data. So the many researchers succeed in capturing vague and imprecise information by fuzzy linear programming problem (FLPP) [1-2],[5].

The concept of a fuzzy decision making was first proposed by Bellman and Zadeh [1]. Recently, much attention has been focused on FLPP [2-4], [7]. An application of fuzzy optimization techniques to linear programming problems with multiple objectives [7] has been presented by Zimmermann. Tanaka et. al. presented a fuzzy approach to multi objective linear programming problems. Negoita has formulated FLPP with fuzzy coefficient matrix, Zhang G. et al. [5] formulated a FLPP as four objective constrained optimization problem where the cost coefficients are fuzzy and also presented it's solution.

In this paper, we provided a method to solve FLPP where both the coefficient matrix of the constraints and cost coefficient are fuzzy in nature.

Each problem, first converted into equivalent crisp linear problems, which are then solved by standard optimization methods.

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II. PRELIMINARIES

Definition.1: A subset A of a set X is said to be fuzzy set if $\mu_A : X \to [0,1]$, where μ_A denote the degree of belongingness of A in X.

Definition 2: A fuzzy set A of a set X is said to be normal if $\mu_A(x) = 1, \quad \forall\ x \in X$

Definition 3: The height of A is defined and denoted as $h(A) = \sup_{x \in X} \mu_A(x)\cdot$

Definition 4: The α -cut and strong α -cut is defined and denoted respectively as

$$\alpha_A = \{x \ / \ \mu_A(x) \ge \alpha \ \}$$
$$\alpha^+{}_A = \{x \ / \ \mu_A(x) > \alpha \ \}$$

Definition 5: Let $\tilde{a}$, $\tilde{b}$ be two fuzzy numbers, their sum is defined and denoted as

$$\mu_{\tilde{a}+\tilde{b}}(z) = \sup \min_{z=u+v} \{, \mu_{\tilde{a}}(u) \ , \ \mu_{\tilde{a}}(v) \ \ \}$$

when $0 \le \lambda \in R$

Definition 6: If a fuzzy number $\tilde{a}$ is fuzzy set A on R, it must possess at least following three properties:

i) $\mu_{\tilde{a}}(x) = 1$

ii) $\{x \in R \ / \ \mu_{\tilde{a}}(x) > \alpha \ \}$ is a closed interval for every $\alpha \in (0,1]$

iii) $\{x \in R \ / \ \mu_{\tilde{a}}(x) > 0 \ \}$ is bounded and it is denoted by $[a_\lambda^L, a_\lambda^R]$

Theorem 1: A fuzzy set A on R is convex if and only if $\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge \min[\mu_A(x_1), \ \mu_A(x_2)]$, for all $x_1, x_2 \in X$ and for all $\lambda \in [0,1]$ where min denotes the minimum operator.

Proof: Obvious.

Theorem 2: Let $\tilde{a}$ be a fuzzy set on R, then $\tilde{a} \in f(R)$ if and only if $\mu_{\tilde{a}}$ satisfies

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 \ , & for \ x \in [m,n] \\ L(x), & for \ x < m \\ R(x), & for \ x > n. \end{cases}$$

where $L(x)$ is the right continuous monotone increasing function $, 0 \le L(x) \le 1$ and $\lim_{x \to \infty} L(x) = 0$, $R(x)$ is a left continuous monotone decreasing function, $0 \le R(x) \le 1$, and $\lim_{x \to \infty} R(x) = 0$.

Proof: Obvious.

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III. FUZZY LINEAR PROGRAMMING PROBLEM

We consider the fuzzy linear programming problem (FLPP) with cost of decision variables and coefficient matrix of constraints are in fuzzy nature.

$$\langle \tilde{c}, x \rangle = f_i(x_j) = f_i(x) = Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j\, x_j$$

(1)

Subject to

$$\sum_{j=1}^{n} \tilde{A}_{ij}\ x_j \le \tilde{B}_i \quad ; \quad 1 \le i \le m, \text{ there exists } x_j > 0$$

$$\mu a_{ij}(x) = \begin{cases} 1 & for \quad x < a_{ij} \\ \dfrac{(a_{ij} + d_{ij} - x)}{d_{ij}} & for \quad a_{ij} \le x \le a_{ij} + d_{ij} \\ 0 & for \quad x \ge a_{ij} + d_{ij} \end{cases}$$

$$\mu_{B_i}(x) = \begin{cases} 1 & for \quad x \le b_{ij} \\ \dfrac{(b_i + p_i - x)}{p_i} & for \quad b_i \le x \le b_i + p_i \\ 0 & for \quad b_i + p_i \le x \end{cases}$$

Let's consider triangular fuzzy numbers A which can be represented by three crisp numbers s, l, r.

$$(1) \Rightarrow \langle \tilde{c}, x \rangle = f_i(x_j) = \text{Max} \sum_{j=1}^{n} \tilde{c}_j\, x_j$$

such that $\sum_{x \ge 0} (s_{ij}\ , l_{ij}\ , r_{ij})\, x_{ij} \le (\ t_i,\ u_i,\ v_i)$

$$0 \le i \le m$$
$$0 \le j \le n$$

where $A_{ij} = \langle s_{ij}\ , l_{ij}\ , r_{ij} \rangle$

$B_{ij} = \langle\ t_i\ , u_i\ , v_i \rangle$ are fuzzy numbers.

Theorem 3: For any two triangular fuzzy numbers $A = \langle\ s_1,\ l_1,\ r_1 \rangle$ and $B = \langle s_2\ , l_2\ , r_2 \rangle$, $A \le B$ if and only if $s_1 \le s_2$, $s_1 - l_1 \le s_2 - l_2$ and $s_1 + r_1 \le s_2 + r_2$

Proof: Obvious

Above problem can be rewritten as

$$\langle \tilde{c}, x \rangle = f_i(x_j) = \text{Max} \sum_{j=1}^{n} \tilde{c}_j\, x_j$$

such that $\sum_{j=1}^{n} s_{ij}\, x_j \le t_i$

$$\sum_{j=1}^{n} (s_{ij} - l_{ij})\, x_j \le t_i - u_i \qquad (2)$$

$$\sum_{j=1}^{n} (s_{ij} + r_{ij})\, x_j \le t_i + v_i \quad , x_i \ge 0$$

where the membership functions of $\tilde{c}_j$ (x) is

$$\mu_{\tilde{c}_j(x)} = \begin{cases} 0 & x < \alpha_j \\ x - \alpha_j & \alpha_j \le x < \beta_j \\ 1 & \beta_j \le x \le \gamma_j \\ \dfrac{\eta_i - x}{\eta_i - \gamma_j} & \gamma_j < x \le \eta_j \\ 0 & \eta_j < x \end{cases}$$

Definition 7: A point $x^* \in X$ is said to be an optimal solution to the FLPP if

$$\langle \tilde{c}, x^* \rangle \ge \langle \tilde{c}, x \rangle \qquad \text{for all } x \in X$$

IV. FUZZY MULTIPLE OBJECTIVE OPTIMIZATION

Consider a multiple objective optimization problem with k fuzzy goals $f_1, f_2 .... f_k$ represented by fuzzy sets $\tilde{F}_i$, i = 1 . . . . . k, and m fuzzy constraints $g_1, g_2, ... g_m$ represented by fuzzy sets $\tilde{G}_j$ j = 1,2 . . . m. By generalizing the analogy from the single objective function, the resulting fuzzy decision is given as

$$\tilde{F}_1 \cap \tilde{F}_2 \cap ..... \tilde{F}_k \cap \tilde{G}_1 \cap \tilde{G}_2 \cap ...... \tilde{G}_m$$

In terms of corresponding membership values for the fuzzy goals and the fuzzy constraints, the resulting decision is

$$\mu_{\tilde{D}}(X) = \min_{i,j} [\mu_{\tilde{F}_i}(X, \mu_{\tilde{G}_j}(X)]$$

An optimum solution X* is one at which the membership function of the resulting decision $\tilde{D}$ is maximum, i.e.

$$\mu_{\tilde{D}}(X^*) = \max\ \mu_{\tilde{D}}(X)$$

V. MULTI OBJECTIVE LINEAR PROGRAMMING PROBLEM WITH FUZZY COEFFICIENTS

In general, multi objective linear programming problem (MOLPP) refers to those LP problems of systems in which multiple objectives are to be controlled.

For above FLPP, the multi objective linear programming problem with fuzzy coefficients can be formulated as

$$\underset{x \in X}{Max} \{ f_1(x), \quad f_2(x), ..., f_k(x) \}$$

Subject to (2)

Where $f_i : R^n \to R^i$

Where R be the set of all real numbers and R^n be an n-dimensional Euclidean space.

By considering the weighting factor, the MOLPP is defined as

$$\underset{x \in X}{Max} \{ w_1 f_1(x), \quad w_2 f_2(x), ..., w_k f_k(x) \}$$

$$\text{i.e. } \underset{x \in X}{Max} \sum_{m=1}^{k} w_m f_m(x)$$

Subject to (2)

VI. NUMERICAL EXAMPLE

Example: We illustrate the method by numerical example.

Solve the following FLPP

$$\text{Max } f_i(x_1, x_2) = \tilde{c}_1 x + \tilde{c}_1 x_2$$

subject to the constraints

$$(3,\ 2,\ 1)x_1 + (6\ ,4\ ,1\ )x_2 \le (13\ ,5\ ,2)$$
$$(4,\ 1,\ 2)x_1 + (6\ ,5\ ,4\ )x_2 \le (7\ ,4\ ,2)$$

Where the membership function of $\tilde{c}_1$ & $\tilde{c}_2$ are

$$\mu_{\tilde{c}1}(x) = \begin{cases} 0\ , & x < 7 \\ x - 7, & 7 \le x < 10 \\ 1, & 10 \le x \le 14 \\ \dfrac{25 - x}{14}\ , & 14 < x \le 25 \\ 0, & 25 < x\ . \end{cases}$$

$$\mu_{\tilde{c}2}(x) = \begin{cases} 0, & x < 20 \\ x - 20, & 20 \le x < 25 \\ 1, & 25 \le x \le 35 \\ \dfrac{40 - x}{5}\ , & 35 < x \le 40 \\ 0\ , & 40 < x \end{cases}$$

Write the above FLPP as Max $f(x_1, x_2) = \tilde{c}_1 x + \tilde{c}_1 x_2$

Subject to constraints

$$\left.\begin{aligned} 3x_1 + 6x_2 &\le 13 \\ 4x_1 + 6x_2 &\le 7 \\ x_1 + 2x_2 &\le 8 \\ 3x_1 + x_2 &\le 3 \\ 4x_1 + 7x_2 &\le 15 \\ 6x_1 + 10x_2 &\le 9 \\ x_1, x_2 &\ge 0 \end{aligned}\right\} \qquad (3)$$

MOLPP: Max

$$(7x_1 + 20x_2,\ 10x_1 + 25x_2,\ 14x_1 + 35x_2,\ \ 25x_1 + 40x_2)$$

Subject to (3)

MOLPP

$$Max\ (w) = \begin{pmatrix} w_1(7x_1 + 20x_2) + w_2(10x_1 + 25x_2) \\ + w_3(14x_1 + 35x_2) + w_4(\ 25x_1 + 40x_2) \end{pmatrix}$$

subject to (3)

Standard optimization technique is used to solve the problem and found solution for different weights.

For example, $w_1 = 0 = w_4,\ w_2 = 1 = w_3$

MOLPP $\quad Max\ (w)\ \ \mathrm{f}(x_1, x_2) = 24x_1 + 60x_2$

Subject to (3)

$$(x_1, x_2) = (0, 0.9)$$

$$Max(w)\ \ \mathrm{f}(x_1^*, x^*_2) = f(0, 0.9) = 0.9\tilde{c}_2$$

$$\mu_{(f(0,0.9))}(x) = \begin{cases} 0 & x \le 18 \\ \dfrac{x - 18}{4.5} & 18 < x \le 22.5 \\ 1 & 22.5 < x \le 31.5 \\ \dfrac{36 - x}{4.5} & 31.5 < x \le 36 \\ 0 & x > 36 \end{cases}$$

Following table lists the solution for above MOLPP for various weights and it also shows that the solutions are independent of weights $(w_i, i = 1,2,3,4)$.

Sr . No.	w_1	w_2	w_3	w_4	(x_1^*, x_2^*)
1	0	1	1	0	(0, 0.9)
2	0	1	0.5	0	(0, 0.9)
3	0.2	0.4	0.5	0.2	(0, 0.9)
4	0.1	0.2	0.3	0.4	(0, 0.9)
5	0	0.3	0	0.4	(0, 0.9)
6	0.2	0.4	0.6	0.8	(0, 0.9)
7	0.5	0	0.5	0	(0, 0.9)
8	0	1	1	0	(0, 0.9)
9	0	0	0	0.5	(0, 0.9)
10	0.3	0.1	1	1	(0, 0.9)
11	0.5	0.5	0.5	0.5	(0, 0.9)
12	0	0	0.5	0.5	(0, 0.9)
13	0.2	0.5	0.5	0.5	(0, 0.9)
14	0.1	0.2	0.3	0.4	(0, 0.9)
15	0	0.2	0	0.2	(0, 0.9)

VII CONCLUSION

We successfully discussed the solution of fuzzy linear programming problem with the help of multi objective constrained linear programming problem where constraint matrix and the cost coefficients are fuzzy quantities and also proved that the solutions are independent of weights.

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