Solving Low-Frequency EM-Ckt Problems Using the PEEC Method

Dipanjan Gope*

Circuit Technology CAD
INTEL Corporation

Albert Ruehli

System Level Design
T.J. Watson IBM Research

Vikram Jandhyala

Electrical Engineering
University of Washington

Dipanjan Gope

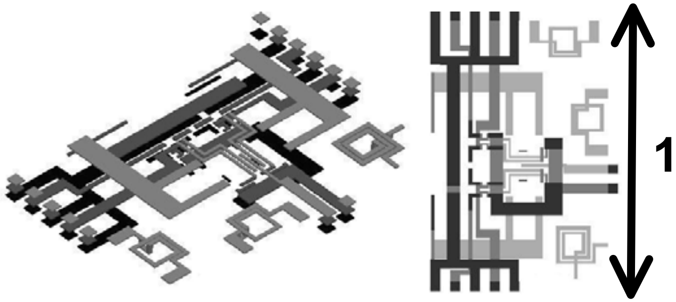
Outline

- **Numerical Problems for Low-Frequency EFIE**
  - Low-Frequency in Circuits: Why Should We Bother?
  - What are the Detrimental Numerical Effects?
- **Existing Solution Methods**
- **PEEC Low-Frequency Solution**
  - Basic PEEC Cell
  - Low-Frequency Strategies
- **EFIE Low-Frequency Solution**
- **Results and Conclusions**

Dipanjan Gope

“Low Frequency” in Circuits

1. Electrically Small Structure



UW VCO Structure

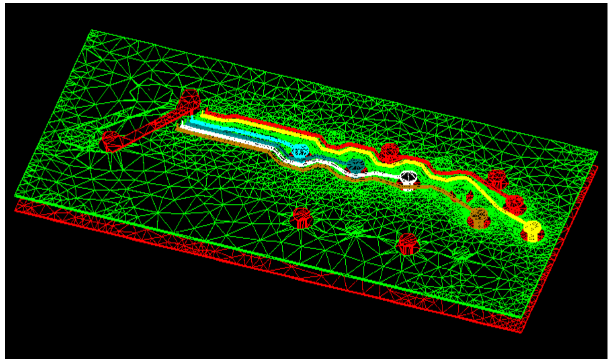
f = 40Ghz
λ = 7.5mm

$$\frac{dimension}{frequency} << 1$$

Electrically Small

On-Chip

2. Local Refined Mesh



Courtesy: Ansoft Corporation

Boards Packages

✓ **Electrostatic**

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

✓ **Magnetostatic**

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

✗ **ElectroMagnetic**

Numerical Problems

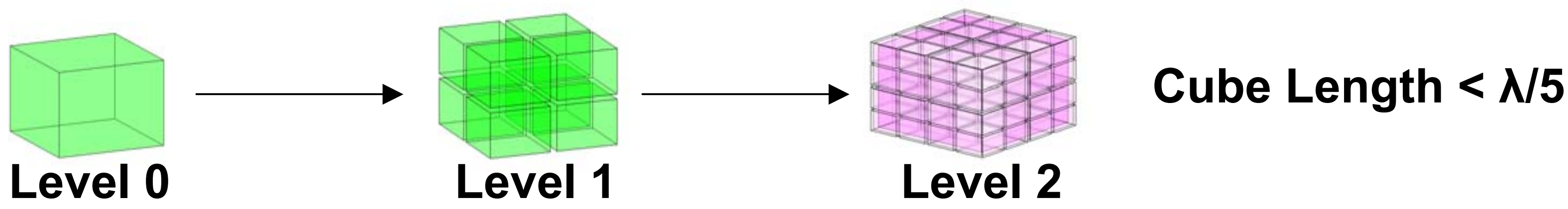
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Dipanjan Gope

“Low Frequency” Problems

- **“Low Frequency” Fast Solver Problem**

  - Traditional FMM: Singularity in Hankel function



  - Solution 1: Low Frequency FMM, Chew *etal.*
  - Solution 2: QR-PEEC fast iterative solver, Ruehli *etal*. EPEP’04

- **“Low Frequency” Mixed Potential Problem**

  - Affects solver convergence: larger number of iterations
  - Affects accuracy even for direct solution

Dipanjan Gope

EFIE Formulation

Frequency domain, Method of Moments

$\mathbf{E}^{s}_{\tan}(\mathbf{J}) + \mathbf{E}^{i}_{\tan} = Z_s \mathbf{J}$	**Electric Field Integral Equation**
$\mathbf{E}^{s}(\mathbf{J}) = -j\omega\mathbf{A} - \nabla\Phi$	**Scattered Electric Field**
$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int_S \frac{e^{-jk\lvert\mathbf{r}-\mathbf{r}'\rvert}\mathbf{J}(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert}ds'$	**Electric Vector Potential**
$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\int_S \frac{e^{-jk\lvert\mathbf{r}-\mathbf{r}'\rvert}\rho(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert}ds'$	**Electric Scalar Potential**
$\nabla_s \cdot \mathbf{J}(\mathbf{r}) + j\omega\rho(\mathbf{r}) = 0$	**Continuity Equation**

$$\mathbf{E}^{s}(\mathbf{J}) = -j\omega\frac{\mu}{4\pi}\int_S \frac{e^{-jk\lvert\mathbf{r}-\mathbf{r}'\rvert}\mathbf{J}(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert}ds' - \nabla\frac{1}{4\pi\varepsilon}\int_S \frac{e^{-jk\lvert\mathbf{r}-\mathbf{r}'\rvert}\rho(\mathbf{r}')}{\lvert\mathbf{r}-\mathbf{r}'\rvert}ds'$$

Vector and Scalar potential from EM Currents

Dipanjan Gope

Mixed Potential Problems

$$j\omega\bar{\mathbf{L}} + \frac{1}{j\omega}\bar{\mathbf{A}}_{N_e\times N_p} \times \bar{\mathbf{P}} \times \bar{\mathbf{A}}^{T}_{N_p\times N_e} = \bar{\mathbf{Z}}$$

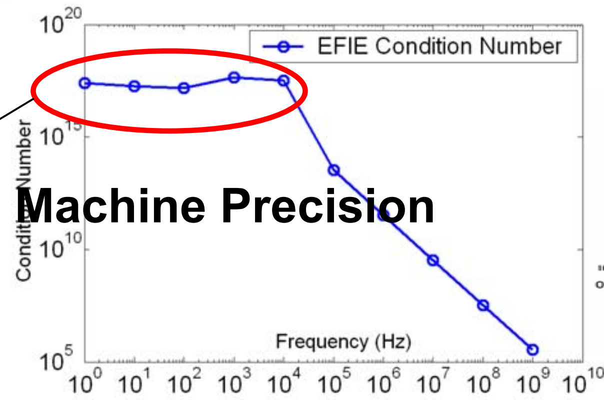
($\bar{\mathbf{L}}$: $N_e \times N_e$; $\bar{\mathbf{P}}$: $N_p \times N_p$; $\bar{\mathbf{Z}}$: $N_e \times N_e$)

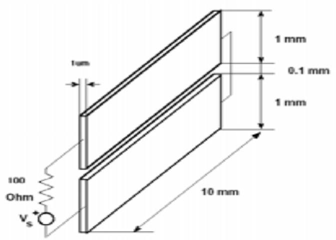
Rank=N_e **Rank=N_p**

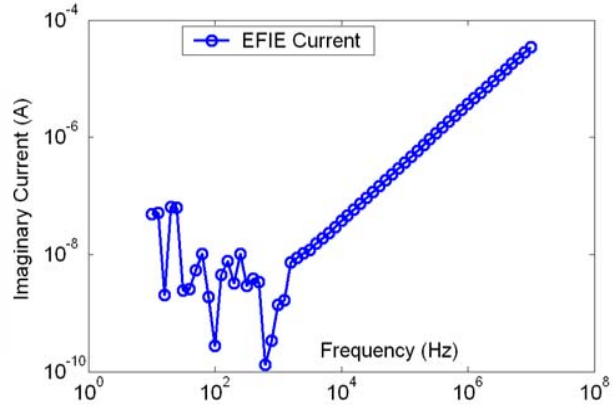
For a closed object: N_e=1.5N_p

Effects

Beyond Machine Precision







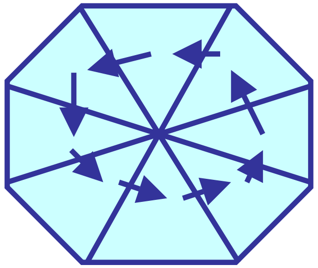
1. Fast Solver Convergence Suffers **2. Direct Solver Result Suffers**

Outline

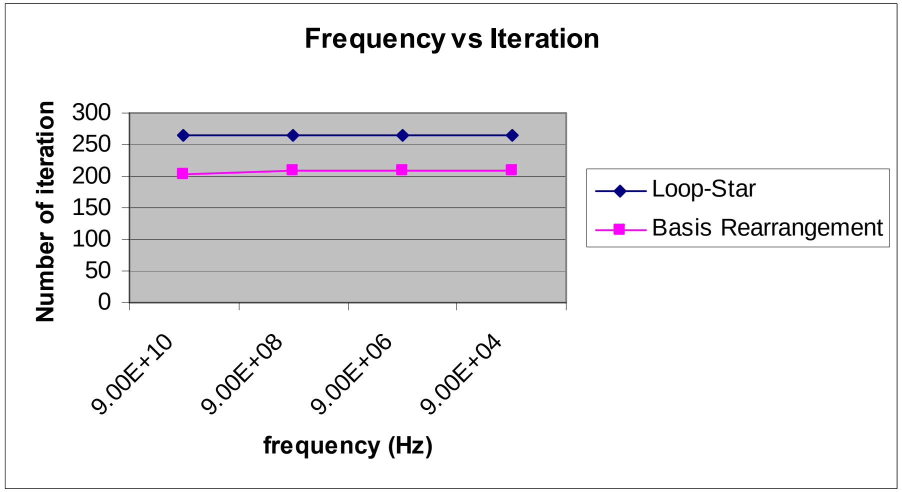
- **Numerical Problems for Low-Frequency EFIE**
  - Low-Frequency in Circuits: Why Should We Bother?
  - What are the Detrimental Numerical Effects?
- **Existing Solution Methods**
- **PEEC Low-Frequency Solution**
  - Basic PEEC Cell
  - Low-Frequency Strategies
- **EFIE Low-Frequency Solution**
- **Results and Conclusions**

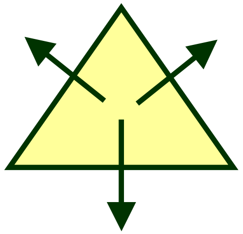
Dipanjan Gope

Loop-Star Decomposition



Loop basis





Star basis

- **Loop basis for solenoidal current (Magneto-static)**
- **Star basis for curl-free current (Electrostatic)**
- **Frequency scaling for improved spectral property**
- **Number of iterations does not scale with frequency**

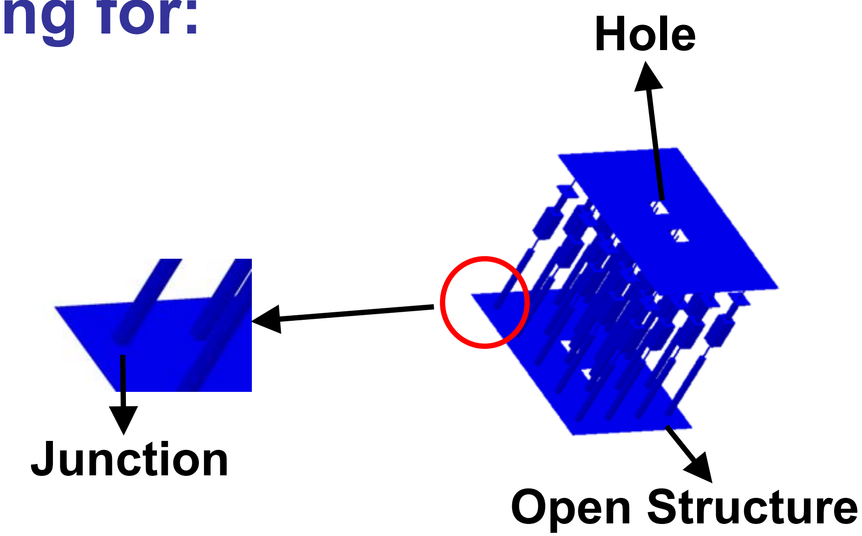
Chew *etal.*

Courtesy: Slide by Swagato Chakraborty

Dipanjan Gope

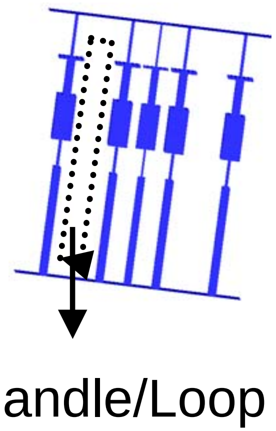
Loop-Star Challenges

- **Loop Detection is Challenging for:**
  - Open structures
  - Structure with holes
  - Structure with handles
  - Structures with junction



More Significant

- **Where to Apply Loop-Star Basis Functions**
  - Detection of mesh where loop-star should be applied
  - Detrimental effects if applied wrongly



Dipanjan Gope

Outline

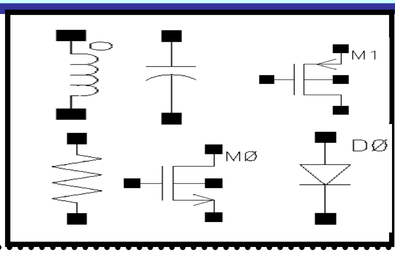
- **Numerical Problems for Low-Frequency EFIE**
  - Low-Frequency in Circuits: Why Should We Bother?
  - What are the Detrimental Numerical Effects?
- **Existing Solution Methods**
- **PEEC Low-Frequency Solution**
  - Basic PEEC Cell
  - Low-Frequency Strategies
- **EFIE Low-Frequency Solution**
- **Results and Conclusions**

Dipanjan Gope

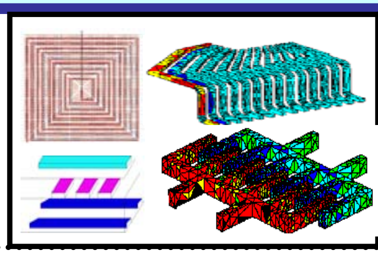
PEEC Single Matrix: EM / Circuit

Elements

Lumped Circuit Elements
e.g. R; L; C; V; I



Distributed Circuit Elements
e.g. Transmission Line



Definition

$t_r >> t_d$

$t_r \sim t_d$

Equations

KVL; KCL

Maxwell's Equations

Solution

SPICE

PEEC

Port Model + SPICE

Dipanjan Gope

intel

PEEC Methodology: SPICE Form

IBM

$$\bar{E}^{i}(\bar{r},\omega)=\frac{\bar{J}(\bar{r})}{\sigma}+j\omega\mu\int_{v'}G(\bar{r},\bar{r}')\bar{J}(\bar{r})dv'+\frac{\nabla}{j\omega}\int_{v'}G(\bar{r},\bar{r}')q(\bar{r})dv'$$

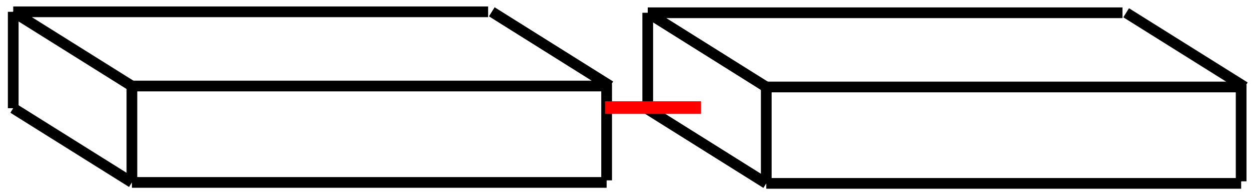
Circuit Model Element Identification

- **KVL: Voltage = R I + sLp I + Q/sC**
- **RHS Term 1: Resistance**
- **RHS Term 2: Partial Inductance**
- **RHS Term 3: Coefficients of Partial Potential**

Dipanjan Gope

PEEC Stick Example

Geometry



Conductor

R1	N1	N2		1.202mOhms	
L1	N2	N3		5.887nH	
C1	N1	0		1.702pF	
K12	L1	L2		1.282nH	0.054ns
F12	N4	0	V1	0.124	0.032ns

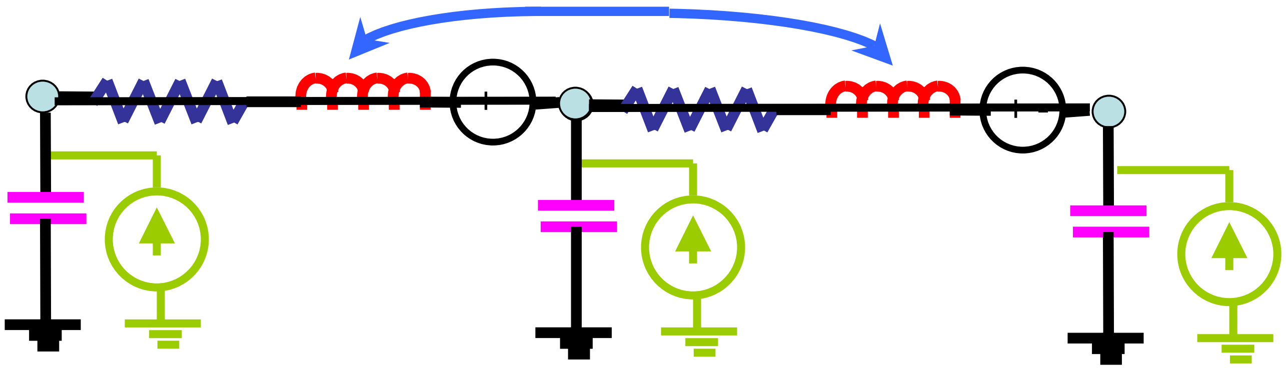
Possible Solution Schemes: .cap, .ind, .ac, .tran

Dipanjan Gope

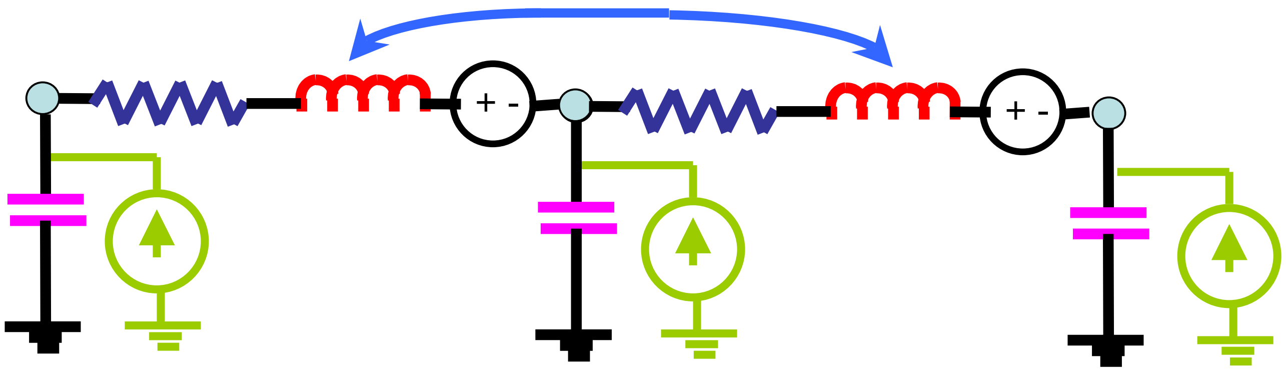
PEEC DC Solution

Continuity Equation $\nabla \cdot J = -j\omega\rho$ NOT Valid

- ElectroStatic: Short Inductors (.cap)



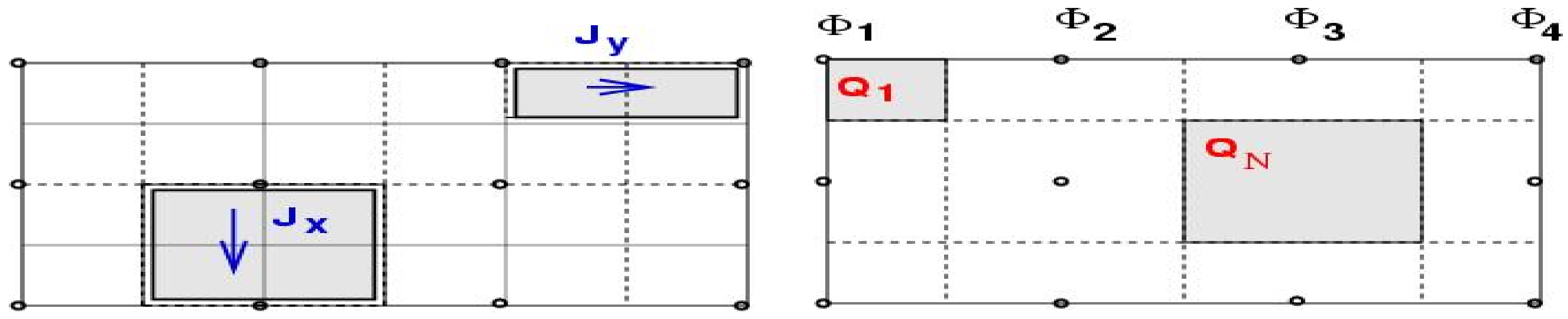
- MagnetoStatic: Open Capacitors (.ind)



Dipanjan Gope

PEEC Low Frequency Solution

Step 1: Separate Charge and Current Basis Functions



- **Charge Basis is Not Derived From Current Basis (Unlike RWG Basis)**

- **Enables Stamping of Scalar and Vector Potentials Differently**
  - Vector potential is stamped in the impedance form (KVL)
  - Scalar potential is stamped in the admittance form (KCL)
  - Both cases ω is in the numerator in matrix elements

In Contrast RWG-EFIE is Completely KVL

$$EFIE :: \left(j\omega \overline{\mathbf{L}}_{N_e \times N_e} + \frac{1}{j\omega} \overline{\mathbf{A}}_{N_e \times N_p} \overline{\mathbf{P}}_{N_p \times N_p} \overline{\mathbf{A}}^T_{N_p \times N_e} \right) < t_{i=0 \ldots N_e}, E_{inc} >$$

Dipanjan Gope

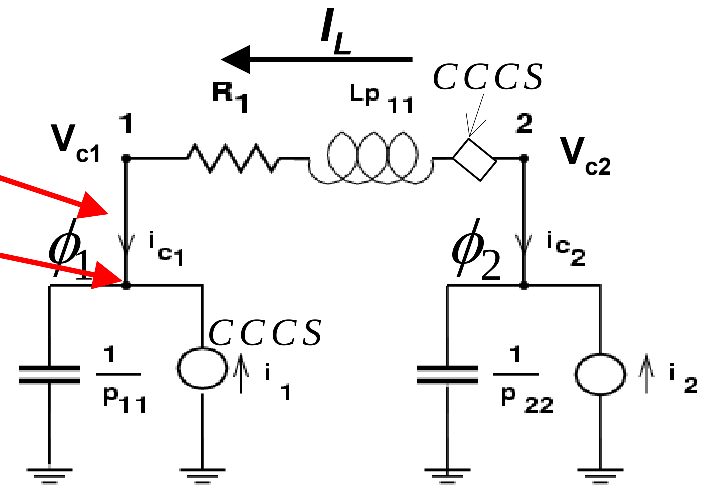
PEEC Low Frequency Solution

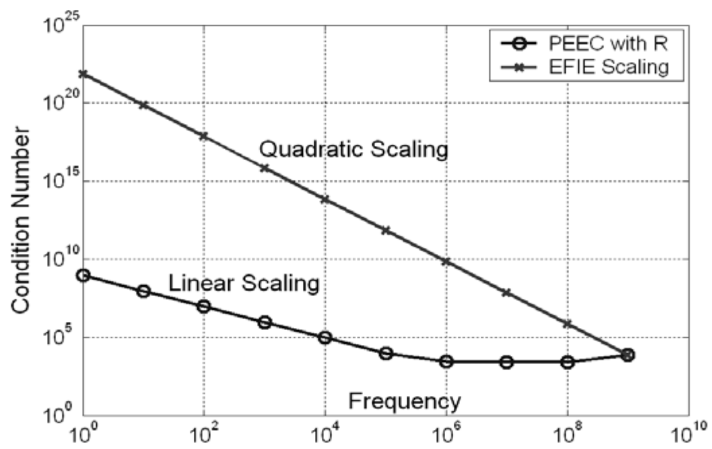
Step 1: Separate Charge and Current Basis Functions

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{s}{p_{11}} & 0 & 0 & -1 & -\frac{p_{12}}{p_{11}} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{s}{p_{11}} & -\frac{p_{21}}{p_{22}} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 & 0 & 0 & -R-sLp_{11} \end{bmatrix} \begin{bmatrix} \phi_1 \\ V_{c1} \\ \phi_2 \\ V_{c2} \\ I_{c1} \\ I_{c2} \\ I_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_s \\ 0 \\ 0 \end{bmatrix}$$

KVL

KCL





- No Extra Memory
- No Extra MatVec Product Time
- No ω in Denominator
- No ω in Off-Diagonal

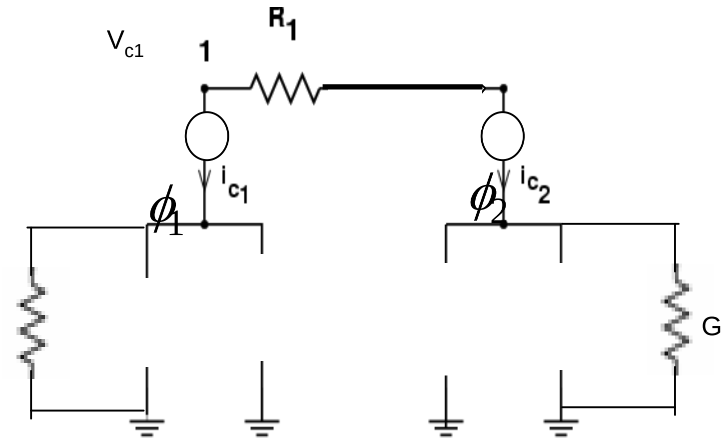
Dipanjan Gope

PEEC Low Frequency Solution

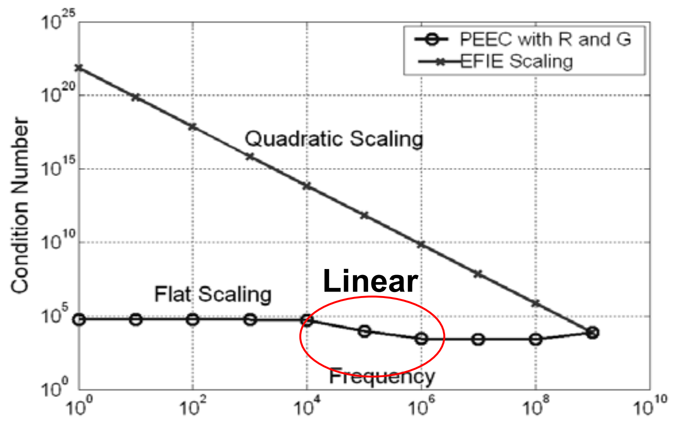
Step 2: Incorporation of Dielectric Loss

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G+\frac{s}{p_{11}} & 0 & 0 & -1 & -\frac{p_{12}}{p_{11}} & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & G+\frac{s}{p_{11}} & -\frac{p_{21}}{p_{22}} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & -R-sLp_{11} \end{bmatrix}$$

At Low Frequencies R and G Dominate

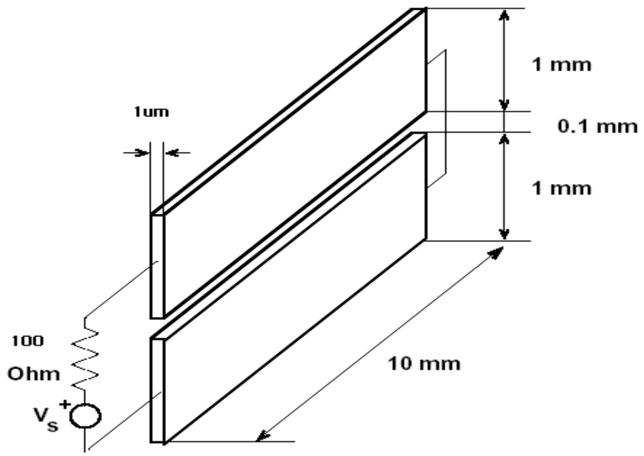


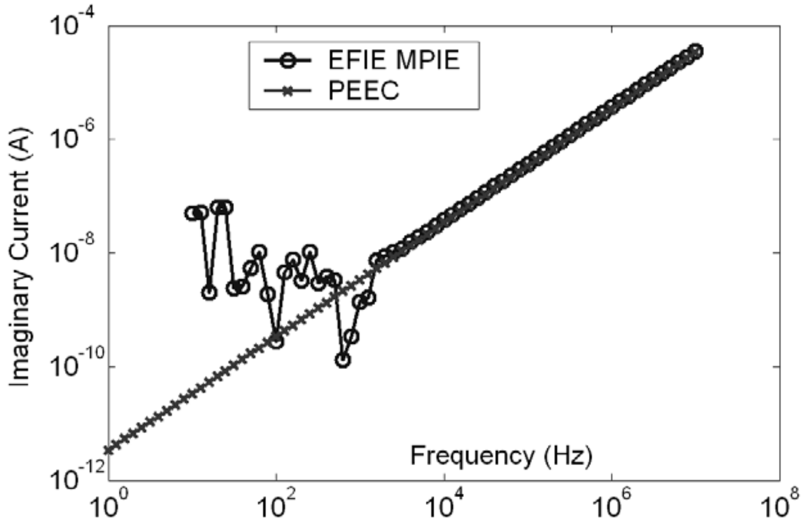
Seam Less Transition to DC

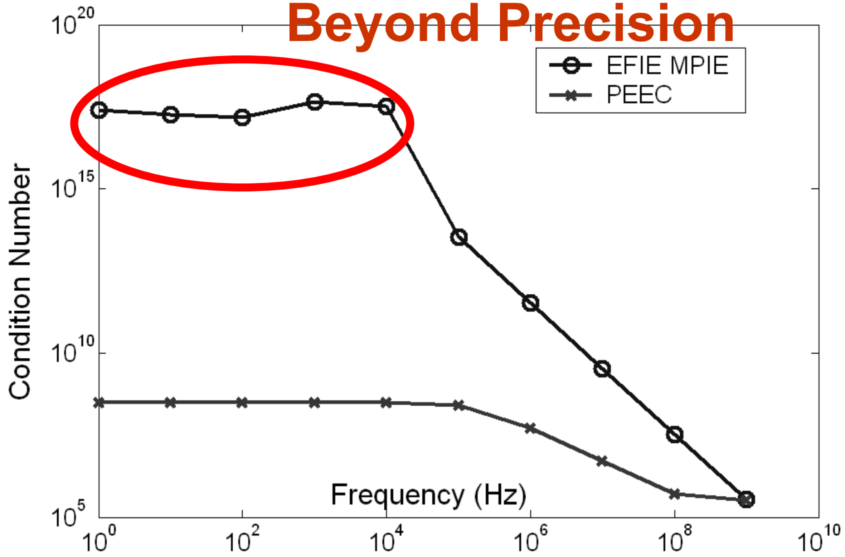


Dipanjan Gope

Transmission Line Example







PEEC Solver Results

Dipanjan Gope

Conclusions

- **Separated Potential Enables Mixed KCL/KVL Form**
- **Loss Enables Flat Conditioning at Low Frequencies**
  - Loss dominates over all effects at low frequencies
- **Seam-Less Transition from .ac to .cap and .ind**
- **Does Not Harm Conditioning for Electrically Large Mesh**
- **No Requirement for Loop Detection**

Highlight