Solving multiobjective optimization problems with decision uncertainty: an interactive approach

Abstract

We propose an interactive approach to support a decision maker to find a most preferred robust solution to multiobjective optimization problems with decision uncertainty. A new robustness measure that is understandable for the decision maker is incorporated as an additional objective in the problem formulation. The proposed interactive approach utilizes elements of the synchronous NIMBUS method and is aimed at supporting the decision maker to consider the objective function values and the robustness of a solution simultaneously. In the interactive approach, we offer different alternatives for the decision maker to express her/his preferences related to the robustness of a solution. To consolidate the interactive approach, we tailor a visualization to illustrate both the objective function values and the robustness of a solution. We demonstrate the advantages of the interactive approach by solving example problems.

Keywords: Multiple criteria decision making, Robust solutions, Interactive methods, Handling uncertainties, NIMBUS, Robustness measure.

1 Introduction

Practical optimization problems often involve multiple conflicting objectives. For these problems, there does not exist a single optimal solution. Instead, there is a set of mathematically equally good Pareto optimal solutions. A solution is Pareto optimal, if none of the objective function values can be improved without impairing at least one of the others. Typically, a decision maker (DM) who is an expert in the problem domain is interested in a single Pareto optimal solution depending on her/his preferences [6] and can be assumed to provide this preference information.

In optimization, uncertainty can originate from different sources and be reflected in different elements of the problems such as decision variables and parameters in objectives or constraints [30]. For example, in portfolio optimization, uncertain future developments can be reflected as parameters in the objective functions as in [21]. Furthermore, uncertainty from possible changes on government policy can be reflected in constraints as in [13] and asset classes cannot be held exactly as planned due to for example change of the regulations can be reflected as uncertainty in decision variables.

In conventional multiobjective optimization methods, the involvement of uncertainty in the problems is usually ignored. As a result, the immunity to uncertainty which we call robustness of solutions is not considered. However, the uncertainty can render the optimized solutions ineffective with undesirable and unexpected degradation on the objective function values. Thus, the consideration on robustness of solutions is as relevant as that of multiple objectives for practical problems.

When robustness is considered, a multiobjective optimization method has to find Pareto optimal solutions without knowing the behavior of the uncertain data exactly. Consequently, a DM has to understand the consequences of the involved uncertainty in addition to considering multiple conflicting objectives simultaneously. In addition, the DM also needs to learn the possible trade-off between the objective function values and robustness.

In recent years, different approaches have been developed sharing the common goal of identifying solutions both with respect to multiple conflicting objectives and being sufficiently immune to the uncertainty (see e.g., [2], [9], and [29]). However, in this paper, we do not assume the availability of probability distribution information as in [2] because such information is not always available. On the other hand, we do not expect any deep understanding on the problem from the DM to judge a fuzzy membership as in [29]. Instead, we aim at supporting the DM to learn about the problem, its attainable solutions, and the consequences of uncertainty and eventually find a most preferred solution.

Different multiobjective optimization methods (see e.g., [17], [26] and [27]), can be classified into a priori, a posteriori, and interactive methods (see e.g., [17]). Interactive methods has demonstrated advantages in supporting a DM to iteratively find a most preferred solution. With interactive methods, the DM does not need to know her/his preferences before knowing the attainable solutions as in so called a priori methods. (S)he is not expected to make a choice among a (large) set of solutions as in a posteriori methods which can be cognitively challenging. Instead, the DM guides the solution process by specifying her/his preferences in each iteration based on a given Pareto optimal solution. During this process, the DM is provided the opportunity to learn about the problem and its attainable solutions. The possibility of learning demonstrates strong potential for us to utilize to achieve the aim. Thus, we concentrate on exploiting interactive methods to support the DM to make a well-informed decision.

As mentioned before, uncertainty can be reflected in different elements of a multiobjective optimization problem. For problems with parameter uncertainty, different so-called robust Pareto optimal solutions (see summary in [14] and [30]) have been defined and some solution methods have been proposed in [5] and [9]. There have been also attempts to support the DM with interactive methods e.g., in [13] and [21]. We concentrate on decision uncertainty [10] in this paper because of the lack of research efforts in supporting the DM to find a most preferred solution for this type of problems.

By incorporation of robustness, we mean the analysis on the consequences of decision uncertainty in the objective function values. In the literature, there exist at least three different strategies to incorporate robustness in solving problems with decision uncertainty, but none of them concentrate on supporting the DM. The first type of strategy is to combine additional objectives, which quantify the robustness of a solution, with the original objectives as in [1] and [11]. The changes in the objective function values due to the uncertainty are optimized simultaneously with the original objectives to compute a set of solutions. From these solutions, the DM is expected to select one based on her/his preferences.

Second, problems with decision uncertainty can be transformed to deterministic ones by modifying the objectives. In [10], the concept of regularization robustness is extended to multiobjective optimization problems to derive a regularized robust counterpart of the uncertain problem. In [8], the original objective functions are replaced by the so-called mean effective objective functions. In [16] and [28], the original objective functions are replaced by their approximated mean and variance functions.

Third, a robustness measure can be used as an additional constraint as in [8], [12] and [15] where only solutions whose measured robustness satisfies predefined thresholds are considered feasible. Alternatively, a set of Pareto optimal or near-Pareto-optimal solutions can be compared based on their measured robustness as in [3] and [25].

Even though the first type of strategy allows the DM to consider multiple objectives and robustness simultaneously, additional objective functions can bring additional cognitive load to the DM. For example, when the deviation of the value of each objective function is combined with the original objectives as in [1], the DM has to consider double amount of objectives simultaneously. Thus, the amount of additional objective functions should be minimized. To support the DM to make a well-informed decision, the information exchange in the interactive solution process should be understandable, i.e., (s)he should understand the provided information and can express her/his preferences conveniently. So the DM should be informed on the objective function values during the solution process. For this purpose, the original objective functions should be preserved. Thus, the second type of strategy is not well fitted for interactive methods. In addition, robustness measure as a constraint as in the third type of strategy does not provide the opportunity for the DM to consider it simultaneously with the objectives and (s)he cannot directly specify her/his preferences. In addition, robustness measure as such should have an understandable meaning to the DM. Thus, with the focus on supporting the DM to learn about the problem and the consequences of uncertainty, we need further developments.

Motivated by the gaps in the literature, we quantify the robustness of solutions with a single understandable robustness measure to capture the consequences of decision uncertainty in the multiple objectives in the problem. Together with the original multiple objectives, we add the robustness measure as an additional objective to give the DM the opportunity of considering robustness and objective function values simultaneously and, thus, balancing between robustness and desirable objective function values. Our goal is not to develop a totally new interactive method but to enhance the existing ones when decision uncertainty is involved in the problem. As an example, we utilize elements of synchronous NIMBUS [20]. But our approach can also be used in for example reference point-based methods [32].

To support the DM to learn about the consequences of the uncertainty during an interactive solution process, a robustness measure should include the following desired properties: 1. The numerical value should reflect how the uncertainties in decision variables can affect the objective function values. Based on the value, the DM can consider how 'robust' a solution is. 2. With the computed numerical value, the DM should be able to specify her/his preferences conveniently. Based on these desired properties, we first identify and analyze the robustness measures in the literature that are closest to them. Then we propose an alternative robustness measure which meets both of the desired properties, and can thus better support the DM in an interactive approach.

The rest of the paper is organized as follows: in Section 2, we present some basic concepts, introduce the NIMBUS method briefly, and discuss robustness measures that are closest to the desired properties. Then in Section 3, we present a robustness measure that can be understandable for the DM and propose the interactive approach, which is followed by numerical examples where we demonstrate the advantages of our approach by solving two problems in Section 4. Finally, we conclude the paper in Section 5.

2 Background

2.1 Multiobjective optimization and decision uncertainty

Deterministic multiobjective optimization problems are defined in the form

$$\begin{array}{ll} \text{minimize or maximize} & \{f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x})\} \\ \text{subject to} & \boldsymbol{x} \in S, \end{array} \tag{1}$$

with objective functions (objectives) $f_i : S \to \mathbb{R}$ to be simultaneously optimized, where $1 \leq i \leq k$ and $k \geq 2$. The decision vectors (which consist of decision variables as their components) $\boldsymbol{x} = (x_1, ..., x_n)^T$ belong to the nonempty feasible region $S \subset \mathbb{R}^n$. Objective vectors $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x}))^T$ consist of objective function values which are the images of the decision vectors. The image of the feasible region is called the feasible objective region $Z = \boldsymbol{f}(S)$. If all the objective functions are minimized, a solution $\bar{\boldsymbol{x}}$ is said to be Pareto optimal if there does not exist another solution $\boldsymbol{x} \in S$ such that $f_i(\boldsymbol{x}) \leq f_i(\bar{\boldsymbol{x}})$ for all $i = 1, ..., k$ and the inequality is strict for at least one index j. If some of the objectives f_i is to be maximized, it is equivalent to minimize $-f_i$.

For (1), the set of Pareto optimal solutions usually contains more than one elements. Mathematically, Pareto optimal solutions are incomparable. The DM is expected to identify the most preferred one among them as the final solution. Only one DM is assumed to be involved in the solution process in this paper. It is often useful for the DM to know the ranges of the objective function values in the set of Pareto optimal solutions. The ideal objective vector $\boldsymbol{z}^* = (z_1^*, ..., z_k^*)^T$ and the nadir objective vector $\boldsymbol{z}^{nad} = (z_1^{nad}, ..., z_k^{nad})^T$ give the bounds of the objective function values. The ideal objective vector is formed by the individual optima of each objective function in the feasible region. The utopian objective vector $\boldsymbol{z}^{**}$, which is strictly better than z^*, is defined for computational reasons. In practice, z_i^{**} is set as $z_i^* - \epsilon$ for $i = 1, ..., k$ if f_i is to be minimized, where $\epsilon > 0$ is a small scalar. The nadir objective vector, which represents the worst objective function values, can be approximated for example by a so-called pay-off table (see e.g., [17] for further details). If the objective function values are with different magnitudes, the nadir and utopian objective vectors can be used to normalize them.

In this paper, we consider multiobjective optimization problems with decision uncertainty. By decision uncertainty, we mean that a computed solution, which we refer to as the base solution $\boldsymbol{x}^b$, cannot be guaranteed to be implemented exactly. Instead, the implementation can involve small perturbations $\boldsymbol{\Delta x}$, i.e., the implemented solution is from the set $\{\boldsymbol{x}^b + \boldsymbol{\Delta x} | \boldsymbol{\Delta x} \in \Omega\}$ where Ω is the set of all possible perturbations in the neighborhood of the base solution. We assume that Ω is a hyperbox and $\boldsymbol{0} \in \Omega$, which does not have to be sysmetric. We refer to the corresponding objective vector $\boldsymbol{f}(\boldsymbol{x}^b)$ as the base objective vector whose components are the base objective function values. The type of uncertain multiobjective optimization problems considered is of the form:

$$\begin{array}{ll} \text{minimize or maximize} & \{f_1(\boldsymbol{x} + \boldsymbol{\Delta x}), ..., f_k(\boldsymbol{x} + \boldsymbol{\Delta x})\} \\ \text{subject to} & \boldsymbol{x} \in S \\ & \boldsymbol{x} + \boldsymbol{\Delta x} \in S, \text{for all } \boldsymbol{\Delta x} \in \Omega. \end{array} \tag{2}$$

In the formulation, $\boldsymbol{x}$ is the decision vector and $\boldsymbol{\Delta x}$ is the unknown possible perturbation within the hyperbox Ω. To solve this problem, we consider all the possible values $\boldsymbol{\Delta x} \in \Omega$ and search for a most satisfactory base solution $\boldsymbol{x}^b$ for the DM. By a most satisfactory base solution, we mean that the DM is satisfied with the base objective function values $(f_1(\boldsymbol{x}^b), ..., f_k(\boldsymbol{x}^b))^T$ and the objective function values when perturbations occur, i.e., $(f_1(\boldsymbol{x}^b + \boldsymbol{\Delta x}), ..., f_k(\boldsymbol{x}^b + \boldsymbol{\Delta x}))^T$ for all $\boldsymbol{\Delta x} \in \Omega$.

2.2 NIMBUS

As mentioned in Section 1, we utilize elements of the NIMBUS method to build our interactive approach. As mentioned in [20], one can always derive a reference point from the preference information utilized in

NIMBUS, and thus, our approach to be proposed can also be used with reference point based methods. In NIMBUS, given the current Pareto optimal solution $\boldsymbol{x}^c$, the DM directs the interactive solution process by specifying preferences as a classification of the objectives. The classification indicates how the current objective function values $\boldsymbol{f}(\boldsymbol{x}^c)$ should change to be more desired by the DM. The DM can classify the objective functions into up to five different classes including:

$I^<$ for those to be improved (i.e., decreased in case of minimizing, increased in case of maximizing),

$I^\leq$ for those to be improved until some desired aspiration level $\hat{z}_i$,

$I^=$ for those that are satisfactory at their current level,

$I^\geq$ for those that may be impaired until a bound ϵ_i, and

$I^\diamond$ for those that are temporarily allowed to change freely.

Each objective is assigned to one of the classes described above. Some objectives must be allowed to be impaired to enable improvements in others because of the nature of the Pareto optimality. If aspiration levels or bounds are used, the DM is expected to provide them.

In the NIMBUS method, new Pareto optimal solutions are computed by solving a scalarized problem, which includes preference information given by the DM in the classifications. In this paper, we use one of the four scalarized problems of the synchronous NIMBUS method [20], which has the form (for minimizing the objectives):

$$\begin{aligned} \text{minimize} \quad & \max_{\substack{i \in I^< \\ j \in I^\leq}} \{ w_i(f_i(\boldsymbol{x}) - z_i^*), w_j(f_j(\boldsymbol{x}) - \hat{z}_j) \} + \rho \sum_{i=1}^{k} w_i f_i(\boldsymbol{x}) \\ \text{subject to} \quad & \boldsymbol{x} \in S \\ & f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{x}^c) \text{ for all } i \in I^< \cup I^\leq \cup I^=, \\ & f_i(\boldsymbol{x}) \leq \epsilon_i \text{ for all } i \in I^\geq, \end{aligned} \tag{3}$$

where $\boldsymbol{x}^c$ is the current Pareto optimal solution, $\boldsymbol{z}^*$ is the ideal objective vector, $\hat{z}_i$ are the aspiration levels for the objective functions in $I^\leq$, ϵ_i are the bounds of allowed impairment for the objective functions in $I^\geq$, $\rho > 0$ is a small scalar bounding the trade-offs, and the coefficients w_i $(1 \leq i \leq k)$ are constants used for scaling the objectives. The value of w_i is based on the estimated ranges, i.e., $\frac{1}{z_i^{nad} - z_i^{**}}$, for normalizing the objective function values.

The DM can compare the two Pareto optimal solutions before and after the classification, so that (s)he can learn how attainable her/his desired changes were. For more information about the method and the proof of Pareto optimality, see [20]. In addition, the NIMBUS method provides the DM an opportunity to generate intermediate solutions and to save interesting solutions during the iterative solution process. The DM can return to a saved solution any time or select one as the most preferred solution from the set of saved solutions.

We shall return to the NIMBUS method and discuss our interactive approach for solving multiobjective optimization problems with decision uncertainty in Section 3. In what follows, we discuss the robustness measures from the literature.

2.3 Robustness measures from the literature

As discussed in Section 1, objective functions are assumed to have a meaning to the DM and, thus, the original objective functions should be preserved to allow the DM to consider their values together with the robustness simultaneously. Furthermore, the DM should also be able to learn their possible trade-offs. For this purpose, naturally, a robustness measure should be used as an additional objective to the problem formulation, i.e., we solve a multiobjective optimization problem by combining the original objectives and a robustness measure as its objectives. By employing an additional objective, the DM can consider balancing between robustness and objective function values.

We have identified three robustness measures in the literature that are closest to the desired properties to be used in the interactive solution process as listed in the introduction. These measures quantify the robustness of a base solution and were originally used as additional constraints, but they can be used as an additional objective as well.

In [8], the measure involves sampling and the difference between the base objective vector and the average function values of samples in the neighborhood of a base solution is used to measure its robustness:

$$R_1(\boldsymbol{x}^b) = \frac{\|\boldsymbol{f}^p(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}^b)\|}{\|\boldsymbol{f}(\boldsymbol{x}^b)\|}, \tag{4}$$

where $\|\cdot\|$ is the Euclidean norm. The so-called mean effective objective vector $\boldsymbol{f}^p(\boldsymbol{x})$ consists of the average objective function values of the samples in the neighborhood. According to the definition, the smaller the value of $R_1(\boldsymbol{x}^b)$ is, the more robust the solution is. We can adopt this measure as an additional objective to be minimized.

In [11], a measure, which is also based on studying samples in the neighborhood of a base solution, is defined for each objective function to capture robustness:

$$f_i^{R_2}(\boldsymbol{x}^b) = \frac{1}{H}\sum_{h=1}^{H} \frac{|\tilde{f}_i(\boldsymbol{x}^h) - \tilde{f}_i(\boldsymbol{x}^b)|}{\|\boldsymbol{x}^h - \boldsymbol{x}^b\|}, \tag{5}$$

where $\boldsymbol{x}^h$ is a sample and H is the total number of samples. The notation $\tilde{f}_i$ is used to indicate that the objective function values are normalized within their ideal and nadir values. This measure studies how much the objective function value changes relative to the perturbation of a base solution. Based on the robustness measure of each objective function, so-called global robustness measures which include all k objectives are defined as $R_2(\boldsymbol{x}^b) = \max_{i=1,\dots,k} f_i^{R_2}(\boldsymbol{x}^b)$, and $R_2(\boldsymbol{x}^b) = \frac{1}{k}\sum_{i=1}^{k} f_i^{R_2}(\boldsymbol{x}^b)$. We can adopt either as an additional objective to be minimized.

In [12] and [15], given a base solution and the maximum acceptable changes from the base objective function values, the radius of the smallest hyper-sphere centered on the base solution is calculated to measure its robustness by solving a single-objective optimization problem:

$$\begin{array}{ll} \text{minimize} & \|\Delta x\|_p \\ \text{subject to} & \max(\frac{|\Delta f_i|}{\Delta f_{0,i}}) = 1. \end{array} \tag{6}$$

This measure studies how much perturbation, i.e., the optimal value of Δx, is allowed in the base solution for the objective function values to be acceptable. The constraint, where Δf_i is a function of Δx, states that the maximum change in the objective function values has to be equal to the pre-specified acceptable level $\Delta f_{0,i}$. The optimized objective function value of (6) is the value of the robustness measure, which we refer to as $R_3(\boldsymbol{x}^b)$. A bigger value $R_3(\boldsymbol{x}^b)$ means that the more robust the base solution is. We can adopt this measure as an additional objective to be maximized.

Table 1: Summary of different robustness measures

measures	$R_1(\boldsymbol{x}^b)$	$R_2(\boldsymbol{x}^b)$	$R_3(\boldsymbol{x}^b)$
parameters	Ω, H	Ω, H	lower and upper bounds of Δx, value of $\Delta f_{0,i}$
randomness involved	random	random	exact and stable

We summarize the characteristics of the three measures in Table 1. The required parameters to compute the robustness measures are presented in the first row. The randomness involved in the computed values of the robustness measures is presented in the second row.

The three measures are based on the study of the neighborhood of a base solution and they require some parameters. As shown in the table, the size of the neighborhood, which is represented by Ω in (2), is commonly required. For measures $R_1(\boldsymbol{x}^b)$ and $R_2(\boldsymbol{x}^b)$, the number of samples (H) in the neighborhood is needed. There were no clear guidelines how these parameters should be set in the papers where the measures were originally proposed. For $R_3(\boldsymbol{x}^b)$, the acceptable levels of change from base objective function values $\Delta f_{0,i}$ are required. This parameter is said to be set by the user, which can be understood as the DM in our context.

Unfortunately, all the existing robustness measures have some shortcomings. As can be seen in the definitions of the measures, the numerical values do not have a direct meaning on how robust a solution is for the DM except the intuitive indication based on if the measure should be minimized or maximized. With the numerical values, the DM cannot formulate and specify her/his preferences conveniently during the interactive solution process. In the interactive solution process, the DM can only specify her/his preferences on the measures based on this intuition which does not help the DM to formulate her/his preferences clearly. The purpose of using an additional objective to incorporate the robustness is to support the DM to find a most preferred solution by simultaneously considering the base objective function values and the robustness of a solution. We can summarize that none of the robustness measures meets our needs well. To communicate the meaning of robustness to the DM in a more understandable way and to allow the DM to formulate and specify preferences conveniently, it is desirable to formulate a new robustness measure. We propose such a measure in the next section.

3 Interactive approach for solving problems with decision uncertainty

3.1 A new robustness measure

As discussed before, we incorporate a robustness measure to the problem formulation by adding an additional objective. None of the measures identified from the literature fully meets the desired properties to be incorporated in an interactive solution process. In this section, we first describe a new robustness measure which is suitable to be used in an interactive approach. Then we propose the interactive approach tailored to solve multiobjective optimization problems with decision uncertainty.

We propose a robustness measure which can deliver the meaning of robustness to the DM in a more understandable way. Our robustness measure investigates the ranges of objective function values in the neighborhood Ω of a base solution $\boldsymbol{x}^b$. The existence of the ranges of the objective function values is a consequence of the uncertainty in the decision variables. In the form of the ranges, the DM can get the information on how the objective function values change. In other words, the ranges characterized by best and worst objective function values describe the variations due to the possible perturbations in the decision variables. For an objective function f_i, the range of its value in the neighborhood can be defined as $r_i(\boldsymbol{x}^b) = \max_{\boldsymbol{\Delta x}\in\Omega} f_i(\boldsymbol{x}^b + \boldsymbol{\Delta x}) - \min_{\boldsymbol{\Delta x}\in\Omega} f_i(\boldsymbol{x}^b + \boldsymbol{\Delta x})$. We refer to these ranges as r_i ranges. As discussed before, we want to introduce only one additional objective not to introduce too much cognitive load. We can use the maximum range, i.e., the upper bound, of all objective functions to measure the robustness of a solution as an objective to be minimized. So our robustness measure is:

$$R_4(\boldsymbol{x}^b) = \max_i[\tfrac{r_i(\boldsymbol{x}^b)}{z_i^{nad} - z_i^{**}}], i = 1, ..., k, \tag{7}$$

where the lower and upper bounds of the neighborhood Ω are provided by the DM when the problem is formulated. As an expert in the application domain, the DM is more likely to know reasonable bounds than others. When compared to the robustness measures discussed in Section 2, Ω has the same meaning as in the measures $R_1(\boldsymbol{x}^b)$ and $R_2(\boldsymbol{x}^b)$. In this measure, the lower and upper bounds do not have to be symmetric around the base solution.

As can be seen in (7), we need to solve $2k$ additional single-objective optimization problems to compute the value. In principle, this can provide the DM exact measurements of the r_i ranges. If approximated r_i ranges can be accepted by the DM, similar sampling techniques as presented in [8] and [11] for measures $R_1(\boldsymbol{x}^b)$ and $R_2(\boldsymbol{x}^b)$ can be applied. For the rest of the paper, we refer to f_i as an active objective function if i gives the maximum for $R_4(\boldsymbol{x}^b)$ in (7).

The computed value of $R_4(\boldsymbol{x}^b)$ is the percentage of the r_i range with respect to the ideal and nadir values of the active objective function. With the help of the ideal and nadir values, this numerical value can tell the DM how much the objective function varies in its own range, which is a concrete expression on the consequences of the uncertainty. The DM can also learn without much effort that the r_i ranges of the other objective functions are smaller than this value. In addition, by computing the value of $R_4(\boldsymbol{x}^b)$, the r_i ranges of all the objective function values in the neighborhood are also available, which we will utilize to support the DM in the interactive solution process. We will discuss how we can utilize the r_i ranges and develop an appropriate visualization to improve the understandability of the robustness measure in the next subsection where we discuss the proposed interactive approach.

3.2 An interactive approach for solving multiobjective optimization problems with decision uncertainty

We incorporate robustness into the problem formulation (1) by adding the measure $R_4(\boldsymbol{x}^b)$ as an additional objective. In [31], Pareto optimality to the original problem of a Pareto optimal solution to a problem formulated with an additional objective is summarized. A solution remains Pareto optimal or not depending on how the objectives conflict with one another in the new problem with an additional objective. So whether the robust solutions found by our approach are Pareto optimal to the original problem depends on the problem itself and the consequences of uncertainty. However, as argued in [4], the robustness of a solution and the corresponding values of the original objectives usually conflict with each other. To gain robustness, sacrifices on the objective function values can be necessary. Furthermore, by learning the trade-offs between the objective function values and robustness, it is a conscious choice for the DM if objective function values are sacrificed.

As mentioned before, we build our interactive approach by utilizing the elements of the synchronous NIMBUS method. We follow the interactive solution process of the NIMBUS method where the DM is expected to specify her/his preferences by classifying the objective functions as described in Section 2.2. Our goal is to support the DM to find a base solution with most satisfactory base objective function values and objective function values when the perturbations occur. This involves differences from the original NIMBUS method, in which the goal is to support the DM to find a most preferred Pareto optimal solution. Because of the specific robustness consideration, we tailor some components of the NIMBUS solution process to support the DM to consider the base objective function values and the robustness of a solution simultaneously.

The numerical value of the robustness measure $R_4(\boldsymbol{x}^b)$ is used to capture the robustness of a base solution. It is the percentage of the r_i range of the active objective function for a base solution in its given neighborhood within the range of that objective. With the information of ideal and nadir values, the DM can combine the numerical value of $R_4(\boldsymbol{x}^b)$ and the ranges of the active objective function to have a concrete understanding on the robustness of the solution.

Based on the definition of $R_4(\boldsymbol{x}^b)$, the r_i ranges (in percentage) of other objective functions are guaranteed to be smaller. This allows the DM to indirectly specify her/his preferences on the r_i range of a specific objective function by providing the desired value for $R_4(\boldsymbol{x}^b)$. By doing so, the DM has specified the desired maximum r_i ranges for all the original objective functions, in which the specific objective function is included. The DM is more likely to learn about these facts without much cognitive effort than learning a numerical value without a direct meaning on the consequences of decision uncertainty. In addition, since the r_i range of each objective function is naturally available with the computation of $R_4(\boldsymbol{x}^b)$, we will utilize this information when we present a solution to the DM.

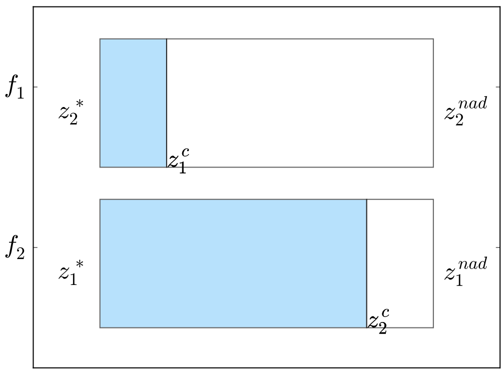

Figure 1: Original IND-NIMBUS visualization

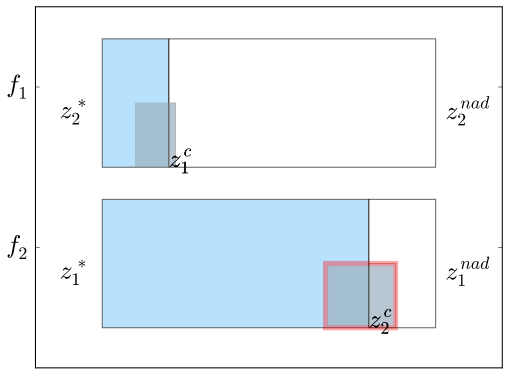

Figure 2: Visualization with robustness information

Visualization can be used to support the DM in studying the trade-offs between optimality and robustness. To visually present a solution to the DM, we tailor a visualization method for presenting the base objective function values and the robustness information simultaneously. An additional component is added to one of the visualizations used in the IND-NIMBUS framework (see [18] and [23]). An example of the visualization in IND-NIMBUS is shown in Figure 1 with two objective functions to be minimized. Each objective function is visualized as a horizontal bar within the range of its ideal and nadir objective values. The colored part of the bar illustrates the current objective function value $\boldsymbol{z}_i^c$, which starts from the ideal value towards the nadir objective value. The DM can classify an objective function e.g., by sliding the endpoint of the colored bar. Instead of adding an additional bar for the value of $R_4(\boldsymbol{x}^b)$, we superimpose the r_i ranges on top of the corresponding bars of the k (original) objective functions.

An example of the tailored visualization method is presented in Figure 2. The r_i range of each objective function is presented as a gray shadow around the current base objective function value. The r_i range indicating the robustness of the current active objective function is highlighted with a frame to inform the DM that (s)he should pay attention to it and can specify her/his preferences. When we visualize a solution as a part of a solution process, the numerical values of the upper and lower bounds of variation in each objective function value will be shown in the corresponding places of the gray shadow. The value of the objective f_{k+1} will be shown in the upper right corner of the highlighted frame. This visualization can further help the DM to understand the robustness of the base solution concretely together with the value of $R_4(\boldsymbol{x}^b)$ because it can illustrate how the uncertainties in the decision variable are reflected in each objective function.

With a solution presented in terms of the base objective function values and the r_i ranges as in Figure 2, the DM can specify her/his preferences for a more desired solution. In the original NIMBUS method, the DM is expected to classify all the objective functions and provide the aspiration levels and bounds for the corresponding classes of objective functions. In our approach, the DM can choose to follow the original NIMBUS method to classify the objective function for robustness in the same way as for the original objective functions. Alternatively, the DM can also choose to classify the original objective functions but specify lower and upper bounds of the r_i range on the current active objective function. In this case, we say that the DM chooses to adjust the r_i range. Once the adjusted r_i range has been specified by the DM, we convert it as a NIMBUS classification by calculating the desired aspiration level. We return later to this technical detail on converting an adjusted r_i range to a proper classification later.

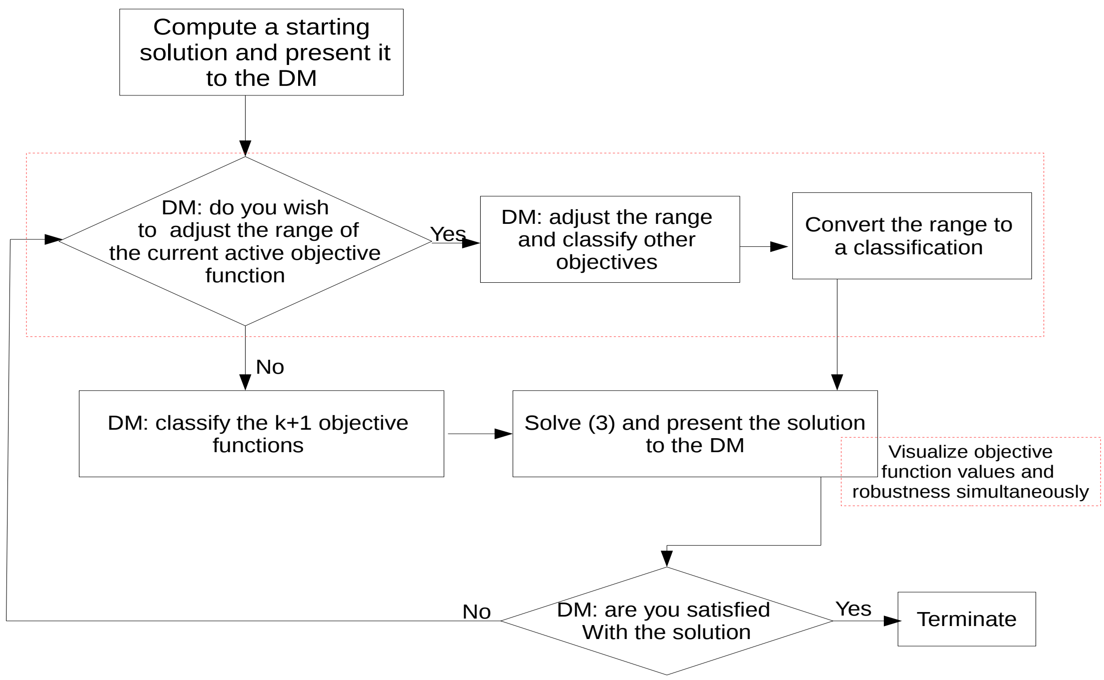

Figure 3: Flowchart of the interactive approach

With the tailored visualization and the multiple available ways of expressing the preferences on the robustness of a solution, our interactive approach, as shown in the flowchart in Figure 3, starts from computing an initial solution and presenting it to the DM with the tailored visualization. The multiple ways to specify preferences on the robustness measure are available for the DM in the highlighted intermediate step. Based on the specified preferences by the DM, we convert the preferences as classifications if necessary, then compute a new solution by solving the scalarized problem (3) and present it to the DM. If the DM is not satisfied with the solution, the solution process continues as in the original NIMBUS method. In this way, the DM iteratively guides the solution process towards a most preferred robust solution.

This interactive approach has four advantages which aim at providing better support to the DM in the solution process. First, the meaning of the numerical value of the robustness measure $R_4(\boldsymbol{x}^b)$ is understandable for the DM, because it is the percentage of maximum possible change in the objective function values with respect to the ideal and nadir values. Second, the r_i range presented to the DM provide an opportunity for the DM to observe and understand how the uncertainties in the decision variables affect the objective function values, i.e., the consequences of uncertainty. In addition, the r_i range are presented together with the base objective function values. So the DM can consider both of them at the same time. Third, we provide multiple alternatives for the DM to specify her/his preferences concerning the robustness of a more desired solution. So in the solution process, the DM can choose a comfortable way in each iteration of the solution process. Fourth, with the robustness measure incorporated as an addition objective function, the DM can find an acceptable balance between the robustness and the objective function values of a solution and gain insights on how they are interdependent.

As mentioned before, the DM can classify all the objectives. If (s)he does so, we can proceed directly by solving (3) for a new solution. Alternatively, the DM can pay special attention to the active objective function and adjust the r_i ranges. In this case, we need to convert the adjusted r_i range to a NIMBUS classification. The close relationship between the desirable aspiration level of an objective function and the classification of it was discussed in [19] and [20]. Here we have $f_{k+1} = R_4(\boldsymbol{x}^b)$. Depending on the adjusted r_i range, we have five different types of conversion:

- The adjusted r_i range is smaller than the current one: it means that the DM wishes to have a more robust solution and f_{k+1} is classified as to be improved to an aspiration level, i.e., $f_{k+1} \in I^{\leq}$ with a value $\hat{z}_{k+1}$, where $\hat{z}_{k+1}$ is the calculated aspiration level based on the adjusted r_i range ;
- The adjusted r_i range is greater than the current one: it means that the DM can accept a less robust solution and f_{k+1} is classified as to be impaired until an upper bound, i.e., $f_{k+1} \in I^{\geq}$ with a value ϵ_{k+1}, where ϵ_{k+1} is the calculated bound based on the adjusted r_i range in the same way as for the aspiration level;
- The adjusted r_i range is the same as the current one: it means that the DM wishes to have a solution as robust as the current one and f_{k+1} is classified as $f_{k+1} \in I^{=}$;
- The DM has adjusted the r_i range to be 0: it means that the DM wishes to have a solution as robust as possible and f_{k+1} is classified as $f_{k+1} \in I^{<}$.
- The DM does not specify any adjustments for the r_i range. This means that (s)he can accept any value in f_{k+1}. So $f_{k+1} \in I^{\diamond}$.

After the conversion, we can use the resulting classification to compute a new solution for the DM.

4 Numerical results

4.1 River pollution problem

In this section, we illustrate the solution process of a multiobjective optimization problem with decision uncertainty with the proposed interactive approach. The river pollution problem considered was originally presented in [22] as a deterministic problem. In the problem, a fishery and a city are polluting water in a river. The city is located downstream from the fishery. Both the city and the fishery have their own pollution treatment plants. We consider the following formulation:

$$
\begin{array}{ll}
\text{maximize} & f_1(\boldsymbol{x}+\boldsymbol{\Delta x}) = 4.07 + 2.27(x_1 + \Delta x_1) \\
\text{maximize} & f_2(\boldsymbol{x}+\boldsymbol{\Delta x}) = 2.60 + 0.03(x_1 + \Delta x_1) + 0.02(x_2 + \Delta x_2) + \frac{0.01}{1.39-(x_1+\Delta x_1)^2} + \frac{0.30}{1.39-(x_2+\Delta x_2)^2} \\
\text{maximize} & f_3(\boldsymbol{x}+\boldsymbol{\Delta x}) = 8.21 - \frac{0.71}{1.09-(x_1+\Delta x_1)^2} \\
\text{minimize} & f_4(\boldsymbol{x}+\boldsymbol{\Delta x}) = -0.96 + \frac{0.96}{1.09-(x_2+\Delta x_2)^2} \\
\text{minimize} & f_5(\boldsymbol{x}+\boldsymbol{\Delta x}) = R_4(\boldsymbol{x}) \\
\text{subject to} & 0.3 \leq x_1, x_2 \leq 1.0, \\
& \text{for all } \Delta x_1 \in [x_1 - 0.1, x_1 + 0.1] \text{ and } \Delta x_2 \in [x_2 - 0.1, x_2 + 0.1],
\end{array}
\tag{8}
$$

where there are four original objectives and the fifth objective function represents robustness. The decision variables x_1 and x_2 represent the proportional amount of biochemical oxygen demanding material to be removed from water in the treatment plants after the fishery and the city, respectively. The more biochemical oxygen demanding material is removed, the more the quality of water will improve. The unknown possible perturbations are represented by Δx_1 and Δx_2 . The information on the neighborhood near the base solution, i.e., lower and upper bounds of the perturbations, is provided by the DM. The first and second objective functions describe the quality of water after the fishery and after the city, respectively, and the third objective function describes the percentage of return on investment at the fishery. The fourth objective represents the addition of tax rate in the city. The fifth objective is the robustness measure presented in (7). We consider uncertainties originating from the operations of the pollution treatment plants. As a result, the amount of removed biochemical oxygen demanding material can involve perturbations from the base values. Consequently, the objective function values can be different from their base values. We solve this problem with the proposed interactive approach interacting with a DM.

The individual optima of the original objectives were calculated to form the ideal objective vector $\boldsymbol{z}^* = (6.34, 3.45, 7.50, 0)^T$. The nadir objective vector was approximated as $\boldsymbol{z}^{nad} = (4.75, 2.85, 0.32, 9.70)^T$. To get started, we set the ideal value of the robustness measure as 0, which means the perturbations of the base solution do not affect the objective function value at all. We set the nadir value as 1, which indicates that the r_i range of the active objective function in the neighborhood is as large as the range between the ideal and nadir values.

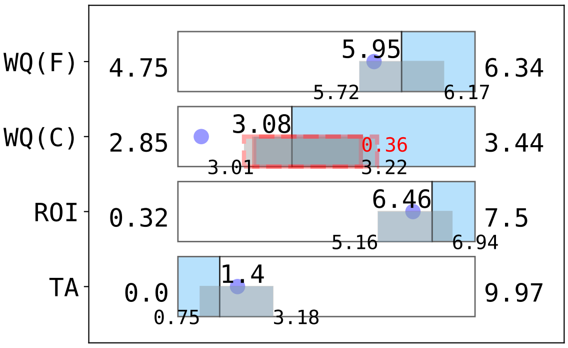

Figure 4: Intial solution

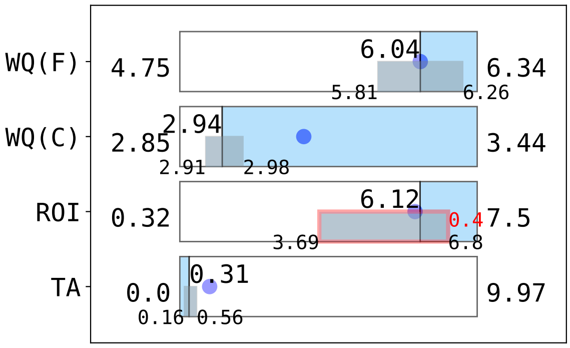

Figure 5: Iteration 1

Initialization: We first introduced our robustness measure to the DM in terms of what the value of f_5 means and what the r_i ranges in the visualization mean. Then we computed and presented an initial solution $\boldsymbol{z}^0 = (5.95, 3.08, 6.46, 1.40, 0.36)^T$ with the tailored visualization method to the DM as shown in Figure 4. The initial solution was calculated as in the NIMBUS method.

In Figure 4, the bars present the water quality after the fishery WQ(F), water quality after the city WQ(C), return on investment of the fishery (ROI), and the additional tax rate in the city (TA). The colored part of a bar illustrates the current value of the corresponding objective (also given numerically) accompanied by the ideal and nadir values at its endpoints. The r_i range, where the values of its lower and upper bounds are presented at the endpoints, is superimposed on top of the corresponding bar to present how uncertainties in the solution can affect the objective function value. For the current active objective function, we highlight its r_i range and mark the current value of the robustness measure in red on the upper right corner. As discussed before, the objective function value of f_5 was not presented in its own bar. Instead, we presented the r_i ranges within the bars of the corresponding objective functions. The DM was asked to choose a preferred way to express her preferences.

The DM chose to adjust the r_i range of the current active objective function f_2. With the adjusted r_i range, we calculated an aspiration level $\epsilon_5 = 0.45$ and converted it to a classification as allowing the value of f_5 to be impaired till 0.45. The adjusted r_i range is represented as a broken line in the first illustration of Figure 4. When considering the initial solution $\boldsymbol{z}^0$, the DM wanted to improve the quality of water after the fishery slightly and reduce the additional tax rate to 2% in the city. At the same time, the quality of water after the city was allowed to be impaired till 2.9 and the return on investment of the fishery was also allowed to reduce till 6%. In the NIMBUS notation, the DM provided the classification for iteration 1: $I^{\leq} = \{f_1, f_4\}$ with aspiration level $\hat{z}_1 = 5.8$, and $\hat{z}_4 = 2$; $I^{\geq} = \{f_2, f_3, f_5\}$ with the bounds $\epsilon_2 = 2.9$, $\epsilon_3 = 6$, and $\epsilon_5 = 0.45$. The aspiration levels and bounds are denoted by dots in Figure 4.

Iteration 1: Based on the classification, a new solution $\boldsymbol{z}^1 = (6.04, 2.94, 6.12, 0.31, 0.40)^T$ was calculated by solving the scalarized problem (3) and presented to the DM as in Figure 5. Based on $\boldsymbol{z}^1$, the DM could see that her preferences in iteration 1 were satisfied. However, she thought that better quality of water after the city should be achieved at the same time of maintaining the same quality of water after the fishery at the current level. In addition, she wished to maintain the current value of the maximum r_i range, i.e., no change to the robustness. Keeping in mind that some objectives have to be impaired in order to achieve better quality of water after the city, the DM allowed the return on investment of the fishery to reduce to 6% and the additional tax rate in the city till 1%. In other words, the DM gave her preference as: $I^{\geq} = \{f_3, f_4\}$ with bounds $\epsilon_3 = 6$, and $\epsilon_4 = 1$; $I^{=} = \{f_1, f_5\}$; $I^{\leq} = \{f_2\}$ with $\hat{z}_2 = 3.1$.

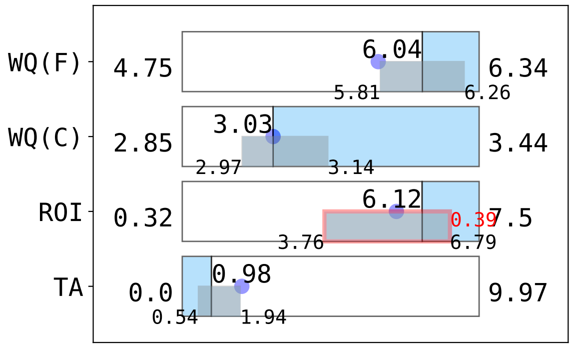

Figure 6: Iteration 2

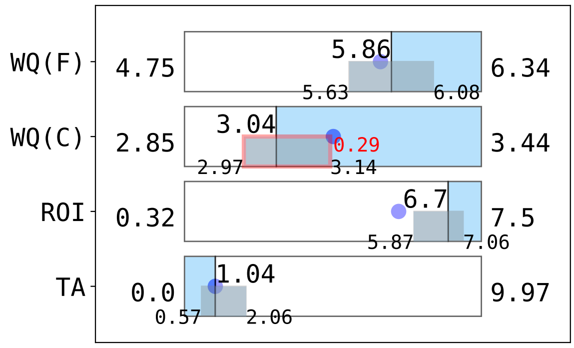

Figure 7: Iteration 3

Iteration 2: According to this classification, a new solution $\boldsymbol{z}^2 = (6.04, 3.03, 6.12, 0.98, 0.39)^T$ was calculated and presented to the DM as in Figure 6. The DM was not satisfied and wanted to make a new classification. Based on $\boldsymbol{z}^2$, the DM noticed that the additional tax rate in the city almost approached her specified upper bound and she did not wish to have worse quality of water after the city. So she decided to keep the current quality of water after the city. As an exploration of a more robust solution, she wanted to reduce the percentage of the r_i range of the active objective function to 0.3 and allow the quality of water after the fishery to decrease to 5.8, return on investment of the fishery to 5.5% and the additional tax rate in the city to increase till 2%. The DM's classification in NIMBUS notation was: $I^{=} = \{f_2\}$; $I^{\leq} = \{f_5\}$ with aspiration level $\hat{z}_5 = 0.3$; $I^{\geq} = \{f_1, f_3, f_4\}$ with bounds $\epsilon_1 = 5.8$, $\epsilon_3 = 5.5$, and $\epsilon_4 = 2$.

Iteration 3: Based on the classification, the new solution computed was $\boldsymbol{z}^3 = (5.86, 3.04, 6.7, 1.04, 0.29)^T$ as visualized in Figure 7. The DM noticed that her preferences were not fully satisfied. So she tried with another classification, i.e., she did not want to have worst quality of water after the city and wanted to reduce the r_i range of the active objective function to 0.25. She continued by allowing the water quality after the fishery to be impaired till 5.8 and the return on investment of the fishery till 5.5%. The DM's classification was: $I^{=} = f_4$; $I^{\leq} = \{f_2, f_5\}$ with $\hat{z}_2 = 3.15$, and $\hat{z}_5 = 0.25$; $I^{\geq} = \{f_1, f_3\}$ with bounds $\epsilon_1 = 5.8$, and $\epsilon_3 = 5.5$.

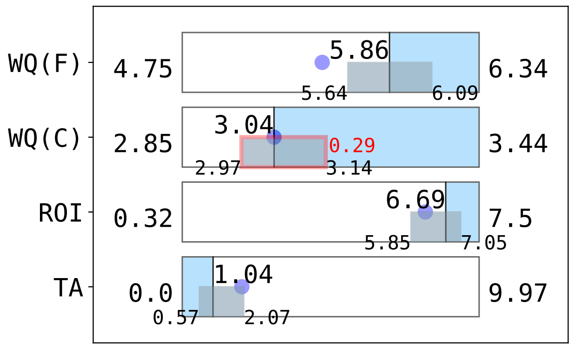

Figure 8: Iteration 4

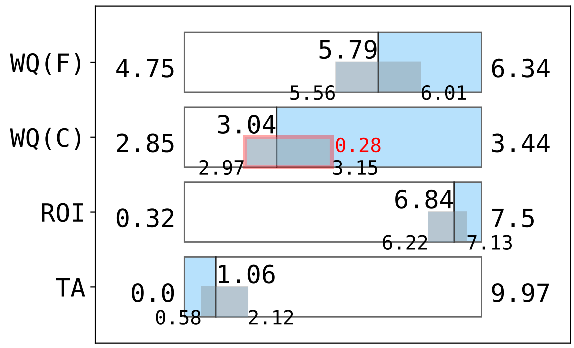

Figure 9: Final solution

Iteration 4: Based on this classification, $\boldsymbol{z}^4 = (5.86, 3.04, 6.70, 1.04, 0.29)^T$ was computed and visualized as in Figure 8. Based on $\boldsymbol{z}^4$, the DM noticed that the quality of water after the city did not improve as she wanted and the r_i range of the active function value did not reduce as she wished either.

So she decided to accept the values of these two objectives. In addition, the return on investment of the fishery approached the bound, but the quality of water after the fishery was much better than the value she could accept. So she thought she could have a better return on investment by having a worse but acceptable quality of water after the fishery. In order to guarantee the operation of the water treatment plant in the city, thus maintaining the quality of water after the city, she decided to allow up to 2% of additional tax in the city. In NIMBUS notation, the DM specified the classification to compute z^5 as: $I^{=} = \{f_2, f_5\}$; $I^{\leq} = \{f_3\}$ with an aspiration level $\hat{z_3} = 6.2$; $I^{\geq} = \{f_1, f_4\}$ with bounds $\epsilon_1 = 5.5$, and $\epsilon_4 = 2$.

Termination: With the classification, $\boldsymbol{z}^5 = (5.79, 3.04, 6.84, 1.06, 0.28)^T$ was computed and presented to the DM as in Figure 9. The DM noticed that the values of return on investment and the additional tax rate were better than she expected, but the quality of water after the fishery was already on the worst acceptable value. At the same time, the quality of water after the city and the r_i range of the active objective function were maintained. So after this iteration, the DM decided to terminate the solution process and accept $\boldsymbol{z}^5$ as the final solution.

During the solution process, the DM was able to understand the consequences of the uncertainty via the robustness measure. Thus the final accepted solution offered a well-informed balance between the base objective function values and the robustness. At the beginning of the solution process, the DM chose to adjust the r_i range of the active objective function because she felt it would be easier. In the later iteration, she could directly classify f_5. The DM noticed that the active objective function changed. She assumed that by altering the r_i range of the water quality after the city, the r_i range of that particular objective function would become worse. But she soon learned that the meaning of the robustness measure specified value for the active objective function is actually an upper bound for the r_i ranges of all the four objectives.

As for the advantages mentioned in Section 3, the DM was able to understand the meaning of the numerical value of the robustness measure. The simultaneously illustrated r_i ranges and the base objective function values helped her to consider both types of information together and then formulate and specify her preferences. She can also easily specify preferences on the robustness of a more desired solution. At the beginning of the solution process, she utilized the possibility to adjust the r_i range to get familiar and work with the robustness measure. By considering robustness as an objective function, the DM learned how the robustness and base objective function values affect each other. Consequently, she utilized this knowledge to find a satisfactory balance between the robustness and the based objective function values of the final solution she accepted.

4.2 Procurement contract selection with pricing optimization for a process network

Next, we illustrate the application of our interactive approach by solving a problem in procurement contract selection with pricing optimization for a process network. We utilize the optimization model presented in [7] as the foundation and augment it with three additional objectives. In the model, procurement contract selection and pricing analytics are combined for multi-period, multi-site tactical production planning. The manufacturer needs to make two key decisions: to select procurement contracts and to set selling prices for products. For the selection of procurement contracts, the manufacturer needs to decide whether to sign or not a particular contract with a supplier for purchasing a type of raw material. For setting the selling prices of final products, the manufacturer is assumed to use the price-response model (see e.g., [24]).

The problem was modeled with a single profit-focused objective in [7]. In this paper, we consider three additional objectives for environmental responsibilities and the maintenance of strategic competence of the manufacturer. The additional objectives include: minimizing the environmental impact scores of selected business partners (i.e., suppliers in this case), minimizing the pollution content emitted from the production process, and maximizing the demand in the market for the main products. Both the pollution content and environmental impact scores are for the consideration of environmental responsibility. Minimizing the pollution content emission is to improve the sustainability of the internal manufacturing process. Minimizing the environmental impact scores of suppliers aims at a responsible choice in business partners. Maximizing the demand in the market is to consolidate the strategic competence in the market.

The processing network considered has been presented in Section 5.1 of [7]. We consider a time horizon with a 3 months period and the manufacturer needed to decide whether or not to sign contracts with two suppliers with different bulk discount contracts and two suppliers with different discount contracts. Differing from [7], where uncertainty due to future developments was incorporated as stochastic parameters, we consider the uncertainty in the production process. It results in perturbations of the amount of raw materials consumed, which affect the objectives of maximizing the profit and minimizing the pollution of the production process. The other two objectives do not involve uncertainty. All data used can be found in [7] and in Appendix A of this paper.

We solved this problem by applying our interactive approach with a real DM. For computing solutions, we used Gurobi® to solve the mixed integer quadratic problem after scalarization. We first calculated the ideal objective vector $\boldsymbol{z}^* = (28.25, 0, 0, 87.88)^T$. The nadir objective vector was approximated as $\boldsymbol{z}^{nad} = (0, 36, 12.31, 0)^T$. To get started, we set the ideal value of the robustness measure as 0 and the nadir value as 1.

We initialized the solution process by first introducing our robustness measure and visualization to the DM. The DM specified that the consumed raw materials in the product process can vary by 8% of their base values. Then we computed and presented an initial solution $\boldsymbol{z}^0 = (28.25, 36.0, 6.22, 53.64, 0.08)^T$. The solution is visualized with our visualization method as illustrated in Figure 10. In the figure, the objective function value of maximizing the profit under uncertainty can exceed its deterministic ideal value. The purpose of presenting the ideal and nadir values of the objective functions is to help the DM to get a general information on the ranges of the values of the base solution. Based on the initial solution, the DM wanted to decrease the environmental score of selected suppliers to 30 and increase the market demand of the main products to 75. At the same time, he also wanted to keep the pollution of the production process at its current value and allow the profit and the robustness of the solution to be impaired until 22 and 0.15 respectively.

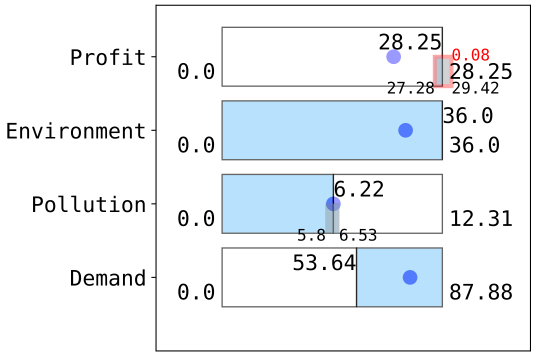

Figure 10: Intial solution

Based on his preferences, a new solution was calculated and presented to him. As the interactive solution process is described in Section 4.1 in details, we here omit the detailed description to avoid repetitions. Instead, we summarize the preferences of the DM and the objective function values of solutions computed in each iteration in Table 2.

Table 2: Iterations of the interactive solution process

Iteration	Solution	Preferences
Initial	$\boldsymbol{z^0} = (28.25, 36.0, 6.22, 53.64, 0.08)$	$I^{\geq} = \{1, 5\}$, $\epsilon = \{22, 0.15\}$, $I^{=}\{3\}$, $I^{\leq} = \{2, 4\}$, $\hat{z} = \{30, 75\}$
1	$\boldsymbol{z^1} = (22.12, 2.0, 6.22, 53.91, 0.073)$	$I^{<} = \{3, 5\}$, $I^{\geq} = \{1\}$, $\epsilon_1 = 20$
2	$\boldsymbol{z^2} = (20.0, 5.0, 3.04, 28.29, 0.05)$	$I^{<} = \{4\}$, $I^{\geq} = \{1, 2, 3, 5\}$, $\epsilon = \{20, 30, 9.5, 0.15, \}$
3	$\boldsymbol{z^3} = (20.0, 13.33, 9.50, 74.48, 0.12)$	$I^{\geq} = \{1, 2, 4, 5\}$, $\epsilon = \{20, 30, 70, 0.15\}$, $I^{\leq} = \{3\}$, $\hat{z}_3 = 8$
4	$\boldsymbol{z^4} = (20.0, 2.0, 8.71, 70.0, 0.10)$	$I^{\geq} = \{1, 2, 4, 5\}$, $\epsilon = \{20, 30, 60, 0.15\}$, $I^{\leq} = \{3\}$, $\hat{z}_3 = 7.5$
5	$\boldsymbol{z^5} = (18.23, 2.0, 7.11, 60.0, 0.095)$	-

After four iterations, the final solution was satisfactory. During the solution process, the DM observed that the first objective (maximizing the profit) was more sensitive to the uncertainty in the production process than the third objective (minimizing the pollution content emitted). The DM understood that this is a property of the problem. This problem is a mixed-integer optimization problem and only the raw materials consumed which are continuous decision variables involved uncertainty. With the help of the suitable solver, our approach was able to handle the preferences of the DM and find solutions accordingly. In addition, our approach helped the DM to understand the consequences of the involved uncertainty and thus, supported him to consider the objective function values and the robustness of solutions simultaneously.

5 Conclusions

In this paper, we focused supporting the DM to simultaneously consider the objective function values and robustness of solutions for multiobjective optimization problems with decision uncertainty. Based on the desired properties for a robustness measure to be used in an interactive approach, we introduced a new robustness measure that can deliver the meaning of robustness in an understandable way to the DM.

We proposed an interactive approach by utilizing elements of the synchronous NIMBUS method which is specifically suitable for solving problems with decision uncertainty. Because of the incorporation of robustness, we modified two components of the interactive NIMBUS solution process. We tailored a visualization method specifically for the new robustness measure and the associated robustness information by superimposing them on top of the bars representing the original objective functions. With this visualization, we can help the DM to consider the objective function values and the robustness of a solution at the same time. We also added a step to provide multiple alternatives for the DM to specify her/his preferences on the robustness of a more desired solution. Even though we built our approach based on NIMBUS, same idea and robustness measure can be applied to other classification based and reference point based methods. We demonstrated the advantages of the interactive approach by solving the river pollution problem and the problem in procurement contract selection with price optimization in a process network. Naturally, this approach can also be used to solve a wider range of problems.

Since we made the information on the r_i ranges of all objective functions available, we can further allow the DM to directly specify preferences on robustness for all or selected objectives in the future. As some of the objectives might be more important in considering robustness than others, we can incorporate the information about the importance also into our robustness measure. Also, as in [21], the obtained solution can be further analyzed to quantify how much worse the solutions are compared to the Pareto optimal solutions of the original problem.

Acknowledgments

We thank Dr. Dmitry Podkopaev for various discussions in constructing the multiobjective version of the procurement contract selection and pricing optimization problem.

A Data for the example problem in Section 4.2

The model of the problem solved is based on [7]. Originally, it has only one objective to maximize profit. We augmented the model with three additional objectives: minimizing the environmental factors, minimizing the emission of pollutant, and maximizing the market demand of the main product. The objective to maximize profit (f_1 in this paper) and the estimation on the demand (f_4 in this paper) as well as the related data can be found in [7]. The objective for responsible selection of suppliers has been inspired by [33]. Using the same notation as in [7], we have

$$f_2(x) = \frac{1}{T}\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{s=1}^{S} G^q_{s,t} y_{s,t}.$$

In the equation, $G^q_{s,t}$ represents the q-th environmental impact score of the supplier s in the planning period t and $y_{s,t}$ is the binary decision variable representing whether the supplier s is selected in the period t. The objective function is to take the average of the aggregation of all the environmental impact scores of all selected suppliers in each period.

For the consideration of pollutant emission, we consider the amount of sulphur dioxide emitted to the air based on the amount of sulphur content in the purchased raw materials. We have

$$f_3(x) = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{P}\sum_{c=1}^{M} E_{j,t} W^c_{j,t}.$$

The emission factor of the process plant j in th time period t is represented by $E_{j,t}$. The notation $W^c_{j,t}$ is the amount of raw material c consumed in the plant j in the period t. The emission factor of the plants in different time periods can be different due to variation of the heating material used. In f_2 and f_3, $G^q_{s,t}$ and $E_{j,t}$ are parameters.

The environmental impact score $G^1_{s,t}$ depends on percentage the supplier s has been paying attention to the environmental protection policies in the time period t:

$$G^1_{s,t} = \begin{cases} 1: & 100\% \\ 2: & \text{more than } 50\% \\ 3: & \text{less than } 50\% \\ 4: & \text{none.} \end{cases}$$

The score $G^2_{s,t}$ depends on the percentage of sustainability of the product of the supplier s in the time period t:

$$G^2_{s,t} = \begin{cases} 1: & 100\% \\ 2: & \text{more than } 50\% \\ 3: & \text{less than } 50\% \\ 4: & \text{none.} \end{cases}$$

The score $G^3_{s,t}$ depends on the percentage of green customers' market share of the supplier s in the time period t:

$$G^3_{s,t} = \begin{cases} 1: & \text{above } 80\% \\ 2: & 60\% \text{ to } 80\% \\ 3: & 40\% \text{ to } 60\% \\ 4: & 20\% \text{ to } 40\% \\ 5: & \text{less than } 20\%. \end{cases}$$

The score $G_{s,t}^4$ depends on the percentage of recycling product design of the supplier s in the time period t:

$$G_{s,t}^2 = \begin{cases} 1: & 100\% \\ 2: & \text{more than } 50\% \\ 3: & \text{less than } 50\% \\ 4: & \text{none.} \end{cases}$$

We used data presented in Table 3 for the emission factors of the processing plants. The scores of the four candidate suppliers in our problem setting are given in Tables 4 -7 and the settings of the contracts are given in Tables 8 and 9.

Table 3: Emission factors of processing plants

Plant	Time period		
	1	2	3
p_1	0.22	0.3	0.24
p_2	0.15	0.18	0.24
p_3	0.21	0.17	0.22

Table 4: Environmental impact scores of supplier 1 (discount contract)

Scores	Time period		
	1	2	3
G^1	3	2	1
G^2	3	2	1
G^3	2	1	2
G^4	1	4	2

Table 5: Environmental impact scores of supplier 2 (bulk discount contract)

Scores	Time period		
	1	2	3
G^1	2	3	2
G^2	3	2	1
G^3	3	1	1
G^4	1	1	2

Table 6: Environmental impact scores of supplier 3 (discount contract)

Scores	Time period		
	1	2	3
G^1	4	1	3
G^2	3	3	4
G^3	1	2	2
G^4	2	3	1

Table 7: Environmental impact scores of supplier 4 (bulk discount contract)

Scores	Time period		
	1	2	3
G^1	1	2	2
G^2	3	2	3
G^3	4	3	1
G^4	4	4	4

Table 8: Discount contract suppliers

Supplier	Price	Discount price	Threshold
S_1	3.15	2.47	20
S_3	3.15	2.58	20

Table 9: Bulk discount contract suppliers

Supplier	Price	Discount price	Threshold
S_2	3.06	2.38	40
S_4	2.95	2.55	40

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