Solving the maths problem in chemistry: the impact of a pre-university maths summer school

D E Shallcross*, N L Allan, K L Shallcross, S J Croker, D M Smith, P W May, and T G Harrison
Bristol ChemLabS
School of Chemistry
University of Bristol
Bristol
BS8 1TS

***d.e.shallcross@bris.ac.uk**

G J Price
Department of Chemistry
University of Bath
Bath
BA2 7AY

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Abstract
Mathematical skills beyond that taught at GCSE level (under 16 in UK) are required to pursue a physical sciences degree in the UK. However, many departments are unable to recruit sufficient students who have both the physical science and Mathematics qualification at A-level (post-16). Therefore, students are admitted with GCSE Mathematics and are taught the mathematical skills during the degree course. In this paper we investigate the impact of running a pre-university mathematics summer school for students about to start a physical sciences degree who have GCSE Mathematics as their highest mathematics qualification. The students are tracked through their first two years of a UK chemistry degree. It is shown that they perform significantly better than similarly qualified students in first year physical chemistry and second year theoretical chemistry units. Reasons for these results are presented.

Introduction
The importance of being equipped with mathematical skills such as calculus, currently not included in a GCSE qualification in mathematics in the UK, for a physical sciences degree is well known[1, 2]. Therefore, there is a strong desire for undergraduates reading degrees in subjects such as chemistry to have successfully studied an A-level (post-16) in mathematics. However, during the 1990s in the UK through to almost the present day, recruiting to a degree in chemistry was hard enough without requiring mathematics A-level. 'The number of students accepted to study for chemistry degrees has mainly decreased since 1994 both in absolute terms and as a proportion of the 18 year old population[3]. While for some institutions it has now become possible to require mathematics A-level for entrance and still maintain entry numbers, for the vast majority this is still impossible and given the introduction of higher fees in 2013 in the U.K. this may never be possible. Therefore, many institutions admit students who have GCSE mathematics (see Shallcross and Walton[2], for a description of the content and an interpretation of what various grades mean) and provide a range of courses to fill in the gaps. In this paper we investigate the effect of running a pre-university mathematics summer school for students about to start a degree in chemistry whose highest qualification in mathematics is a GCSE. Students who took the course were about to start a degree in chemistry at some 12 different universities. However, in this study, only the students who went to study at Bristol (11 out of 30) were tracked through the first two years of their degree and the impact on their results in all areas of chemistry were analysed relative to students with equivalent entry qualifications, who did not attend the summer school.

Details about the summer school
The summer school was run in the second week of September 2008, starting at 2 pm on the Monday and finishing at 1 pm on the Friday. The morning sessions ran from 9.30 am to 1 pm and the afternoon session ran from 2 pm to 5 pm with both sessions having a 30 minute break. All students applying to Bristol to read chemistry who did not have A-level mathematics were invited to attend (around 200 students) the summer school regardless of whether they were eventually coming to Bristol to read for a degree. There were 35 applicants leading to 30 attendees (5 dropped out before the summer school started) and these students came from all over England and Wales. There were no applicants from Scotland or Northern Ireland, although students from these areas were invited. Since the maximum number that could be accommodated was estimated to be 40 there was no need for any selection process or to split the summer school into two. It was known that many of the invitees were not intending to come to Bristol to read for a degree and there was no attempt to select out only Bristol bound students. Of these 30 attendees, 10 were female, 20 were male and of the 11 who were about to come to Bristol, 7 were male and 4 were female. The other 19 summer school attendees were about to start degrees in chemistry at 12 other UK universities. Through funding from the Royal Society of

Chemistry's CFOF project[4] it was possible to cover the cost of accommodation (at a reduced rate, including breakfast) for the week, all lunches, teas and coffees, bench fees and administrative support (~ £175 per student). The students just had to cover the cost of their own evening meals, transport to and from Bristol and had free evenings throughout. There were two types of session; the first was a workshop (4 of these), where a tutor would introduce a topic for no more than 20 minutes and then there would be problems to solve with four tutors (2 academics and 2 postgraduates) available to help students work through them. This would be followed by a short plenary where common mistakes were discussed. Then a new topic would be introduced and the workshop would continue. In all these sessions there was an emphasis on providing a relevant (here chemical) context to the mathematics introduced, something noted by several researchers as being a key to effective cognition of mathematical tools[5-13]. The second session type was a practical one (four of these), either in the teaching laboratories (three) or in a computer laboratory (one). These sessions were designed to allow students to apply basic mathematics used in the laboratory, e.g. yield and purity, moles calculations, logarithms, graph plotting and the exponential function (Beer-Lambert Law) and to collect data to be used in calculus sessions e.g. rates of reaction. It was also felt that 5 days spent in a seminar room working through mathematics problems may not be conducive to learning and so the practical sessions were an important part. When setting up the timetable we decided to have the first 25% of the time as mathematics workshops, the next 50% as practical applications of these tools and the final 25% as an introduction to calculus. The actual timetable was:

Timetable

Monday pm	Basic algebra, orders of magnitude, rearranging equations, applications to chemistry.
Tuesday am	Further algebra, indices, quadratic equations, functions (log, exp, trigonometry).
Tuesday pm	Basic statistics, error analysis with some applications.
Wednesday am	Practicals to emphasise error analysis and basic algebra.
Wednesday pm	Use of Excel in physical chemistry (simulating spectra, functions etc).
Thursday am	Practicals to support the idea of the exponential function (Beer-Lambert Law) and rates of reaction.
Thursday pm	Introduction to calculus (gradients of graphs and functions).
Friday am	Further calculus, differentiation and simple integration.

Worksheets from the course and practical scripts can be obtained from the authors on request. There was a short welcome and introduction to the course on the Monday and a short multiple-choice test using hand-held voting pads, providing instant feedback. This test was repeated at the end of the course.

Results of before and after summer school test

Part of the introduction and plenary of the course was taken up with running an interactive quiz, that was in part to determine what aspects of the course were successful and in part to determine what could be improved from an administrative viewpoint. The questions and their pre and post summer school responses are provided in table 1. In both cases 25 students took part out of the 30 attendees (5 had long journeys and were either arriving later or leaving early).

It is interesting to inspect table 1 and see that in many cases there was a perceived increase in confidence and ability after the summer school, particularly in rearranging equations, using indices and in using standard form. However, equally interesting was the mixed post-response in certain types of mole calculation (concentration and gas volume type) and calculating percentage errors. It emerged during the week, particularly in practical sessions that students thought they knew how to do these type of calculations, but realised that they did not. Most were able to overcome their misconceptions, but some were still struggling at the end of the week. Those that did master these techniques commented that they had not done many practicals where they had to do these type of calculations and that it was good to have had the practical sessions, which helped to reveal the deficiency and also give some context to the problem.

We also asked some mathematics questions at the start of the course, the pre-summer school scores were very low, averaging 24% for algebra and 0% for calculus (there is no calculus in GCSE specifications for mathematics). The same test was given again at the end and the scores rose sharply to 96% for algebra and 76% for calculus. These increases are reassuring but the long-term impact of the summer school was important to assess and are investigated in the next section.

Data collected for Bristol Students

Eleven students from the summer school went on to read for a degree at Bristol. All these students took an in-house run mathematics course in their first year, where they were joined by a further 29 students (who were invited to attend the summer school but declined) who also did not have an A-level in mathematics to make a class total of 40. The results of the first and second year exams in all subjects in chemistry at Bristol for these 40 students (11 attending the summer school and 29 who did not) were collected and analysed, inter-compared and compared also with the rest of the students in the cohort and are shown in table 2.

Table 1: Pre and post summer school responses to the same questions and some post summer school questions

Question	Agree Strongly	Agree	Neutral	Disagree	Disagree Strongly
I can rearrange mathematical equations easily	Pre: 3 Post: 12	4 12	14 1	4 0	0 0
I can do moles by mass calculations easily	Pre: 14 Post: 17	11 8	0 0	0 0	0 0
I can do moles by concentration calculations easily	Pre: 12 Post: 17	13 6	0 2	0 0	0 0
I can do moles by gas volume calculations easily	Pre: 5 Post: 6	14 11	4 4	2 4	0 0
I am confident with standard form number representation	Pre: 10 Post: 18	11 6	4 1	0 0	0 0
I am confident calculating percentage errors	Pre: 2 Post: 6	9 8	8 7	6 3	0 1
I am confident converting between units	Pre: 4 Post: 10	15 11	5 4	0 0	0 0
I can plot graphs and error bars	Pre: 5 Post: 8	6 9	6 6	6 2	2 0
I am confident using indices	Pre: 4 Post: 17	13 8	8 0	0 0	0 0
Post only					
Algebra useful Statistics useful Excel useful Practical useful Calculus useful Recommend Summer Sch.	20 7 8 15 21 24	5 11 10 8 4 1	0 4 6 2 0 0	0 3 1 0 0 0	0 0 0 0 0 0

Comments on data in table 2

First, it is striking that the cohort of students who attended the summer school did well across the board in the end of year 1 examinations. They were on a par in inorganic chemistry with the whole year and above average in Physical and Organic Chemistry. In all cases they performed better than those that did not attend the summer school. They also did better than the non summer school students in their own mathematics examination at the end of year 1. This latter result is striking as the percentage of A* and A grades at GCSE mathematics was higher in the non summer school cohort than the cohort that attended. There is no suggestion that the students' organic chemistry or even their inorganic chemistry mark benefitted from attendance at the summer school. Therefore, it is possible that the students who attended the summer school were more motivated and harder working than those that did not and would have gained higher marks anyway. It is interesting to note that on GCSE grade alone the attendees were weaker than the non-attendees and this may have been a contributory factor to their willingness to attend over those that did not.

Table 2: Mean examination results for (a) students without mathematics A level attending the summer school (b) students without maths A level who didn't attend the summer school (c) all students

Year 1	**Mathematics**	**Organic**	**Inorganic**	**Physical**
a) Summer School Attendees	60.6	69.3	63.3	69.8
b) Non Summer School Attendees	53.2	61.1	56.8	57.9
c) Year average		65.5	63.7	64.5
Year 2	**Theoretical**	**Organic**	**Inorganic**	**Physical**
a) Summer School Attendees	70.5	64.4	60.4	55.9
b) Non Summer School Attendees	58.6	67.6	60.2	55.3
c) Year average	67.2	66.5	61.5	57.5

We interviewed all the students who attended the summer school at the end of year 1, after their examinations and asked them to comment on the usefulness of attending the summer school. Here are some common themes that emerged from these discussions:

> *The summer school allowed us to make friends ahead of arriving at University and that helped to get us off to a good start.*
>
> *It was good to experience Halls of Residence ahead of time and to spend a week getting used to Bristol.*
>
> *It was very useful to go through the algebra at the start of the course and revise all the stuff we had learned at GCSE but had forgotten in the last two years. The mixture of academics and postgraduates was good and the relaxed style was good.*
>
> *All the algebra we covered was important in year 1 chemistry.*
>
> *We were worried about calculus and still have problems, but going through the basics and using chemical examples made it easier to understand.*
>
> *The laboratory sessions were fun, they broke up the week and looking back, it was a good way to reinforce the mathematics we were covering.*

There is a possibility that the summer school cohort was simply hard working and that with or without the summer school they would have done well. However, given the comments made in interview, it is clear that that week of refreshing the mathematics they knew was very important and useful. For some students it is more than two years since they studied mathematics and the first term at university can be very hard if you are trying to catch up. The introduction of calculus in the context of chemical examples, e.g. rates of reaction, first graphically then mathematically seemed to work well too.

Did this improvement persist into year 2? There was no statistical difference between the exam results of the students in the summer school and non summer school groups in year 2 in Inorganic and Physical Chemistry, with the non summer school cohort improving dramatically in organic chemistry and the summer school cohort appearing to drop down in performance (a fact that is beyond the scope of this paper). It should be noted that Physical Chemistry in year 2 at Bristol does contain mathematics, but also a fair amount of Physics. Several of the students who did not attend the summer school had taken A-level Physics, whereas none of those that attended the summer school had. However, the Theoretical Chemistry Unit is very mathematical and here the summer school attendees did exceptionally well compared with the non attendees and the rest of the Chemistry class. So had the summer school transformed these attendees into brilliant mathematicians? The simple answer is no. What the summer school did was to allow the students to hit the ground running and to take in and understand more of the mathematics they were presented with in their first year course relative to the ones who did not attend. While the latter were still trying to remember the basics, the former could concentrate on understanding new material. The first year mathematics course is an excellent primer for the theoretical course in year 2. However, previously no group from the non A-level mathematics cohort, taking this course has ever averaged a higher mark than the year average and so this result in year 2 was extremely noteworthy.

Reflections

Foster and Tall[14] reflect on the fact that less successful mathematics students will tend to cling to known procedures and have a rigid view of symbols, whereas successful students develop flexible ways of using them. Gray and Tall[15] and Saxe[16] argue that 'poor' mathematics students are simply doing a harder version of mathematics by not seeing the relationships and patterns. Boaler[5] and Lave[10] would argue that even 'successful students' sometimes cannot translate their mathematical knowledge to a new context, such as a chemical problem very easily. Skemp[17] suggests that much teaching in school mathematics is instrumental, i.e. students are shown procedures, which is easier to teach. Whereas, what is ultimately far better would be a relational approach to teaching, where students develop schema that allow them to be able to move from the starting point to the end point *via* numerous routes.

For some students it is more than two years since they studied mathematics and the first term at university can be very hard if you are trying to catch up.

Both the summer school and the first year mathematics in-house course were designed to develop a range of schema. In addition, all problems come out of a chemistry context. Students on this mathematics course often seem to find a new lease of life being taught mathematics (a subject they have generally found difficult or have avoided beyond GCSE) in the context of a subject they have generally excelled in, Chemistry. We have not converted these students into outstanding mathematicians but we have opened up to them representations in mathematics[18] that they can use more effectively than those they have learned in school. For example, a classic problem in algebra is the notion that the letters chosen are arbitrary[19] and the general ability to recognise underlying mathematics when presented in word form[20].

More data are needed without doubt to convince that a summer school can have an impact. However, there is enough evidence from the analysis of this project to suggest that it could be very effective. Not only as a refresher course, but also as a way to allow new students to get a head start and become familiar with their University setting ahead of time, even to make friends early. Such additional aspects were emphasised as being important in the end of year interviews. The latter aspect argues for a physical summer school compared with a virtual (on-line) or web-based course for students to follow pre-University, although there is evidence that these are also successful[21, 22]. However, successful web-based courses require a considerable investment of time in development[23, 24] to be appropriate and so any concept of saving time and resources by running an on-line course will only occur after some time compared with a face-to-face run course.

Run as a co-ordinated regional or national program, a series of mathematics pre-university summer schools around the country may have a considerable positive impact on Physical Sciences teaching in the U.K. Without further funding it has not been possible to run more summer schools beyond this pilot program, but is something that should be considered by HE funders.

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