



## Solving the Paradox of Multiple IRR's in Engineering Economic Problems by Choosing an Optimal $\alpha$ -cut

B. Moradi, H. Shakeri, S. NamdarZangeneh \*

B. Moradi, Alzahra University

H. Shakeri, Amirkabir University of Technology

S. NamdarZangeneh, Faculty member of Industrial Engineering Department, Alzahra University

### KEYWORDS

Internal Rate of Return,  
Uncertainty,  
Fuzzy numbers,  
Borrowing or Investing in  
a project

### ABSTRACT

Until now single values of IRR are traditionally used to estimate the time value of cash flows. Since uncertainty exists in estimating cost data, the resulting decision may not be reliable. The most commonly cited drawbacks to using the internal rate of return in evaluation of deterministic cash flow streams is the possibility of multiple conflicting internal rates of return. In this paper we present a fuzzy methodology for solving problems of multiple IRR in any type of streams. Utilization of fuzzy cash flow allows modeling of uncertainty in estimating cost data. The approach of  $\alpha$ -cut is to decrease the range of the final fuzzy set by increasing the degree of membership. For each fuzzy IRR in an optimum  $\alpha$ -cut, and an obtained present value of each stream, it is possible to decide on acceptance or rejection of a project according to the type of each stream (borrowing or investing). The upper bound of  $\alpha$ -cut is the worst case for borrowing and the lower bound of  $\alpha$ -cut is the worst case for investing. It is shown that both the internal rate of return and the present value are important in decision making and by analyzing the sensitivity of these values relative to the  $\alpha$ -cut variation, one can see the behavior of the project and choose a narrower fuzzy range.

© 2012 IUST Publication, IJIEPR, Vol. 23, No. 1, All Rights Reserved.

### 1. Introduction

In order to model insufficient information, fuzzy set concept can be employed to deal with the uncertainty in the cash flow analysis. In real life, decisions sometimes have to be made under the context of incomplete knowledge. It is very likely that decision makers give assessments based on their knowledge, experience, and subjective judgment when estimating cost data. If the cost data are not completely known, linguistic terms are frequently used to make the estimations. A linguistic variable is a variable whose values are not numbers but words or sentences in a

natural or artificial language [10]. Linguistic terms could be properly represented by the approximate reasoning of fuzzy set theory and produce fuzzy numbers by replacing the single-valued estimation for any vague cost data.

When the numbers and linguistic terms produce a data set, a fuzziness is required to show that mathematically. Depending on the degree of uncertainty, different degrees of fuzziness is used to express a variable. For that cause an appropriate membership function needs to be employed and whenever the given data is expressed in fuzzy numbers, the results will also be fuzzy.

One of the first problems that every economic analyst must solve is to decide whether the project is accepted or rejected based on the given cash flow. When the given data doesn't have any uncertainty, and sufficient

\* Corresponding author: Sodabeh Zangeneh

Email: [szangeneh@alzahra.ac.ir](mailto:szangeneh@alzahra.ac.ir)

Paper first received April 07, 2011, and in revised form Sep. 27, 2011.

information exists, the traditional approaches are capable of reaching the appropriate decisions.

In traditional cash flow analysis, present worth, annual worth, internal rate of return, and benefit-cost ratio are the most widely used approaches to perform the economic analysis. Unfortunately, decision makers rarely have enough information to perform the proper economic analysis. Moreover, the traditional economic analysis uses single-value estimates therefore a small change in the cost data may cause completely reversed decisions. And as a result, a good alternative may have been ignored due to a minor error in estimating the cost data.

In case of multiple rates of return, each rate has a meaningful interpretation with regard to the type of investment stream. It does not matter which rate is used for acceptance or rejection of the cash flow stream, as long as one identifies the underlying investment stream as a net investment or a net borrowing. When we say it does not matter which rate is used, we mean that regardless of which rate is chosen, the cash-flow acceptance or rejection decision will be the same, and consistent with net present value [2].

The discount rate often used in capital budgeting let the net present value of all cash flows equal to zero. When it is used appropriately, it can be a valuable aid in project acceptance and selection.

In some cases, several zero NPV discount rates may exist, and therefore there is no unique IRR. An IRR is unique if one or more years of net investments (negative cash flow) are followed by years of net revenues. But if the signs of the cash flows change more than once, there may be several IRRs. when several IRR's are found, no rational means for determination of the most appropriate IRR for economic desirability is found [9].

In order to solve the problem of inflexibility and incompleteness in using single-valued estimates for cost data, the fuzzy set concept can be employed to deal with the uncertainty in the cash flow analysis. To solve this problem we first present fuzzy methodology based on fuzzy cash flow and two kinds of streams in projects that lead to valid acceptance and rejection of the project. Then we propose the code of this methodology in MATLAB package.

Lorie, et al [11] showed that a unique IRR does not exist for some cash flow streams. The appeal of the IRR lies in its interpretation as a rate of return. A half-century has passed since the work of Lorie, et al [11] and Hirshlifer [4]. The notion described in Jensen, et al [12] remains unchanged.

The work of Brealey et al [13] remains one of the best graduate textbooks on corporate finance and the authors continue to argue for the superiority of NPV partly on the basis of deficiencies of IRR, now called pitfalls. However, multiple or nonexistent internal rates frustrate this interpretation. The consensus is that multiple internal rates constitute a severe drawback(White et al [14]) such as incorrect

conclusions (Canada et al [15] and Sullivan et al [16]), difficult to explain or interpret properly ([17]and [14]), invalid and not useful[1], meaningless ([18], [19], [20], [21] and [16]), inaccurate, ambiguous and contradictory [19].

When there are no real-valued internal rates of return, then the IRR approach must be abandoned. One must examine cash flow stream carefully to rule out the possibility of multiple or nonexistent rates ([23], [1], [22] and [21]).

Gordon [2] pointed out that even when there are multiple internal rates, their interpretation as rates of return can still be useful, valid, meaningful, correct, explainable, and non-contradictory.

Buckley et al [24] proposed the fuzzy extension of mathematics of finance. He develops the fuzzy extension of the elementary compound interest problems, as the future value, the present value and the internal rate of return (IRR) of a cash flow. Carlsson et al [6] considered the IRR decision rule in capital budgeting problems with fuzzy cash flows. Kuchta [6] proposed the fuzzy rate of return analysis and applications.

The point of the present paper is to develop a method of analyzing multiple uncertain IRR problems, with the interpretation of the fuzzy results and proposal of a more general criteria for deciding on acceptance and rejection of project. A fuzzy modelling is proposed for solve the problem of IRR based on the two kind of investment streams in mixed investments projects. This is demonstrated by analyzing an  $\alpha$ -cut of each set fuzzy data to determine the optimum  $\alpha$ -cut. We choose an optimal range of fuzziness for the given uncertain set of data. In the last section few examples are demonstrated to show the effectiveness of this method.

## 2. Economic Problems With Multiple IRR

It is very likely that decision makers give assessments based on their knowledge, experience, and subjective judgment when estimating cost data. If the cost data are not completely known, linguistic terms such as: around, approximately or about are used to describe the parameters of cash flow. Because of underlying uncertainty, parameters must be modeled as fuzzy numbers. For that some kind of fuzzy numbers such as triangular and trapezoidal are utilized.

Selecting any one of this membership functions needs a view of the amount of uncertainty which we want to take into account.

It is important to state that for more complex fuzzy numbers the procedure is the same as the one presented in this article and the triangular assumption is sufficient.

As expected a set of fuzzy input leads to a fuzzy output and the analyst must interpret the fuzzy results. The fuzzy parameters utilized in this research are the internal rate of return and the net present value.

### 3. Fuzzy Internal rate of Return

#### 3.1. The Internal Rate of Return

A cash flow stream is a finite or infinite sequence  $x = (x_0, x_1, \dots, x_T)$  of monetary values. The amount received initially is  $x_0$ , and the amount received after period  $t$  is  $x_t$ . For a finite stream  $x = (x_0, x_1, \dots, x_T)$ , we assume that the horizon,  $T$ , is chosen so that  $x_T \neq 0$ . The net present value,  $PV(x|r)$ , of a cash flow stream  $x$ , with an interest rate of  $r$  is given by:

$$PV(x|r) = \sum \frac{x_t}{(1+r)^t} \tag{1}$$

It is defined for proper interest rates  $r > -1$ . For a cash flow stream  $x$ , let  $IRR(x)$  be the set of all interest rates  $r$  which make  $PV(x|r) = 0$ . (Note that  $IRR(x)$  cannot contain  $1$  because  $PV(x|r) = -1$  is undefined.) For finite streams  $x = (x_0, x_1, \dots, x_T)$ , the present value function  $PV(x|r)$  is a  $T$ -degree polynomial in  $(1+r)^{-1}$ , so  $IRR(x)$  can contain anywhere from  $0$  to  $T$  distinct values. If  $r \in IRR(x)$ , then we will call  $r$  an internal rate of return for  $x$ .

As is well known, for conventional cash flows, the cash flow are negative for the first few periods and positive thereafter. In this cases the internal rates of return exist and this rates are unique, this can be depicted in Figure 1. It is known that if there is a reinvestment in a project, then its IRR may become ill-defined and IRR may have more than one solution. This can be depicted in Figure 2.

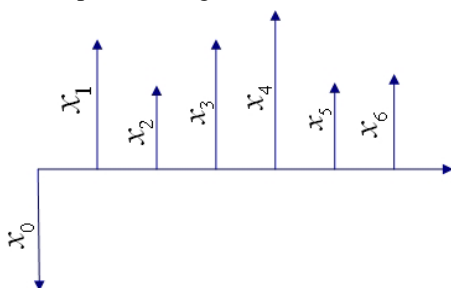


Fig. 1. Conventional cash flows cash flow

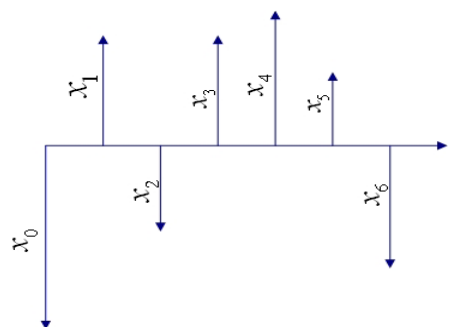


Fig. 2. Nonconventional cash flows

#### 3.2. Fuzzy Internal Rate of Return

When the initial data (cash flows) is fuzzy, the manipulation takes place on fuzzy numbers, and the output will also be fuzzy numbers too. In this section, fuzzy IRR will be introduced.

A triangular fuzzy number has a lower bound, an upper bound and of course a center bound which has the biggest membership share.

The membership function of fuzzy IRR's is the same as a fuzzy cash flow. So the output IRR has a lower bound, a centre bound and an upper bound. In fact, each bound needs a separate calculation.

##### 3.2.1. Applying an $\alpha$ -Cut

As mentioned in the previous sections, the fuzzy concept is employed to model uncertainty. It is important to keep in mind that enlarging the range of fuzzy numbers is not always a good idea. It may bring more uncertainty to the results. So, It is desirable to reduce the range on the cost data.

**Why  $\alpha$ -cut:** For this situation, decision makers can apply the  $\alpha$ -cut to reduce the range. Depending on different cases, decision makers can choose different values of  $\alpha$  (from  $0$  to  $1$ ) to reduce this range. If the analyst is very confident with each set of cost data, a higher value of  $\alpha$  may be assigned. Next implementation will be a function of  $\alpha$ -cut. For example the net present value  $PV(x|r)$  of a cash flow stream  $x$  with an interest rate of  $r$  and a specific  $\alpha$ -cut is given below:

$$PV(x|r, \alpha) = \sum \frac{x_t(\alpha)}{(1+r(\alpha))^t} \tag{2}$$

To find an appropriate  $\alpha$ -cut a thorough sensitivity analysis is required. This issue is discussed in section 5. Because the upper and lower bounds of the membership function are the worst case for any decision making problem, our calculation are focused on them.

##### 3.2.2. Complications of Several Fuzzy IRRs

Existence of multiple fuzzy IRRs depends on the type of fuzzy cash flow stream. The cash flow stream is classified as conventional and non-conventional investments. A conventional investment has only one sign change through the cash flow, and this guarantees a single IRR. In a non-conventional investment, signs are interspersed through the cash flow.

Non-conventional investment is classed as pure and mixed investment. This classification depends on the unrecovered investment balance stream. The unrecovered investment balance ( $F_t(k(a))$ ) can be positive, negative or zero at time  $t = 0$ . If it is negative then one can conclude that the firm has committed

$F_t(i)$  to the project (the firm has money invested in the project).

If the unrecovered balance stream is positive at time  $t$  then the firm has over recovered investment and is actually borrowing from the project during the project period  $t$  to  $t+1$  and we have a borrowing stream. Finally  $F_t(i) = 0$  means that the firm exactly recovered its invested funds.

Let  $F_t$  = unrecovered investment balance at  $t$  time, and  $N$  = project life, then we have:  $F_0 = x_0$  at  $t = 0$  and  $F_t$ :

$$\begin{aligned} F_0(\alpha) &= -x_0 \\ F_1(\alpha) &= (1+k(\alpha))F_0 + x_1(\alpha) \\ &\vdots \\ F_N(\alpha) &= F_T(\alpha) - (1+k(\alpha)) + x_T(\alpha) \end{aligned} \tag{3}$$

For the exact capital recovery,  $F_t$  must equal to zero [1]. By application of these equations for the fuzzy IRR computation, we let NPV equal to zero the calculations will be implemented on lower, upper and the center part separately.

$$\begin{aligned} NPV_c(x|K, \alpha) &= \sum_t \frac{x_{tc}}{(1+K_c)^t} = 0 \\ NPV_l(x|K, \alpha) &= \sum_t \frac{x_{tl}(\alpha)}{(1+K_l(\alpha))^t} = 0 \\ NPV_u(x|K, \alpha) &= \sum_t \frac{x_{tu}(\alpha)}{(1+K_u(\alpha))^t} = 0 \end{aligned} \tag{4}$$

$x_l(\alpha)$  and  $x_u(\alpha)$  can be derived straightforwardly. Its relationships are as follow:

$$\begin{aligned} x_l(\alpha) &= x_l(\alpha = 0) + (x_c - x_l(\alpha = 0))\alpha \\ x_u(\alpha) &= x_u(\alpha = 0) + (x_c + x_l(\alpha = 0))\alpha \end{aligned} \tag{5}$$

By solving equation 5, one can find the roots for any  $\alpha$ -cut and for each of the bounds[10].

**4. Investing and Borrowing from a Project**

As depicted in Figure 3, at time 0, the firm invests \$1000 invest on a project and therefore the unrecovered balance,  $F_t$ , for the firm equals to \$-1000, but the project has \$1000 as investment. In time 1 we compute  $F_1 = \$+4000$  which means in time 1 the firm has over recovery (the firm borrows

from the project), so the project loses \$4000 at time 1 and  $c_1 = \$-4000$ , therefore  $F_t = -C_t$ .

Then we can calculate investment stream (from the firm's view) as exactly equal to the negative of the unrecovered investment:  $F_t = -C_t$ .

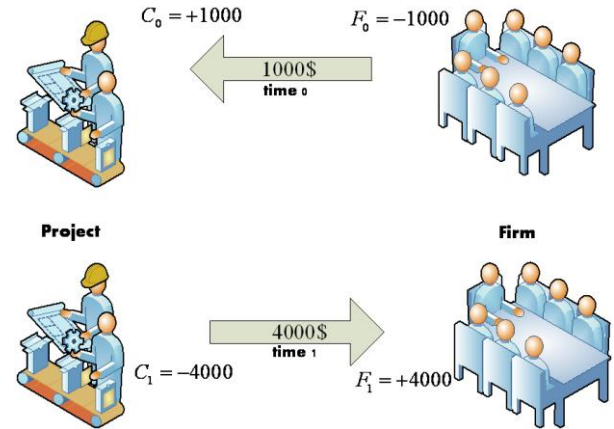


Fig. 3. Investing and borrowing from project

$$\begin{aligned} C_0(\alpha) &= -x_0 \\ C_1(\alpha) &= -((1+k(\alpha))x_0(\alpha) + x_1(\alpha)) \\ C_2(\alpha) &= -((1+k(\alpha))^2 x_0 + (1+k(\alpha))x_1(\alpha) + x_2(\alpha)) \\ &\vdots \\ C_{T-1}(\alpha) &= ((1+k(\alpha))^{N-1} x_0 + (1+k(\alpha))^{N-2} x_1(\alpha) + \\ &\quad + (1+k(\alpha))x_{T-2}(\alpha) + x_{T-1}(\alpha)) \\ C_T(\alpha) &= (1+k(\alpha))^{-1} x_N(\alpha) \end{aligned} \tag{6}$$

**4.1. Pure or Mixed Investment with Fuzzy Cash Flow**

A pure investment has a project investment balance calculated at the project's internal rate of return,  $k$ . The balances are either zero or negative throughout the projects life,  $N$ . Pure investment also has a single IRR despite the sign changes, and the firm doesn't borrow from the project. The firm recovers its investment  $F_T(k(\alpha) = 0)$  at the end of project life, earning interest at the IRR value,  $k$ , in the interim periods. In contrast a mixed investment can have several IRR's. Mixed investment can be defined as any investment that is not a pure one. So a mixed investment is a project for which  $F_t(k(\alpha)) > 0$  for some values of  $t$ , and  $F_t(k(\alpha)) \leq 0$  for all other values of  $t$ . A mixed investment contains unrecovered and over recovered investment balances (from stand point of the firm). A mixed investment stream contains both investing and borrowing streams (from stand point of the project). During the projects life, the firm is both an investor in the project and a borrower from the project [1].

For an investment stream  $C$ ,  $C_t < 0$ , signifies that the investor borrows an amount  $F = -C_t$  at time  $t$  from the project, and repays the loan at the rate of  $k(\alpha)$  on period hence. If  $C \geq 0$ , the firm has money invested in the project and we have an investment stream. According to [2] in either of the cash flows we have:

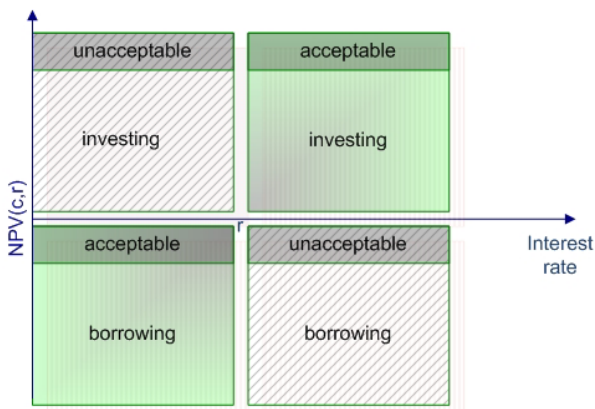
$$NPV(q, r, \alpha) = \frac{(k(\alpha) - r)}{(1 + r)} NPV(c, r, \alpha) \tag{7}$$

If  $x$  is a investment stream, then  $NPV(q|r) \geq 0$  if and only if  $k \geq r$ . Also If  $x$  is a pure borrowing stream, then  $NPV(q|r) \geq 0$  if and only if  $k \geq r$ .

Investment streams,  $c$ , with both positive and negative components are known as mixed investments. Accordingly, we distinguish two kinds of mixed investment streams.

**4.1.1. Decision Making**

If the present value of the investment  $PV(c|r)$  is positive at the market rate of interest,  $r$ , then investor is investing funds in the project in question, and it is called a net investment. For net investments, the investor anticipates returns of  $k$  on  $c$  exceed the market rate of  $r$ . However, if  $PV(c|r)$  is negative, then the investor is borrowing funds from the project, it is called net borrowing. An investor would anticipate a net borrowing as long as the repayment rate  $k$  for the loan is less than the market rate  $r$ . Graphically, this can be depicted as shown in Figure 4.



**Fig. 4. Decision Making based on derived IRR and  $PV(x|r)$**

**5. Choosing an Optimum  $\alpha$ -Cut and Sensitivity Analysis**

In the analysis of fuzzy cash flow, the interval of the final fuzzy set serves as the basis for the final decision making. If this interval is reasonably small, the preferred alternative can be selected with more

distinction. On the other hand, if the final fuzzy set covers a wide range after the analysis, the final decision will be difficult to reach. For selecting an optimal interval, an optimal  $\alpha$ -cut needs to be selected. So it is necessary to exert a sensitivity analysis on this problem. The purpose of the sensitivity analysis is to assess the effects of the variations of the cost data on the final fuzzy set. Based on the sensitivity analysis results, decision makers can pay more attention to define the fuzzy set carefully to reduce the final range of the fuzzy set.

As stated before, one common method to reduce the range of a fuzzy set is called the  $\alpha$ -cut, where  $\alpha$  is the degree of membership. By properly assigning a value to  $\alpha$ , the range of the fuzzy set can be reduced. This research will explore the feasibility of using  $\alpha$ -cut to reduce the range of the fuzzy numbers.

A numerical example will be applied to illustrate the concepts of sensitivity analysis and  $\alpha$ -cut. One must notice that the sensitivity analysis is unique to every single economic problem. The result of the sensitivity analysis can only provide an indication as which cost data is more sensitive in general.

**5.1. Implication of the Sensitivity Analysis**

By comparing the PV or IRR with the most certain ( $\alpha = 1$ ) PV or IRR, the percentage change can be served as an indicator of sensitivity for the  $\alpha$ -cut. The  $\alpha$ -cut which caused the most percentage change, is defined as the most sensitive one. By Comparison among derived data, and their deviations, one can choose an optimal  $\alpha$ -cut and proceed with that.

**6. Results and Discussion**

Some classic examples are considered in this section. Table 1 shows the examples.

**Tab. 1. Examples of Projects with Uncertain Cash Flow**

Example	year	Lower Bound	Center Bound	Upper Bound
example 1 (market rate of interest=20%)	0	-3000	-3000	-3000
	1	13500	15000	16000
	2	-13500	-15000	-16000
example 2 (market rate of interest=10%)	0	-1000	-1000	-1000
	1	5500	6000	6500
	2	-10500	-11000	-11500
example 3 (market rate of interes=10%)	0	-1000	-1000	-1000
	1	3800	4000	4500
	2	-3800	-4500	-4500



For all of the examples, the procedure proposed in the former sections are applied. First internal rates of return will be computed for each of the bounds then for each roots of the IRR, Present Value and other assesment are derived.

**Example 1**

In this example, IRR roots for  $\alpha = 1$  are 38.20% and 261.8% . The data for  $\alpha = 0$  (the biggest interval of approximation) are given in table 2.

In this case, the cash flow has multiple IRR's, and for each of them, a separate investment is obtained.

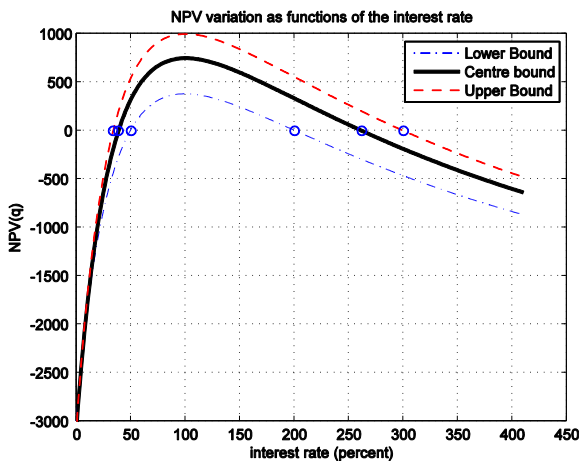


Fig. 5. Present values of bounds as functions of the interest rate (example 1)

By checking the signs of each present value in table 2, we realize that if the internal rate is lower or higher than the market rate, for each corresponding investment stream c, decision can be made. In this example, all of the IRR roots are exceeding the market rate and by checking the sign of the present value for each, one can see that the corresponding investment streams are net borrowings from the project. Since the repayment rates k exceed the market rate, the project is undesirable. So, regardless of which internal rate we use, the conclusion is the same and consistent with the present value evaluation at the market rate: "Project is undesirable".

Tab. 2. IRR roots and calculated  $PV(c|r)$  for  $\alpha$ -cut=0 (example 1)

	Lower Bound		Center Bound		Upper Bound	
	IRR	PV (c r)	IRR	PV (c r)	IRR	PV (c r)
Root1	50%	-4500	38.2%	-6045.1	33%	-7000
Root2	200%	-750	261.8%	-454.91	300%	-333

The analysis of  $\alpha$ -cut and the sensitivity analysis are shown in figure 3. By assessing figure 3, the best option for the value of  $\alpha$  will be an average one (because of the inverse sensitivity effect on IRR and present value). Based on the importance of both IRR and PV in decision making, it is important to give credit to the behavior of these parameters.

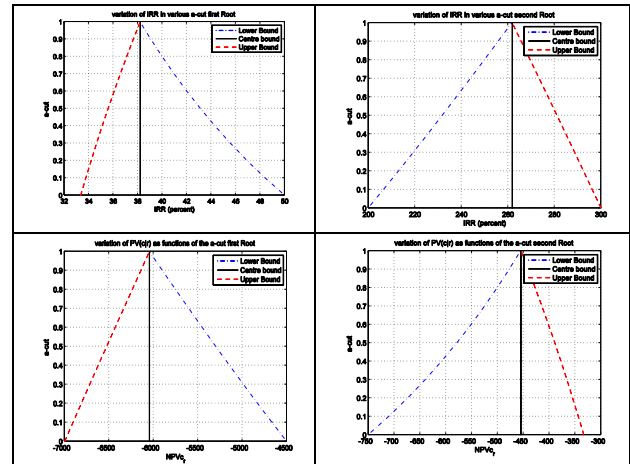


Fig. 3. IRR and Present Value variation for each Root (example 1)

In example one, because of the smooth behavior of Present Value one can choose a higher  $\alpha$  . If decision makers have more confidence with the fuzzy set of an alternative, a higher value can be assigned to the final fuzzy set. In order to minimize the disturbance caused by the fuzzy range increments, decision makers should focus on the most sensitive cost data. Because division and power factor are involved in the fuzzy number operation rules, the percentage differences are changing faster in increasing direction rather than in decreasing direction for the results.

**Example 2**

This example is an uncertain version of a classical problem that was discussed in [4] and later in [2] thoroughly. In this paper the effect of uncertainty on this example is illustrated. As it is depicted in figure 6, the ordinary problem without any uncertainty (center bound in the figure) has three internal rates of return 0% , 100% , 200% .

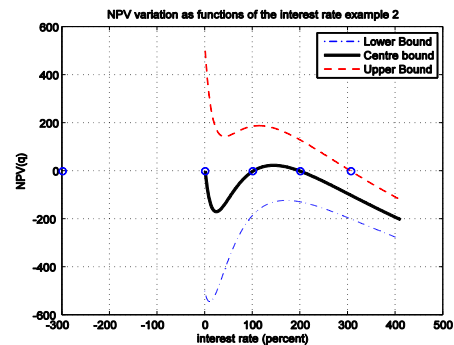


Fig. 6. Present values of bounds as functions of the interest rate (example 2)

For the uncertain case the data are given in table 1. In figure 6, the lower bound, the center bound, the upper bound and the present value of the cash flow stream are shown. As It is shown, the upper bound stream has one internal rate of return and the lower bound stream has no roots. In this example, IRR roots and Present values for  $\alpha$ -cut=0 (the biggest interval of approximation) are given in table 4.

**Tab. 4. IRR roots and calculated PV(c|r) for  $\alpha$ -cut=0 (example 2)**

	Lower Bound		Center Bound		Upper Bound	
	IRR	PV (c r)	IRR	PV (c r)	IRR	PV (c r)
Root1	:	:	0%	1413.2	:	:
Root2	:	:	100%	-157.02	:	:
Root3	:	:	200%	-74.38	3.6.37%	107

By checking the sign of its present value (1413.2 > 0), the corresponding investment stream is indeed a net investment. However, since  $k=0$  falls short of the market rate, the project is undesirable.

On the other hand, at internal rates 100% or 200% , the corresponding investment streams are net borrowings from the project, having negative present values causing both ones to be undesirable projects.

The most interesting part of this example is the different behaviour of each bound. Referring to table 4, the nearest roots of upper bound IRR and Center bound IRR, are 200% from center and 306.37% from upper bound but the sign of the Present Value is different and the type of investing is different.

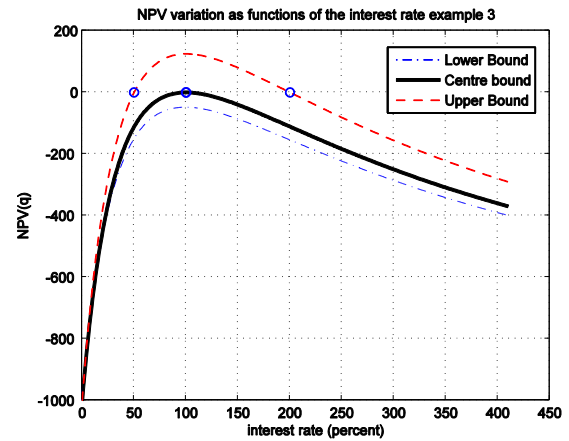
For the center bound, it is a borrowing stream and the project is rejected, but for the other bound, the stream is investing and it is desirable. So it is not always safe to approximate an uncertain fuzzy number to a crisp number.

**Example 3**

This example is like the former one and is an uncertain version of a classical problem that was discussed in [5] and [2]. The effect of uncertainty will be discussed here. As in figure 7, the ordinary problem without any uncertainty (center bound in the figure) has one internal rates of return  $k = 1000\%$  .

For the uncertain case, as it was in figure 7, one can see the lower bound, the center bound and the upper bound Present value of cash flow stream. It is depicted that the upper bound stream has two internal rate of return and the lower bound stream has not any roots. In this

example IRR roots and Present values for  $\alpha$ -cut = 0 or the biggest interval of approximation are presented in the table 5.



**Fig. 7. Present values of bounds as functions of the interest rate (example 3)**

In this case all of the roots exceed the market rate, and the present value is negative for all of the IRR's. But caution is needed in this case, because the first root of upper bound IRR is close to the market rate and one can encounter a problem if this root falls

**Tab. 5. IRR roots and calculated PV(c|r) for  $\alpha$ -cut=0 example 3**

	Lower Bound		Center Bound		Upper Bound	
	IRR	PV (c r)	IRR	PV (c r)	IRR	PV (c r)
Root1	:	:	100%	-818.181	50%	-1727
Root2	:	:	100%	-818.181	200%	-363.6

**7. Conclusion**

By selecting a simple fuzzy number to model an uncertain number, a global approach for uncertain economic problems has been proposed. In this approach by data analysis and assessing their sensitivity to  $\alpha$ -cut, an optimum  $\alpha$  can be selected. With this approach, decision makers can have more space to define each cost data and by reducing the range of fuzzy number, they can obtain more information from the final result.

Both Internal rate of return and Present value are important in decision making and by analysing sensitivity of these two relative to the  $\alpha$ -cut variation, one can see the behavior of the project. For comparison of the competing projects, this study may be helpful and the amount of uncertainty has an obvious effect on the comparison. Being aware of the type of investment

(borrowing or investing) is necessary and it is important to compare two streams with the same investment type (borrowing/investing).

For modeling linguistic estimations with more detail, one can use a more complex membership function like trapezoidal membership functions.

### References

- [1] Bussey, *The Economic Analysis of Industrial Projects*, Prentice Hall, Englewood Cliffs, New Jersey, 1992.
- [2] Hazen, G.B., *A New Perspective on Multiple Internal Rates of Return*, *The Engineering Economist*, volume 48, number 1, 2003.
- [3] Dorota Kuchta: *Fuzzy Rate of Return Analysis and Application*, *Fuzzy Engineering Economics with Applications*: 97-104, 2008.
- [4] Hirshleifer, J., *On the Theory of Optimal Investment Decisions*, *Journal of Political Economy*, Vol. 66, 1958, pp. 329-352.
- [5] Herbst, A., *The Unique, Real Internal Rate of Return: Caveat Emptor!*, *Journal of Financial and Quantitative Analysis*, Vol. 13, No. 2, 1978, 1970, pp. 363.
- [6] Carlsson Christer, Fuller Robert: *On Fuzzy Internal Rate of Return*, *Turku Centre for Computer Science, TUCS technical report No. 211. Finlandia*, 2000.
- [7] Buckley J.J., *Solving Fuzzy Equations*, *Fuzzy sets and systems*, 50, 1-14. 1992.
- [8] Buckley J.J., Qu, Y., *Solving Linear and Quadratic Fuzzy Equations*, *Fuzzy sets and systems* 38, 43-59, 1990.
- [9] Thuesen, G.J., Fabrycky, W.J., *Engineering Economy*, Prentice-Hall, Englewood Cliffs, New Jersey., 1989
- [10] Zadeh, L.A., *The Concept of a Linguistic Variable and Its Application to Approximate Reasoning*, Part 1, 2, 3. *Information Science*, Vol. 8, 1975, P. 199-249; Vol. 9, 1976, pp. 43- 58.
- [11] Lorie, J.H., Savage, L.J., *Three Problems in Rationing Capital*, *The Journal of Business*, Vol. 28, No. 4, 1955, pp. 228-239.
- [12] Jensen, M., Smith, C., *The Theory of Corporate Finance: A Historical Overview*, *The Modern Theory of Corporate Finance*, McGraw-Hill Inc., New York, pp. 2-20 Available at SSRN: <http://ssrn.com/abstract=24416>, 1984.
- [13] Brealey, R., Myers, S., Allen, F., *Principles of Corporate Finance (9th ed.)*, McGraw-Hill, 2009.
- [14] White, J.A., Case, K.E., Pratt, D.B., Agee, M.H., *Principles of Engineering Economic Analysis*, Wiley, New York, 1998.
- [15] Canada, J.R., Sullivan, W.G., White, J.A., *Capital Investment Analysis for Engineering and Management*, Prentice-Hall, Upper Saddle River, New Jersey, 1996.
- [16] Sullivan, W.G., Bontadelli, J.A., Elin M., Wicks, E.P., DeGarmo : *Engineering Economy*, Prentice-Hall, Upper Saddle River, New Jersey, 2000.
- [17] Blank, L.T., Tarquin, A.J., *Engineering Economy*, Engineering Economy, McGraw-Hill, New York, 1989.
- [18] Stevens, G.T., Jr. *Economic and Financial Analysis of Capital Investments*, Wiley, New York, 1979.
- [19] Steiner, H. M.M., *Engineering Economic Principles*, McGraw-Hill, New York, 1992.
- [20] Young D., *Modern Engineering Economy*, Wiley, New York, 1993.
- [21] Eschenbach, T.G., *Engineering Economy: Applying Theory to Practice*, Irwin, Chicago, 1995.
- [22] Fleischer, G.A. *Introduction to Engineering Economy*, PWS Publishing, Boston, 1994.
- [23] Teichrow, D., Robichek, A.A., Montalbano, M., *An Analysis of Criteria for Investment and Financing Decisions under Certainty*, *Management Science* 12, 1965, 151-179.
- [24] James, J. Buckley, Douglas Tucker: *Extended Fuzzy Relations: Application to Fuzzy Expert Systems*, *International Journal of Approximate Reasoning* 1(2): 1987, 177-195.