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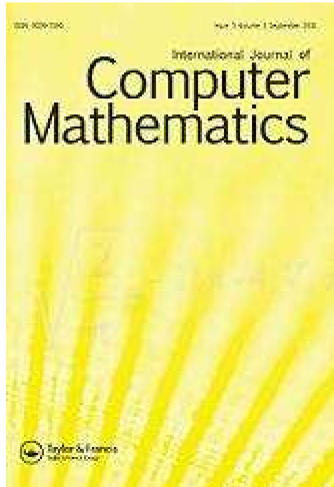

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Solving third and fourth order partial differential equations using GFDM. Application to solve problems of plates-cmmse10.

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Abstract

This paper describes the generalized finite difference method to solve second-order partial differential equation systems and fourth-order partial differential equations. This method is applied to solve problem of thin and thick elastic plates.

Key words: meshless methods, generalized finite difference method, moving least squares, plates, instructions
MSC 2000: 65M06, 65M12, 74S20, 80M20

1 Introduction

The Generalized finite difference method (GFDM) is evolved from classical finite difference method (FDM), also called meshless finite difference method. The bases of the GFD were published in the early seventies. [10] was the first to introduce fully arbitrary mesh. He considered Taylor's series expansions interpolated on six-node stars in order to derive the finite difference (FD) formulae approximating derivatives of up to the second order. [12] suggested that additional nodes in the six-point scheme should be considered and an averaging process for the generalization of finite difference coefficients applied. The idea of using an eight node star and weighting functions to obtain finite difference formulae for irregular meshes, was first put forward by [11] using moving least squares (MLS) interpolation and an advanced version of the GFDM was given by [2]. [4] reported that the solution of the generalized finite difference method depends

on the number of nodes in the cloud, the relative coordinates of the nodes with respect to the star node, and on the weight function employed.
An h-adaptive method in GFDM is described in [1, 5, 7, 8]. [9] reported improvements of GFDM and comparison with other meshless method.
The papers [6, 13] shows the application of the GFDM in solving parabolic and hyperbolic equations and advection-diffusion equation.
This paper describes how the GFDM can be applied to solve second-order partial differential equation systems and fourth-order partial differential equations. This method is applied to solve problem of thin and thick elastic plates.
The paper is organized as follows. Section 1 is an introduction. Section 2 describes the GFDM obtaining the explicit formulae. Section 3 describes the application of GFDM to Plates. In Section 4 some numerical results and the comparison with other methods are included . Finally, in Section 5 some conclusions are given.

2 The generalized finite difference method

Let us to consider a problem governed by

$$\alpha_1\frac{\partial U}{\partial x}+\alpha_2\frac{\partial U}{\partial y}+\alpha_3\frac{\partial^2 U}{\partial x^2}+\alpha_4\frac{\partial^2 U}{\partial y^2}+\alpha_5\frac{\partial^2 U}{\partial x\partial y}+\alpha_6\frac{\partial^3 U}{\partial x^3}+\alpha_7\frac{\partial^3 U}{\partial x^2\partial y}+ \\ \alpha_8\frac{\partial^3 U}{\partial x\partial y^2}+\alpha_9\frac{\partial^3 U}{\partial y^3}+\alpha_{10}\frac{\partial^4 U}{\partial x^4}+\alpha_{11}\frac{\partial^4 U}{\partial x^3\partial y}+\alpha_{12}\frac{\partial^4 U}{\partial x^2\partial y^2}+ \\ \alpha_{13}\frac{\partial^4 U}{\partial x\partial y^3}+\alpha_{14}\frac{\partial^4 U}{\partial y^4}=f(x,y) \quad in \quad \Omega \quad (1)$$

with boundary condition

$$\beta\frac{\partial U}{\partial n}+\gamma U=g(x,y) \quad in \quad \Gamma \quad (2)$$

where $\Omega \subset R^2$ with boundary Γ; $\alpha_i, (i=1,\cdots,14)$, β and γ are constant coefficients; and f, g are two known smoothed functions.
The intention is to obtain explicit linear expressions for the approximation of partial derivatives in the points of the domain.
First of all, an irregular cloud of points is generated in the domain. On defining the composition central node with a set of N points surrounding it (henceforth referred as nodes), the star then refers to the group of established nodes in relation to a central node. Each node in the domain have an associated star assigned [1, 2, 4, 8, 11].
If u_0 is an approximation of fourth-order for the value of the function at the central node (U_0) of the star, with coordinates (x_0, y_0) and u_j is an approximation of fourth-order for the value of the function at the rest of nodes, of coordinates (x_j, y_j) with

$j = 1, \cdots, N$, then, according to the Taylor series expansion

$$U_j = U_0 + h_j \frac{\partial U_0}{\partial x} + k_j \frac{\partial U_0}{\partial y} + \frac{h_j^2}{2} \frac{\partial^2 U_0}{\partial x^2} + \frac{k_j^2}{2} \frac{\partial^2 U_0}{\partial y^2} + h_j k_j \frac{\partial^2 U_0}{\partial x \partial y} + \\ + \frac{h_j^3}{6} \frac{\partial^3 U_0}{\partial x^3} + \frac{k_j^3}{6} \frac{\partial^3 U_0}{\partial y^3} + \frac{h_j^2 k_j}{2} \frac{\partial^3 U_0}{\partial x^2 \partial y} + \frac{h_j k_j^2}{2} \frac{\partial^3 U_0}{\partial x \partial y^2} + \frac{h_j^4}{24} \frac{\partial^4 U_0}{\partial x^4} + \frac{k_j^4}{24} \frac{\partial^4 U_0}{\partial y^4} + \\ + \frac{h_j^3 k_j}{6} \frac{\partial^4 U_0}{\partial x^3 \partial y} + \frac{h_j^2 k_j^2}{4} \frac{\partial^4 U_0}{\partial x^2 \partial y^2} + \frac{h_j k_j^3}{6} \frac{\partial^4 U_0}{\partial x \partial y^3} + \cdots \quad (3)$$

where $h_j = x_j - x_0; k_j = y_j - y_0$.
If in equation 3 the terms over fourth order are ignored. It is then possible to define the function

$$B(u) = \sum_{j=1}^{N} [(u_0 - u_j + h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} + \frac{h_j^2}{2} \frac{\partial^2 u_0}{\partial x^2} + \frac{k_j^2}{2} \frac{\partial^2 u_0}{\partial y^2} + h_j k_j \frac{\partial^2 u_0}{\partial x \partial y} + \\ + \frac{h_j^3}{6} \frac{\partial^3 u_0}{\partial x^3} + \frac{k_j^3}{6} \frac{\partial^3 u_0}{\partial y^3} + \frac{h_j^2 k_j}{2} \frac{\partial^3 u_0}{\partial x^2 \partial y} + \frac{h_j k_j^2}{2} \frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{h_j^4}{24} \frac{\partial^4 u_0}{\partial x^4} + \frac{k_j^4}{24} \frac{\partial^4 u_0}{\partial y^4} + \\ + \frac{h_j^3 k_j}{6} \frac{\partial^4 u_0}{\partial x^3 \partial y} + \frac{h_j^2 k_j^2}{4} \frac{\partial^4 u_0}{\partial x^2 \partial y^2} + \frac{h_j k_j^3}{6} \frac{\partial^4 u_0}{\partial x \partial y^3}) w(h_j, k_j)]^2 \quad (4)$$

where $w(h_j, k_j)$ is the denominated weighting function.
If the norm 4 is minimized with respect to the partial derivatives the linear equation system is obtained

$$\boldsymbol{A D_u} = \boldsymbol{b} \quad (5)$$

where

$$\boldsymbol{A} = \begin{pmatrix} \sum_{j=1}^{N} h_j^2 w^2 & \sum_{j=1}^{N} h_j k_j w^2 & \sum_{j=1}^{N} \frac{h_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_j k_j^2}{2} w^2 & \sum_{j=1}^{N} h_j^2 k_j w^2 & \sum_{j=1}^{N} \frac{h_j^4}{6} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^3 k_j^2}{4} w^2 \\ & \sum_{j=1}^{N} k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^2 k_j}{2} w^2 & \sum_{j=1}^{N} \frac{k_j^3}{2} w^2 & \sum_{j=1}^{N} h_j k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j}{6} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^2 k_j^3}{4} w^2 \\ & & \sum_{j=1}^{N} \frac{h_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^2 k_j^2}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j}{2} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{12} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^4 k_j^2}{8} w^2 \\ & & & \sum_{j=1}^{N} \frac{k_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j k_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j^2}{12} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^2 k_j^4}{8} w^2 \\ & & & & \sum_{j=1}^{N} h_j^2 k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^4 k_j}{6} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^3 k_j^3}{4} w^2 \\ & & & & & \sum_{j=1}^{N} \frac{h_j^6}{36} w^2 & \dots & \sum_{j=1}^{N} \frac{h_j^5 k_j^2}{24} w^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & S & & Y & & M & \dots & \sum_{j=1}^{N} \frac{h_j^4 k_j^4}{16} w^2 \end{pmatrix} \quad (6)$$

and

$$\boldsymbol{D_u} = \left\{ \frac{\partial u_0}{\partial x} \quad \frac{\partial u_0}{\partial y} \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^2 u_0}{\partial y^2} \quad \frac{\partial^2 u_0}{\partial x \partial y} \quad \frac{\partial^3 u_0}{\partial x^3} \quad \dots \quad \frac{\partial^4 u_0}{\partial x \partial y^3} \quad \frac{\partial^4 u_0}{\partial x^2 \partial y^2} \right\}^T \quad (7)$$

$$\boldsymbol{b} = \left\{ \begin{array}{c} \sum_{j=1}^{N}(-u_0+u_j)h_j w^2 \\ \sum_{j=1}^{N}(-u_0+u_j)k_j w^2 \\ \sum_{j=1}^{N}(-u_0+u_j)\frac{h_j^2}{2} w^2 \\ \sum_{j=1}^{N}(-u_0+u_j)\frac{k_j^2}{2} w^2 \\ \sum_{j=1}^{N}(-u_0+u_j)h_j k_j w^2 \\ \sum_{j=1}^{N}(-u_0+u_j)\frac{h_j^3}{6} w^2 \\ \vdots \\ \sum_{j=1}^{N}(-u_0+u_j)\frac{h_j^2 k_j^2}{4} w^2 \end{array} \right\} \tag{8}$$

and solving system 5 the explicit difference formulae are obtained as in [5, 8].
On including the explicit expressions for the values of the partial derivatives in 1 the star equation is obtained as

$$-m_0 u_0 + \sum_{j=1}^{N} m_j u_i = f(x_0, y_0) \tag{9}$$

with

$$m_0 = \sum_{j=1}^{N} m_j \tag{10}$$

The coefficients $m_0 = m_0(N, h_j, k_j, w)$ and $m_j = m_j(N, h_j, k_j, w)$ of the star equation 9 depends of the following factors ([1, 4, 8]):

- The number of nodes of the star, N. Working in 2D with fourth order partial differential equations, the minimum number of nodes of the star is 14, then the system of equations 5 (where $\boldsymbol{A}$ is symmetrical matrix) can be solved by using the Cholesky method.
  The results improve as the number of nodes in the star increases [4].
- Selection of the nodes of the star. Similarly to the results reported in [4, 8]. The results improve when using the four quadrants criterion for the selection of nodes of the star.
- The weighting function w. Similarly to the results reported in [4, 8] the potential function $w = \frac{1}{(dist)^3}$ has been used, where *dist* is the distance between the central node and the considered node in the star.

If this process is carried out for each node of the domain a linear equations system is obtained, where the unknowns are the values u_i. On solving this system of equations, the approximated values of the function in the nodes of the domain are obtained and the partial derivatives may easily be calculated using 5.

3 Application of GFDM to Plates

3.1 Thin Elastic Plates

The partial differential equation, frequently called Lagrange's equation, which relates the rectangular coordinates, the load, the deflections, and the physical and elastic constants of a laterally loaded plate, is well known. Its application to the solution of problems of bending of plates is justified if the following assumptions or hypotheses are met: **a**) the plate is composed of material which may be assumed to be homogeneous, isotropic, and elastic, **b**) the plate is of a uniform thickness which is small as compared with its lateral dimensions, **c**) the deflections of the loaded plate are small as compared with its thickness. The additional differential expressions relating the deflections to the boundary conditions, moments, and shears are equally well known [3]:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{q(x,y)}{D}; \quad D = \frac{Et^3}{12(1-\nu^2)} \tag{11}$$

where $w(x, y)$ is deflection function in each point of the plate, $q(x, y)$ is intensity of pressure in each point, normal to the plane of the plate, ν is Poisson's ratio for the material of the plate, E is Young's modulus for the material of the plate and t is the thichness of plate.

3.2 Thick Elastic Plates

The partial differential equations are:

$$\begin{cases} \dfrac{t^3}{12}\boldsymbol{H^T C_f H \theta} + t\boldsymbol{C_c}(\boldsymbol{\nabla} w - \boldsymbol{\theta}) = \boldsymbol{0} \\ -\boldsymbol{\nabla^T}(t\boldsymbol{C_c})\boldsymbol{\theta} + \boldsymbol{\nabla^T}(t\boldsymbol{C_c})\boldsymbol{\nabla} w = -q \end{cases} \tag{12}$$

where

$$\boldsymbol{H} = \begin{pmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} \end{pmatrix}; \quad \boldsymbol{C_f} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{pmatrix}$$

$$\boldsymbol{C_c} = \frac{\alpha E}{2(1+\nu)}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \boldsymbol{\theta} = \begin{pmatrix} \theta_x \\ \theta_y \end{pmatrix}$$

$w = w(x, y)$ is deflection function, θ_y and $-\theta_x$ are the rotations of the cross sectional plane about the x- and y-axes, respectively.

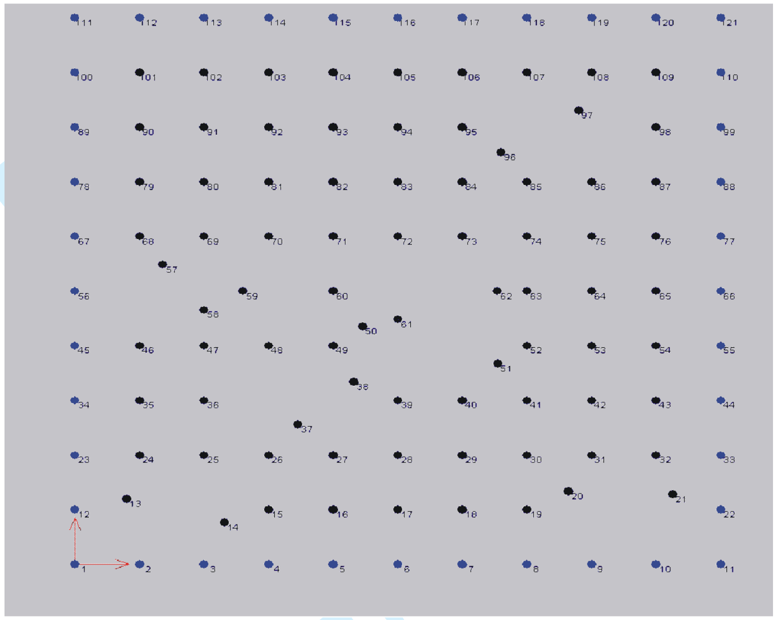

Figure 1: Irregular cloud of nodes (121 nodes)

4 Numerical Results

4.1 Academic Examples

This section provides some of the numerical results when solving partial differential equations in a square domain of unit side, with Dirichlet boundary conditions, using star of 24 nodes, the four quadrants criterion for the selection of nodes of the star and the weighting function

$$\Omega(h_j, k_j) = \frac{1}{(\sqrt{h_j^2 + k_j^2})^3} \tag{13}$$

The global exact error can be calculated as

$$Global \quad exact \quad error = \frac{\sqrt{\frac{\sum_{i=1}^{N} e_i^2}{N}}}{exac_{max}} \tag{14}$$

where N is the number of nodes in the domain, $exac_{max}$ is the maximum exact value of function in the domain, e_i is the exact error in the node i.

4.1.1 Example 1

Application of the GFDM to solve the partial differential equation

$$\triangle^2(U) = 0 \tag{15}$$

with boundary conditions

$$\begin{cases} U(0,y) = y^4 & if \quad 0 \le y \le 1 \\ U(1,y) = 1 + y^4 - 6y^2 & if \quad 0 \le y \le 1 \\ U(x,0) = x^4 & if \quad 0 \le x \le 1 \\ U(x,1) = x^4 + 1 - 6x^2 & if \quad 0 \le x \le 1 \end{cases}$$

The cloud of points employed was irregular and is given in fig. 1. The analytical solution is

$$U(x,y) = x^4 + y^4 - 6x^2y^2 \tag{16}$$

The global error is: 0.00001471%

4.1.2 Example 2

Application of the GFDM to solve the partial differential equation

$$-\frac{\partial^3 U}{\partial x^3} + \frac{\partial^3 U}{\partial y^3} + \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \tag{17}$$

with boundary conditions

$$\begin{cases} U(0,y) = y^3 & if \quad 0 \le y \le 1 \\ U(1,y) = 1 + y^3 - 3y - 3y^2 & if \quad 0 \le y \le 1 \\ U(x,0) = x^3 & if \quad 0 \le x \le 1 \\ U(x,1) = x^3 + 1 - 3x^2 - 3x & if \quad 0 \le x \le 1 \end{cases}$$

The cloud of points employed was irregular of 121 nodes and is given in fig. 1. The analytical solution is

$$U(x,y) = x^3 + y^3 - 3x^2y - 3xy^2 \tag{18}$$

The global error is: 0.0001769%

4.1.3 Example 3

Application of the GFDM to solve the systems

$$\begin{cases} \dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 V}{\partial x \partial y} = 0 \\ \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 U}{\partial x \partial y} = 0 \end{cases} \tag{19}$$

with boundary conditions

$$\begin{cases} U(0,y) = \sin y & if \quad 0 \le y \le 1 \\ U(1,y) = e \sin y & if \quad 0 \le y \le 1 \\ U(x,0) = 0 & if \quad 0 \le x \le 1 \\ U(x,1) = e^x \sin 1 & if \quad 0 \le x \le 1 \end{cases} \qquad \begin{cases} V(0,y) = \cos y & if \quad 0 \le y \le 1 \\ V(1,y) = e \cos y & if \quad 0 \le y \le 1 \\ V(x,0) = e^x & if \quad 0 \le x \le 1 \\ U(x,1) = e^x \cos 1 \sin 1 & if \quad 0 \le x \le 1 \end{cases}$$

The cloud of points employed was irregular and is given in fig. 1. The analytical solution is

$$U(x,y) = e^x \sin y; \quad V(x,y) = e^x \cos y \tag{20}$$

The global errors are: $errorU = 0.0000425\%$; $errorV = 0.0000464\%$

4.2 Plates

4.2.1 Thin Elastic Plates

In this section we are going to solve Eq. 11 in two different cases: fixed plate and simply supported plate.
Tables 1 and 2 show the results, (using star of 24 nodes, the four quadrants criterion for the selection of nodes of the star and the weighting function 13), of the maximum displacement at the node located at $(0.5, 0.5)$, using regular meshes of 49, 81, 289 and 441 nodes, of a 1×1 thin plate $(\nu = 0.3)$, (t=0.05), with the edges completely fixed (movements and rotations constrained), with uniform load and with point load at $(0.5, 0.5)$. The error is evaluated using the following formula

$$error = \frac{|displacement - exact.max.displacement|}{exact.max.displacement} \times 100 \quad (21)$$

Table 1: Fixed plate with an uniform load of $q = \frac{Et^3}{12(1-\nu^2)}$.

Exact. max. displacement= 0.001260[**3**]		
nodes	displacement	% error
49	0.001328	5.39
81	0.001297	2.94
289	0.001275	1.19
441	0.001265	0.39

Table 2: Fixed plate with a point load of $P = \frac{Et^3}{12(1-\nu^2)}$ at the point $(0.5, 0.5)$.

Exact. max. displacement= 0.005600[**3**]		
nodes	displacement	% error
49	0.005436	2.93
81	0.005488	2.00
289	0.005568	0.57
441	0.005600	0.00

Tables 3 and 4 show the results of the maximum displacement at the node located at $(0.5, 0.5)$, using regular meshes of 49, 81, 289 and 441 nodes, of a 1×1 thin plate $(\nu = 0.3)$, (t=0.05), simply supported (movements constrained at the edges), with an uniform load and with a point load at $(0.5, 0.5)$.

Table 3: Simply supported plate with an uniform load of $q = \frac{Et^3}{12(1-\nu^2)}$.

Exact. max. displacement= 0.004062[**3**]		
nodes	displacement	% error
49	0.004282	5.42
81	0.004191	3.17
289	0.004079	0.66
441	0.004073	0.27

Table 4: Simply supported plate with a point load of $P = \frac{Et^3}{12(1-\nu^2)}$ at the point $(0.5, 0.5)$

Exact. max. displacement= 0.01160[**3**]		
nodes	displacement	% error
49	0.01095	5.60
81	0.011136	4.00
289	0.011456	1.24
441	0.0115	0.86

In the cases of uniform load (Tables 1 and 3) we apply at each point the load corresponding to the area of influence of this point.

4.2.2 Thick Elastic Plates

In this case for modeling the second order pde's Eq. 12 we use star of 8 nodes, the four quadrants criterion for the selection of nodes of the star and the weighting function 13. Table 5 shows the results of the maximum displacement at the node located at $(0.5, 0.5)$ of a 1×1 thick plate with its boundaries completely fixed and uniform load, using the GDFM with a regular mesh of 961 nodes. The results are provided for different values of the thickness of the plate.

4.2.3 Comparison of results with other methods

The following figure 2 shows the displacement of the node located at $(0.5, 0.5)$ for the fixed 1×1 plate, as the thickness is increased. The results obtained from the GFDM have been compared with the ones obtained using a finite element commercial software. In order to better understand the differences, two models have been created.
The first finite element model uses 2,500 shell elements with six degrees of freedom per

Table 5: Fixed plate with an uniform load of $q = \frac{Et^3}{2(1-\nu^2)}$.

Maximum displacement	
thickness	displacement
0.05	0.001177
0.1	0.00144
0.2	0.00216
0.3	0.003236
0.4	0.004724

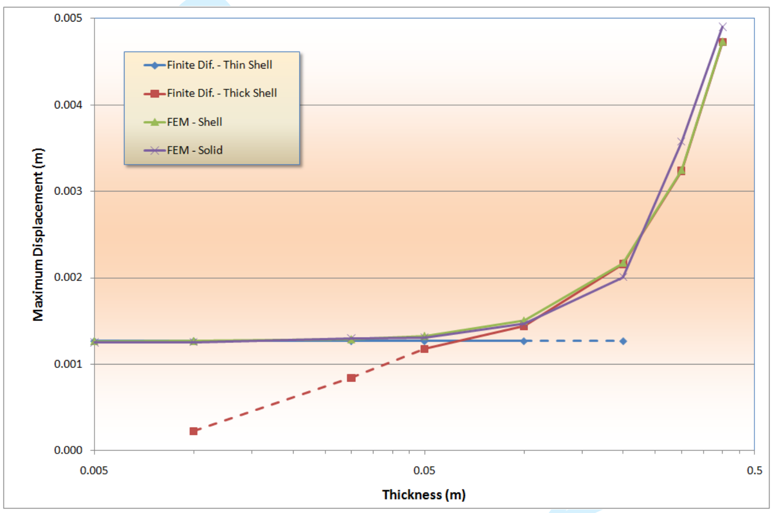

Figure 2: Comparison between different numerical methods at the node $(0.5, 0.5)$ for the fixed 1×1 plate as the plate thickness is increased

node. The element used is a 4-node element suitable for analyzing thin to moderately-thick shell structures.
The second finite element model uses 25000 brick 8-node elements with three degrees of freedom per node.
Figures 3 and 4 show the deformation of a fixed irregular shaped plate with a point load on the node located at $(0.5, 0.5)$ using the GFDM and the finite element method.

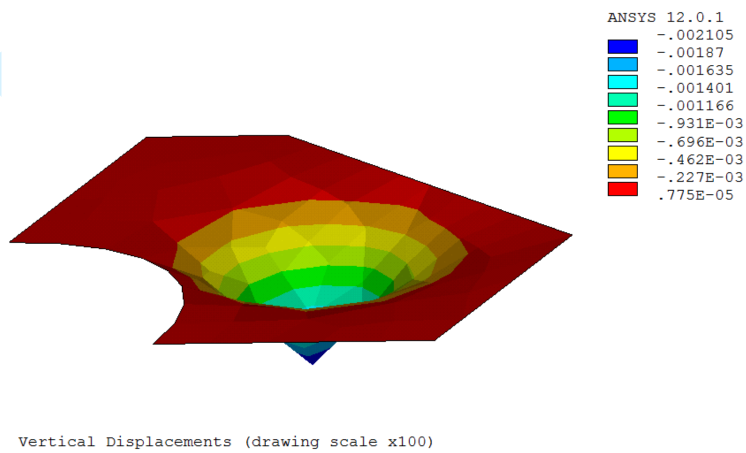

Figure 3: Deformed plate using ANSYS finite element software [14]

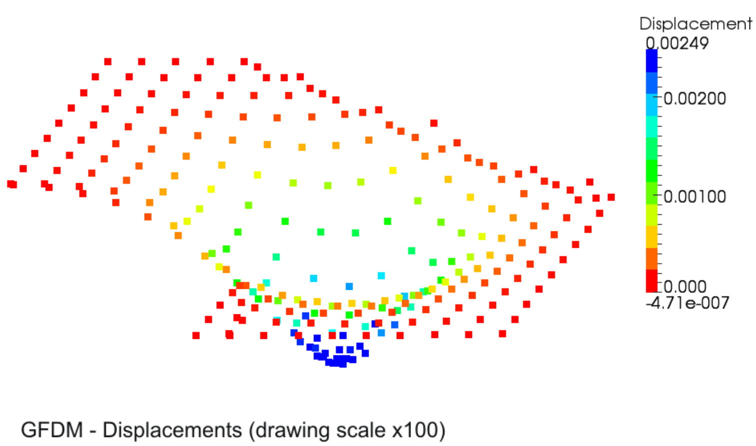

Figure 4: Deformed plate using GFDM

5 Conclusions

The GFDM has been used to obtain the solution of up to fourth order differential equations. The method has been applied to solve thin and thick plates.
A series of academic examples have been tested to compare the GFDM results with the analytical results. It has been observed that accurate results can be obtained.
A 1.0×1.0 square plate with a point load and uniform loads and with fixed or simply supported edges has been analyzed, varying the number of nodes. The obtained solution has been compared with the analytical solution. Even though the numerical solution approaches the theoretical solution as the number of nodes increases, with a number of nodes ≥ 400 an accurate result is provided.
An analysis has been carried out varying the thickness of the plate and comparing the results with a finite element commercial software. For a range between 0.05 and 0.10 of the thickness/length ratio of the plate, the results are similar (7% and 13% difference respectively). For a ration greater than 0.10 the thick plate formulation should be used, while for a ration below 0.05 the thin plate formulation provides much more accurate results. This confirms the validity of the applied procedure.
Finally, an irregular plate with a point load has been checked, comparing the results obtained with the GFDM with the ones obtained by the commercial finite elements software [14]. Both the maximum displacement and the deformed shape agree with the two methods.

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