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Some Applications of Intuitionistic Fuzzy Sets for the Determination of a Sociometric Index of Acceptance

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Abstract: *A simple use of intuitionistic fuzzy sets to determine and process the information describing the level of acceptance of people in a social group (e.g., pupils in a school class) is presented in the paper. An algorithm of processing linguistic assessments is given that leads to aggregated opinions and conclusions in the form of linguistic evaluation.*

Keywords: *Sociometry, index of acceptance, intuitionistic fuzzy sets, linguistic evaluations.*

1. Introduction

R e n č o v á [7] has developed the idea of delimitation of a sociometric index of acceptance in a group of persons (e.g., pupils in a school class), described first by H e i n z o v á and B e l k o [3]. This index, denoted as I, is counted as the difference of the value of the membership function and the non-membership function of a intuitionistic fuzzy set.

The Intuitionistic Fuzzy Set A in the universum X is understood as

$$A = \{(x, \mu_A(x), \nu_A(x)): x \in X\},$$

where μ_A and ν_A are functions from X in a closed interval [0, 1], and for every $x \in X$ holds $\mu_A(x) + \nu_A(x) \leq 1$.

The values $\mu_A(x)$ and $\nu_A(x)$ are, respectively, the degree of membership and the degree of non-membership of element x to the set A.

The corresponding values of membership and non-membership functions are obtained based on questionnaire completed by all members of the group. The participant survey is designed to put the names of all persons; sign + denotes acceptance of a person and sign –, signifying lack of acceptance. One may also consider not to place any sign so that he could not specify their relationship to the person. When we have collected and compared data from surveys obtained for the individual x values of $A(x)$ and $N(x)$ which are the number of the members of the group accepting person x, and number of members of the group that do not accept this person.

The author notices that the values $\frac{A(x)}{n}$ and $\frac{N(x)}{n}$, where n is the group size, i.e., standardized values of $A(x)$ and $N(x)$, can be interpreted as the values of the membership function and non-membership function of an intuitionistic fuzzy set.

On the basis of these values index I is calculated given by the formula $I(x) = \frac{A(x)}{n} - \frac{N(x)}{n}$, being an aggregation of subjective impressions relating to the receipt of the person x by the participants of the group.

The introduction of the coefficient I is caused by the need for measurement of the satisfaction of the participation in the social group. It could also be important to point out all persons accepted by the majority of the group (*persona grata*, *VIP*), persons able to take the leadership, and also outsiders or persons clearly not accepted by the group (*black sheep*).

2. Comments and doubts

Every person has to fill the questionnaire. In the cited article [7] it has not been explicitly clarified whether the student is asked to fill a questionnaire about acceptance of him/herself. You can guess that the answer is yes. But independently of his/her response, it does not imply any information of the acceptance of a person in the group and perhaps (especially in a small group size) it affects the unit value index of I. In addition, it can be difficult for a person to answer the question, "Do I accept myself in a group?", not mistaking it with the question "Do I accept myself (in general)?" Especially the teenagers may have a problem with this. Deciding to skip the self-assessment we should pay particular attention to proper design of the questionnaire providing respondent's anonymity.

The question *accept*/*not accept,* with the additional possibility *I do not know* can safely be extended to the question about some kind of power of acceptance. From our experience we know certainly that some people in the group are hardly accepted by us and for others it is hard to imagine the existence of this group. The survey bipolar ± both receive a plus.

Intuitionistic fuzzy sets are a tool, the use of which may support the person expressing opinion about the group or a person, undertaking some decisions about this group. One may consider what value for the researcher may have aggregation (averaging) of all opinions in the group. It would be then an index showing the strength of the compactness (coherency) of the group and it would be possible to

compare groups (student classes). Compared with the corresponding value in another group can show you the consistency (cohesion) of this group. This group index would be the average opinion of a person about colleagues. The higher the value is, the more colleagues accept each other (in general).

Index I proposed by Renčová is identical for the same difference in the values of $A(x)$ and $N(x)$. So it does not distinguish, for example (the extreme) where $A(x_1)=N(x_1)=0$ and $A(x_2)=N(x_2)=[n/2]$. From the perspective of sociology these situations are different. The person x_2 raises in the group much more interest and emotions than the person x_1, which is some kind of a "neutral element". These persons would also be perceived differently in the case of distribution of the group into subgroups.

The calculation of index I is easy and its value is relatively easy to interpret (but one would think what type of the scale which measures the values of $I(x)$ is meaningful). However, we have the impression that making almost immediately a defuzzyfication, results in losing the major advantage of fuzzy sets which is vagueness and possibility of processing. One can have doubts whether vagueness is an advantage. But we have to remember that this type of imprecision (fuzziness) is a feature of many terms of the natural language – including the term *accept*.

3. A suggestion of modifications

Some psychologists research suggests that it is often convenient to assess options (people, objects, activities) using natural language expressions. In many cases, the graduation of assessments is permissible and even important. Graduation is associated with the gradation of adjectives defining a particular feature and its intensity. Something can be good, better, the best. A person may be more or less accepted. When designing the survey it would be appropriate to give the surveyed persons the choice of several degrees of "intensity of acceptability".

M i l l e r [6] proved that the number of evaluations of the variant cannot be too big. The average person can clearly distinguish between no more than 7±2 levels of severity of the parameter. With more degrees the adjacent levels begin to overlap and can no longer be entirely clearly distinguished (Miller's Law, Miller's magical number 7).

The collecting and processing opinions would thus be as follows.

We give all the members x_i (i = 1, ..., n) of group X a questionnaire to fill, in which the respondents must indicate for each person in the group (except him/herself), one of the seven responses (linguistic evaluations) given below:

1) strongly accepted,
2) accepted,
3) rather accepted,
4) rather not accepted,
5) not accepted,
6) definitely not accepted,
7) I cannot determine whether I accept this person or not. (I don't now).

In the polls we get $n^2–n$ responses (n–1 responses from each of the n respondents). After collecting we assign to them the Intuitionistic Fuzzy Values (IFV) given in Table 1.

Table 1. Intuitionistic counterparts of the linguistic evaluations. Source: Author's proposal

Linguistic evaluation given by x_i about person x_j	IFV, $F_i = \langle \mu_{i,j}, \nu_{i,j} \rangle$
Strongly Accepted (SA)	$\langle 1, 0 \rangle$
Accepted (A)	$\langle 0.66, 0 \rangle$
Rather Accepted (RA)	$\langle 0.33, 0 \rangle$
Rather Not Accepted (RNA)	$\langle 0, 0.33 \rangle$
Not Accepted (NA)	$\langle 0, 0.66 \rangle$
Definitely Not Accepted (DNA)	$\langle 0, 1 \rangle$
I Don't Now (IDN)	$\langle 0, 0 \rangle$

Any person x_j, j = 1, ..., n, obtains n–1 intuitionistic fuzzy evaluations. After some aggregation these evaluations form the index $I_j = \langle a_j, b_j \rangle$. It is not possible to uniquely determine the aggregation. The aggregations corresponding to a standard product and sum operations on the intuitionistic fuzzy sets does not seem appropriate due to the fact that even one answer *I don't know* makes $\langle a_j, b_j \rangle = \langle \min\limits_{i \neq j} \mu_{i,j}, \max\limits_{i \neq j} \nu_{i,j} \rangle = \langle 0, a \rangle$ and $\langle a_j, b_j \rangle = \langle \max\limits_{i \neq j} \mu_{i,j}, \min\limits_{i \neq j} \nu_{i,j} \rangle = \langle b, 0 \rangle$, where the values $\mu_{i,j}$ and $\nu_{i,j}$ are, respectively, the degree of membership and degree of non-membership of the IFV values (opinion of i-th person about j-th person). At this point, a lot of the remaining information, acquired in surveys is lost.

Similarly, we lose certain information in the case of even one answer (RA), (A) or (SA) and aggregations of a sum-type and in the case of even one answer (RNA), (NA) or (DNA) and aggregations of a product-type.

For IFV values, specified in Table 1, it seems reasonable to use the averaging operation @ given by A t a n a s s o v [2, p. 138] in the form

$$\langle a, b \rangle \,@\, \langle c, d \rangle = \left\langle \frac{a+c}{2}, \frac{b+d}{2} \right\rangle.$$

This operation, generalized to more than two arguments, allows to determine the formula of the intuitionistic index

$$I_j = \langle a_j, b_j \rangle = \left\langle \frac{\sum\limits_{i \neq j} \mu_{i,j}}{n-1}, \frac{\sum\limits_{i \neq j} \nu_{i,j}}{n-1} \right\rangle.$$

In fact, the values of membership and non-membership functions are simply the arithmetic means of the aggregated values. The aggregation in the form of an arithmetic mean has of course also advantages and disadvantages.

The value of I_j can be interpreted as the truth-value of the sentence "the person x_j is accepted by others in the group X", in the intuitionistic fuzzy logic. Equivalently: the values of a_j and b_j are the values of membership and non-membership functions of element x_j to A_X; intuitionistic fuzzy set of people,

accepted in the group (class) X. An index is obtained, therefore, which is a medium level of acceptance by the other group members.

For index $I_j = \langle a_j, b_j \rangle$ the value $\pi_j = 1 - a_j - b_j$ can be determined, called the hesitation margin (hesitancy degree, intuitionistic fuzzy index, degree of uncertainity) of elements x_i in IFS A_X. This value informs the investigator about the clarity of evaluation of j-th person. The lower it is, the stricter (more precise) the person is though perhaps extreme, as judged by the group. On the contrary, a higher hesitation margin indicates weak clarity of the acceptance (or weak clarity of the non-acceptance) in the group. It may reflect the low status of the person in the group. Giving, besides the index I_j, the degree of indeterminacy π_j we also obtain important information about the perception of j-th person.

4. Fuzzines and what next?

The resulting fuzzy indicator I_j can be subjected to some kind of defuzzyfication giving a clear declaration of acceptance or non acceptance of a person. A relatively simple transformation will be defuzzyfication based on the idea of Intuitionistic Fuzzy Tautology (IFT). This means that, if for $\langle a_j, b_j \rangle$ holds $a_j \geq b_j$, we can say that a person is accepted in the group (the inequality means that the person is at least equally acceptable as it is not acceptable). Unfortunately, this type of assessment causes loss of previously acquired information about the intensity of acceptance.

An alternative way is to give information about the acceptance in the form of linguistic evaluation. Assuming seven linguistic assessments, given earlier, we should not expect that the index I_j will be equivalent to some of them. We can then try to determine which of these assessments will be the most adequate.

There exist two basic ways to determine the adequacy of the Intuitionistic Fuzzy Sets (or IFSs). The first one is based on a measure of the distance of sets, the second, on the measure of similarity between two intuitionistic fuzzy sets. In general they are not equivalent.

Let $I_j = \langle a_j, b_j \rangle$ and $F_k = \langle \mu_k, \nu_k \rangle$ be intuitionistic fuzzy values.

Normalized Hamming distance of the I_j and F_k is given by the formula

$$l_{\text{IFS}}(I_j, F_k) = 0.5\left(\left|a_j - \mu_k\right| + \left|b_j - \nu_k\right| + \left|\pi_j - \pi_k\right|\right) \in [0, 1].$$

The lower the value $l_{\text{IFS}}(I_j, F_k)$, the more opinion I_j corresponds to F_k.

Kacprzyk and Szmidt [4], on the basis of the above, suggested a similarity index Sim (similarity measure) of the intuitionistic fuzzy sets given by formula

$$\text{Sim}(I_j, F_k) = l_{\text{IFS}}(I_j, F_k^C) - l_{\text{IFS}}(I_j, F_k) =$$
$$= 0.5\left(\left|a_j - \nu_k\right| + \left|b_j - \mu_k\right| - \left|a_j - \mu_k\right| - \left|b_j - \nu_k\right|\right) \in [-1, 1].$$

where $F_k^C = \langle \nu_k, \mu_k \rangle$ is the standard complement of F_k.

The greater the Sim value is, the greater I_j opinion is more similar to F_k. The greater the Hamming distance from the reference value F_k is, the greater the distance from the value of complement of the pattern is.

Note that Sim(I_j, F_k) = Sim(F_k, I_j) therefore, it does not matter whether we use the complement of F_k or I_j.

In connection with the seven linguistic assessments used for each person x_j, we get seven Sim(I_j, F_k) values. Choosing the maximum among them we get the answer to which linguistic evaluation the aggregate rating I_j is most similar.

5. Numerical example

Assume that in the group of five persons we have obtained the following opinions of i-th person about j-th, where $i, j = 1, ..., 5$.

Table 2. Linguistic evaluation in the group. Source: sample data

	x_1	x_2	x_3	x_4	x_5
x_1	–	SA	RA	A	A
x_2	IDN	–	NA	A	RA
x_3	A	A	–	A	SA
x_4	A	SA	IDN	–	SA
x_5	A	RA	IDN	A	–

The resulting aggregate fuzzy values are given in Table 3.

Table 3. Aggregeted fuzzy values. Source: own study

	x_1	x_2	x_3	x_4	x_5
μ	0.495	0.7475	0.0825	0.66	0.7475
ν	0	0	0,165	0	0
π	0.505	0.2525	0.7525	0.34	0.2525

Measures of similarity Sim of the variants towards the various linguistic evaluations are given in Table 4.

Table 4. Similarity measures of the aggregated values to the linguistic evaluations. Source: own study.

x_i	**SA**	A	RA	RNA	NA	DNA	IDN
x_1	0,495	0,495	0,33	–0,33	–0,495	–0,495	0
x_2	0,7475	0,66	0,33	–0,33	–0,66	–0,7475	0
x_3	–0,0825	–0,0825	–0,0825	0,0825	0,0825	0,0825	0
x_4	0,66	0,66	0,33	–0,33	–0,66	–0,66	0
x_5	0,7475	0,66	0,33	–0,33	–0,66	–0,7475	0

The given values show that the aggregated opinions of the individuals are most similar: in the case of x_1 to SA and A, in the case of x_2 to SA, in the case of x_3 to RNA, the NA and DNA together, in the case of x_4 to SA and A, and finally in the case of x_5 to SA.

At first glance we can see the discrepancies that disqualify this manner. Three major disadvantages of the method, seen in this example, are:

1) the evaluations of all persons are equally similar to the evaluation IDN,

2) the average rating of person x_4 is exactly counterpart of the opinion of accept (A), while Sim recognizes the same resemblance to accept (A) and strongly accept (SA),

3) it is difficult to accept the same similarity of x_3 up to three evaluations.

Because of these drawbacks, we use the method with Hamming distance only taking into account the values of parameters μ, ν, and π. Justification of the use of all three parameters was presented in the paper of Kacprzyk and Schmidt [5]. Hamming distances $l_{\mathrm{IFS}}(I_j , F_k)$ are shown in Table 5.

Table 5. Hamming distances between aggregated values and linguistic evaluations. Source: own study

x_i	**SA**	A	RA	RNA	NA	DNA	IDN
x_1	0.505	0.165	0.165	0.495	0.66	1	0.495
x_2	0.2525	0.0875	0.4175	0.7475	0.7475	1	0.7475
x_3	0.9175	0.5775	0.2475	0.165	0.495	0.835	0.2475
x_4	0.34	0	0.33	0.66	0.66	1	0.66
x_5	0.2525	0.0875	0.4175	0.7475	0.7475	1	0.7475

From the above table we read that the opinion about person x_1 is closest to evaluations A and RA, together. The opinions about x_2, x_4 and x_5 are closest to evaluations A, and the opinion about x_3 – to RNA.

One can probably consider the disadvantages of this method of obtaining the linguistic evaluation. However, it is devoid of obvious flaws of Sim method.

We note that the intuitionistic fuzzy value of average of all opinions in the group is equal to AvAll= $\langle \mu, \nu, \pi \rangle$ = = $\langle 0.5465, 0.033, 0.4205 \rangle$ and is most similar (Sim) to the strongly accept (SA), and the nearest, in the l_{IFS} sense, to the accept (A).

6. Conclusion

The paper presents a method of appointing a sociometric index that indicates the level of acceptance of the members of the social group. To determine the level of the acceptance, linguistic values are used. Intuitionistic counterparts of the linguistic assessments were proposed. It was shown how to obtain the aggregate (average) opinion and its linguistic equivalent. The biggest doubts may arise about the data from Table 1. IFV therein affects obviously the final evaluation of each person but the universal way of their determination is not known. Despite these shortcomings we believe that the use of intuitionistic fuzzy sets can be a convenient and appropriate tool to describe the levels of acceptance in a group.

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