Some aspects of the pure theory of corporate finance: bankruptcies and take-overs: reply

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This paper reaffirms the earlier argument that when individuals differ in their expectations concerning the returns to investments, there will be an optimal debt-equity ratio, at a sufficiently high debt level. It assumes that in the judgment of lenders there is a finite probability of bankruptcy. The terms at which individuals as well as firms can borrow depends on their indebtedness and perceptions of potential lenders. Stapleton's analysis rests on the unacceptable assumption that individuals can borrow an arbitrary amount at the riskless rate, even though the firm in which they have all their wealth invested cannot. A new proof of the potentiality of productive inefficiency in the presence of bankruptcy is also presented.

■ Stapleton has raised an interesting point in his comment on my paper.[1] When there is an incomplete set of securities, demand curves for different securities will be downward sloping. The evaluation of the security in the market depends on the evaluation placed on the security by the marginal purchaser of the marginal unit of the security. These demand curves will in general depend on the distribution of wealth, attitudes towards risk, and assessments of the returns to various securities. In the analysis of Section 3 of my paper I had assumed that the marginal purchaser of the firm's securities was an individual of type *a*, i.e., somebody who was optimistic about the fortunes of the firm, and the marginal purchaser of the firm's bonds was an individual of type *b*, somebody who was pessimistic about the fortunes of the firm. This would be the case so long as the total value of the firm exceeded the net worth of the individuals of type *a*. In that case, the calculations showing the dependence of the valuation of the firm on the debt-equity ratio of the firm were correct. The more subtle question was, how do I know who is the marginal individual purchasing each security, that in this case the valuation of the firm exceeds the net worth of individuals of type *a*?

This is a more complicated question to which Section 4 of my paper was devoted. One approach is to make some hypothesis concerning market segmentation or separability, i.e., the bondholders and stockholders are different individuals. For

[1] See [3].

instance, that is the interpretation of my analysis provided by Rubinstein and by Scott.[2] But both the assumption of market segmentation and the assumptions on no borrowing and no short sales can be dispensed with; they were introduced, with some distaste, to simplify the analysis. A more complete analysis would have introduced the possibility of individual borrowing and short sales, but with the terms of those contracts determined by the lender's subjective estimation of the individual's capacity to carry out those contracts. Thus, if individuals have no assets of their own (or wage income) other than that described in the model, then the terms at which equity owners could borrow would essentially be the same as those at which the firm could borrow: firm bonds would default in exactly the same states of nature that the corresponding individual borrowings would default. Obviously, if the individual has other sources of income and other liabilities, firm debt and individual debt are not perfect substitutes. The equilibrium would then be described by, say, *a* type individuals borrowing on personal account from *b* type individuals, and individuals of type *b* would sell short some securities to individuals of type *a*. If individuals of type *b* were risk neutral, they would like to sell an indefinite amount of securities short. But individuals of type *a* do not believe that individuals of type *b* can meet those commitments. Thus, an unsecured short sale would sell at a discount, and the discount would increase the more the short sales, until the point is reached where the individual of type *b* no longer wishes to sell any further securities short. (Risk aversion obviously puts another limit on the amount of short sales.) These possibilities for borrowing by the individuals of type *a* obviously affect their demand for equities and affect the demand for the firm's bonds as well. It is still the case, however, that there is an optimal debt-equity ratio for the firm.

Let me now relate these remarks to Stapleton's comments. It is trivial that if all the shares and debt of a firm are owned by the same kind of individual, the decomposition into debt and equity makes no difference. The case analyzed by Stapleton, where individuals can borrow an arbitrary amount at the riskless rate, even though the firm in which they have all their wealth invested cannot, is one such case. It requires, however, a peculiar kind of misperception on the part of the lenders—one which is inconsistent with the intent of the analysis.

Another result of the paper which may have been the subject of some misunderstanding is that relating to the productive efficiency of the economy with bankruptcy. Consider first the case of whether there is multiplicative uncertainty in the returns (gross of interest costs, but net of all other costs), i.e., the returns to a one-period investment of I can be written as $\Theta h(I)$, where Θ is the stochastic variable. Let $\hat{r}$ be the nominal rate of interest and B the number of bonds. The net return is then

$$\max\{\Theta h(I) - (1 + \hat{r})B_1, 0\}. \tag{1}$$

We let $F(\Theta)$ be the cumulative distribution of Θ, so π, the probability the firm will not go bankrupt, is

[2] In [1] and [2], respectively.

$$\pi = 1 - F\left(\frac{(1 + \hat{r})B}{h(I)}\right).$$

Note that the ratio of B to $h(I)$ completely determines the pattern of returns of the risky bonds:

$$1 + \tilde{r} = \min\left\{(1 + \hat{r}), \frac{\Theta h(I)}{B}\right\}. \quad (2)$$

Thus, $\hat{r}$ will simply be a function of $h(I)/B$. An individual who owns a firm, who is risk neutral, and can obtain a return of ϕ on some other security maximizes[3]

$$h(I)\int_{(1+\hat{r})B/h(I)} \Theta dF(\Theta) - (1 + \hat{r})B\pi + \phi(B + W_0 - 1), \quad (3)$$

where W_0 is his initial wealth. Hence

$$h'\int_{(1+\hat{r})B/h} \Theta df - \frac{(1 + \hat{r})B\pi}{h} = \phi\left(1 - \frac{Bh'}{h}\right) \quad (4)$$

and

$$\hat{r}'\pi + \frac{(1 + \hat{r})\pi h}{B} = \phi h/B. \quad (5)$$

Equation (5) can be solved for h/B. Since all firms within the risk class will then have the same pattern of returns for their risky bonds, they will all have to pay the same nominal rate of interest, and it is immediate that the market value will be proportional to $h(I)$. Hence h/B is proportional to the debt-equity ratio, so that (5) can be interpreted as saying that all firms within the risk class will have the same debt equity ratio.[4] But then (4) implies that h' will be the same for all firms within the risk class: there is productive efficiency.

On the other hand, if there is multiplicative uncertainty on output, but not on returns, or if there is marginal multiplicative uncertainty on returns but not inframarginal multiplicative uncertainty, there will in general not be productive efficiency. Assume, for instance, that we can write net returns of the jth firm in a given risk class as

$$\Theta h^j(I) + c^j.$$

Productive efficiency clearly requires that if there are two firms producing, which are identical except for their values of c^j and the shape of the h^j function,

$$h^{j\prime} = h^{i\prime}.$$

But now, the return on the risky bond is no longer just a function of B/h:

$$(1 + \tilde{r}^j) = \min\left\{(1 + \hat{r}), \frac{\Theta h^j(I) + c^j}{B^j}\right\}.$$

When the implications of this are traced through the remaining

[3] ϕ is the maximum return he can obtain on any other security; it is the opportunity cost of capital. Again we restrict short sales and individual borrowing.

[4] This is not precisely correct; since $\hat{r}'$ is not necessarily one-signed, there may be more than one solution to the maximization problem.

analysis, it is apparent that there will not in general be productive efficiency.

References

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