Some Aspects of Wind Power Statistics

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(Manuscript received 15 April 1976, in revised form 24 January 1977)

ABSTRACT

Some of the problems of wind power statistics are examined. The exact relationship between the mean wind speed and the mean of the cube of the wind speed is discussed. The Weibull probability density function, a good model for wind speed distributions, leads to a Weibull model for the distribution of the cube of the wind speed. This model facilitates the computation of the mean and the standard deviation of the total wind power density, the usable wind power density, and the wind power density during the hours when an aerogenerator is operating. The Weibull model is applied to data from three Oregon wind power sites located in rugged terrain. It is concluded that the mean and standard deviation of the wind speed are the minimum statistics necessary for wind power estimates, that the Weibull model for the wind power density has many computational advantages, and that the existing wind power studies based solely on the total mean wind power density omit much valuable information about the wind power potential of a site.

1. Introduction

In recent years increasing attention has been paid to the development of techniques for determining the availability of wind power resources. In the extensive wind power climatology studies (Reed, 1975; Barber *et al.*, 1977) which depict annual and seasonal mean total wind power values for the United States, the mean total wind power was computed by cubing the wind speeds in frequency tables obtained from the National Climatic Center. There is an error inherent in using these data caused by the small number of frequency classes available for many stations (Barber *et al.*, 1977).

Other investigators have sought simple wind speed distributions which could be parameterized solely by the mean wind speed. Wentink (1974) has investigated methods of fitting a Planck distribution (with parameter $f = 3$) to his Alaskan data. The use of this distribution in meteorology originated with Dinkelacker (1949). Court (1975, personal communication) has been experimenting with the Rayleigh distribution (a chi distribution with 2 degrees of freedom) which has also been used in other countries (e.g., Narovlyanskii, 1968; Baynes, 1974). Whenever such distributions can be used, it is easy to determine the mean of the cube of the wind speed and therefore the mean total wind power.

Crutcher and Baer (1962) showed that the bivariate normal distribution is adequate for most wind samples; however, the univariate distribution which is derived from the bivariate normal distribution results in a complicated expression involving the summation of the products of Bessel functions (Smith, 1971). It is difficult to use this distribution in wind power applications unless it can be simplified to the Rayleigh under a very restrictive set of assumptions.

The Pearson Type III (gamma) family has often been used to describe wind speed distributions (Putnam, 1948; Sherlock, 1951). The Weibull distibution is a special case of the generalized gamma distribution. For wind speed distributions the Weibull is often a practical, if somewhat empirical, alternative to the simpler one-parameter distributions such as the Rayleigh. (The Rayleigh is itself a special case of the Weibull.)

The Weibull distribution has been fit to both upper air data (Baynes and Davenport, 1975) and surface wind speed data (e.g., Wentink, 1976; Justus *et al.*, 1976). Baynes (1974) has used the Weibull distribution in a theoretical analysis of specific output (plant factor). Specific output is the proportion of the time that an aerogenerator will deliver its full rated power. Justus *et al.* (1976) have been successful in fitting the Weibull distribution to 135 National Climatic Center wind summaries and then computing mean potential output estimates using numerical integration techniques. They also interpreted their results in terms of plant factors.

The Weibull distribution is a useful tool for wind power analysis, and it is intended that this rather exemplary treatment will acquaint the reader with the unique versatility of the Weibull distribution. There are several important questions which are beyond the scope of this paper. These include 1) how to draw an independent sample from a series of wind speed observations which are often correlated at different lag times, 2) how the wind power varies with time, 3) how best to handle wind speed distributions which are too deformed or heterogeneous for even a two-parameter distribution

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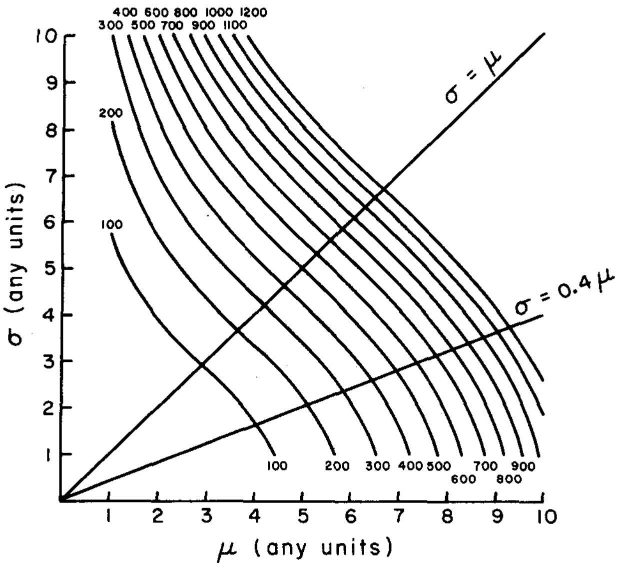

FIG. 1. Isopleths of constant speed cubed assuming that the wind speed distribution is not skewed (i.e., symmetrical).

to adequately fit, and 4) how to determine the variation of wind power with height.

2. Estimation of mean wind power density

In theory, the instantaneous power density (W m^{-2}) available in a flow of air through a unit cross-sectional area normal to the flow is simply

$$P=\tfrac{1}{2}\rho V^3, \tag{1}$$

where V is the instantaneous wind speed (m s^{-1}) and ρ the density of air ($\sim$1.23 g m^{-3} at sea level). The expectation of P (i.e., the mean power density per unit area) is then

$$E(P)=\tfrac{1}{2}E(\rho V^3). \tag{2}$$

Wind power estimates have been based on the assumption that the density is not correlated with the wind speed.[2] In this case, Eq. (2) becomes

$$E(P)=\tfrac{1}{2}E(\rho)E(V^3). \tag{3}$$

The mean air density is then estimated from the U. S. Standard Atmosphere, and the problem of determining the mean wind power density at a given location is reduced to determining the mean of the speed cubed.

The mean of the speed cubed can be determined exactly, once it is recognized as just the expectation of the third moment of the wind speed about zero, i.e.,

$$E(V^3)=\sigma^3[\sqrt{\beta_1}+3\mu/\sigma+(\mu/\sigma)^3], \tag{4}$$

where μ, σ and $\sqrt{\beta_1}$ are the mean, standard deviation and skewness of the wind speed distribution, respectively.

[2] The error introduced by this assumption on a constant pressure surface is probably less than 5% (Barber, 1976, personal communication).

If the skewness is negligible (i.e., if the distribution is nearly symmetrical) Eq. (4) reduces to

$$E(V^3)=3\mu\sigma^2+\mu^3. \tag{5}$$

For this case, isopleths of constant mean speed cubed in the μ, σ plane are depicted in Fig. 1. The diagonal line is for $\sigma=\mu$. Since we expect *a priori* that the coefficient of variation σ/μ for the wind speed will be less than unity, it is the lower right half of this figure that is useful for wind power estimates. The standard deviation and skewness are not generally functions of the mean only. Fig. 1 indicates that although the mean speed cubed and therefore the mean wind power density increase with the mean wind speed, the standard deviation is necessary in order to compare the wind power potential of various sites. Reed (1974) estimated that the error was at least 40% without some specification of the standard deviation of the wind speed distribution, but the precise way the higher moments entered the problem has not been clearly understood.

Actual wind speed data are positively skewed, and the mean speed cubed estimates of Fig. 1 should have, according to Eq. (4), the following correction term added:

$$C(\sigma,\sqrt{\beta_1})=\sigma^3\sqrt{\beta_1}. \tag{6}$$

Fig. 2 shows the correction term $C(\sigma,\sqrt{\beta_1})$ as a function of the standard deviation over a typical range of skewness associated with wind speed distributions. The skewness can be conveniently estimated if certain assumptions can be made about the wind speed frequency distribution. This is one of the advantages of the Weibull model.

3. The Weibull model

The Weibull distribution is a unimodal, two-parameter family of distribution functions which has been successfully fitted to wind speed distributions (i.e., Justus *et al.*, 1976; Wentink, 1976). This probability density function has the form

$$f_X(x)=acx^{c-1}\exp(-ax^c);\quad a>0,\ c>0,\ x>0, \tag{7}$$

where c is called the shape parameter and $a^{-1/c}$ is a scale parameter. If $c=2$, Eq. (7) reduces to the Rayleigh distribution. The Weibull distribution has its mean and variance in terms of gamma functions

$$\mu=(1/a)^{1/c}\Gamma(1+1/c), \tag{8}$$

$$\sigma^2=(1/a)^{2/c}[\Gamma(1+2/c)-\Gamma^2(1+1/c)], \tag{9}$$

while the expectation of its third noncentral moment is

$$E(X^3)=a^{-3/c}\Gamma(1+3/c). \tag{10}$$

The standard form of Weibull distribution is found by setting $a=1$. The effect of variation in the shape parameter is the shown in Fig. 3. For $0<c<1$, this distribution has its mode at zero and is a decreasing func-

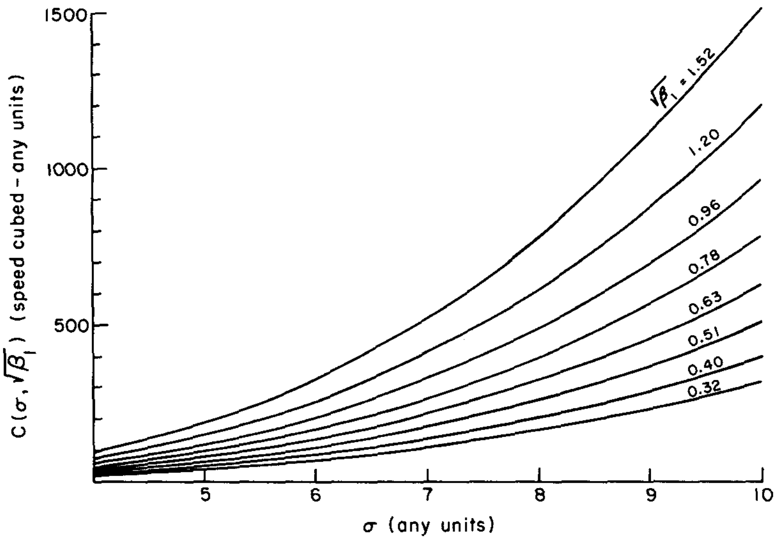

FIG. 2. The correction term as a function of the standard deviation of the wind speed over a typical range of skewness values for wind speed distributions.

tion of x. For $c=1$, the distribution is exponential. At $c=3.5$, the distribution is approximately normal.

The Weibull parameters can be estimated by the method of maximum likelihood or by fitting the Weibull curve to a cumulative distribution function (Justus *et al.*, 1976); but, if both the sample mean and standard deviation are known, the shape parameter can be most conveniently estimated by the nomogram of Kotel'nikov (Johnson and Kotz, 1970), part of which is shown as Fig. 4.

The parameter a can then be calculated from the equation for the mean [Eq. (8)]. The Gamma function tables found in most standard references are sufficient for this purpose.

The skewness of a Weibull distribution is a function only of the shape parameter c. Johnson and Kotz (1970) provide the values given in Table 1.

Therefore, if the mean and the standard deviation of the wind speed are known, then all the parameters needed to compute the mean of the cube of the wind speed using Eq. (4) can be specified. The experience of Wentink (1976) and Justus *et al.* (1976) indicates that the parameter c will vary from 1.1 to 2.6 with an average value of about 2.0.

For the Weibull distribution, the standard deviation is a linear function of the mean, and the slope is determined solely by the shape parameter (Fig. 5). Therefore, in situations in which this model is applicable, the

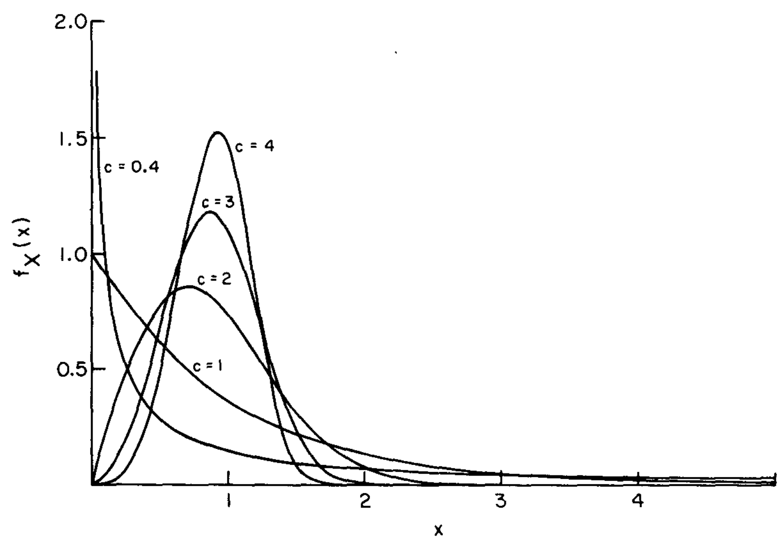

FIG. 3. The standard form of the Weibull distribution.

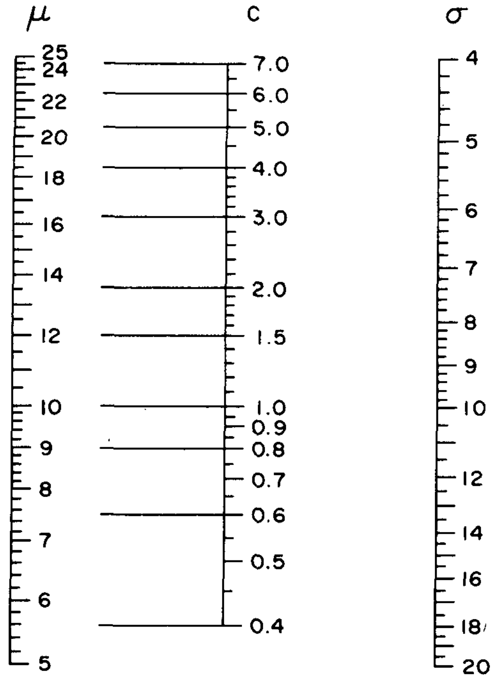

FIG. 4. Kotel'nikov's nomogram for the Weibull distribution. Draw a straight line between the appropriate estimates of μ and σ. The estimate of the shape parameter is determined by the intersection of this line and the c scale. After Johnson and Kotz (1970).

region of interest in Fig. 1 will above the line $\sigma=0.4\mu$ ($c\approx 2.7$).

4. The probability density function for the total wind power density

For cases where the Weibull model adequately describes the wind speed distribution, it is possible to determine the frequency distribution of the speed cubed. With the transformation $Y=X^3$, Eq. (7) becomes

$$f_Y(y)=a(c/3)y^{(c/3)-1}\exp(-ay^{c/3}). \tag{11}$$

Eq. (11) is recognizable as a Weibull function with parameters a and $c/3$; the mean and variance are therefore known to be

$$E(Y)=a^{-3/c}\Gamma(1+3/c), \tag{12}$$

$$\mathrm{var}(Y)=a^{-6/c}[\Gamma(1+6/c)-\Gamma^2(1+3/c)]. \tag{13}$$

Using Eq. (8), these equations simplify to

$$E(Y)=\mu^3[\Gamma(1+1/c)]^{-3}\Gamma(1+3/c), \tag{12a}$$

$$\mathrm{var}(Y)=\mu^6[\Gamma(1+1/c)]^{-6}[\Gamma(1+6/c)-\Gamma^2(1+3/c)]. \tag{13a}$$

Eq. (12) is the mean of the probability density function for the speed cubed. Note that it is the same as (10). Eq. (13) is the variance of the speed cubed and, therefore, proportional to the variance of the total power density in the wind. Eqs. (12a) and (13a) state that the mean speed cubed and its standard deviation are both equal to the cube of the mean multiplied by a function of both the mean and the standard deviation of the wind speed. To this author's knowledge, no one has examined any models for the frequency distribution of the total wind power density or attempted to estimate its dispersion about its mean value.

The effect of increases in the mean wind speed on the mean of the speed cubed and the standard deviation of the speed cubed is shown in Fig. 6. As with any Weibull function, the coefficient of variation depends only on the shape parameter, in this case the shape parameter for the speed cubed distribution ($c/3$). Table 2 shows that the standard deviation of the cube of the speed can

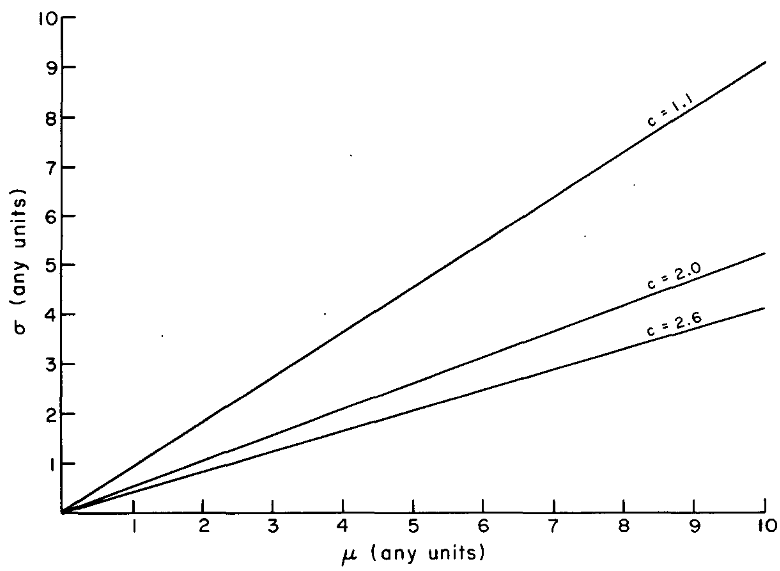

FIG. 5. The standard deviation of a Weibull distribution as a function of its mean for different values of the shape parameter c.

TABLE 1. Skewness of Weibull distributions.

c	$\sqrt{\beta_1}$
1.2	1.52
1.4	1.20
1.6	0.96
1.8	0.78
2.0	0.63
2.2	0.51
2.4	0.40
2.6	0.32
2.8	0.24

be expected to be larger than its mean but by an amount that rapidly decreases with increasing values of the shape parameter of the wind speed distribution.

Fig. 7 shows the frequency distribution for the speed cubed for a range of shape parameters assuming the mean wind speed is 5 (any units). Fig. 8 illustrates the effect of changes in the mean wind speed for a constant shape parameter ($c=2$). Each figure is divided into three sections. Section I has low values of the cube of the wind speed ($<3.6^3$). Section II has the intermediate levels between 3.6^3 and 8.0^3, and Section III is the tail of the distribution.

Variations in both the shape parameter c and the mean speed affect the percentage of the speed cubed (and therefore power density) represented by these three sections. Section I is relatively insensitive to changes in the value of c; however, increasing mean speeds substantially reduce the percentage of the speed cubed in this section. In Section II the area is increased by increases in the value of c, and its centroid is shifted toward higher values of speed cubed by increases in the mean wind speed. Finally, in the tail of the distribution (Section III) the area is decreased by increasing the shape parameter and decreasing the mean wind speed.

TABLE 2. The coefficient of variation for the speed cubed for different values of the shape parameter for the wind speed distribution.

c	σ/μ
1.1	3.65
2.0	1.55
2.6	1.53

Although evaluation of a wind power site will be affected by the percentage of power density represented by each section, generally the sites with the lower values of the shape parameter and higher mean wind speeds will have the highest total mean power density.

The cumulative distribution function for the speed cubed has a particularly neat form

$$F_Y(y)=1-\exp(-ay^{c/3}). \tag{14}$$

The power duration curve which is routinely used in wind power studies is just the function $1-F_Y(y)$ plotted with the ordinate and the abscissa reversed (Fig. 9).

5. Usable power density

The total power in the wind cannot be extracted by an aerogenerator. For example, the NASA 100 kW Plumbrook unit (Justus *et al.*, 1976) begins generating electricity at its cut-in speed of $V_0=3.6$ m s^{-1}. It has a rated speed of $V_1=8.0$ m s^{-1} at which the maximum possible power (100 kW) is generated. The unit must be furled (shut down) at wind speeds greater than V

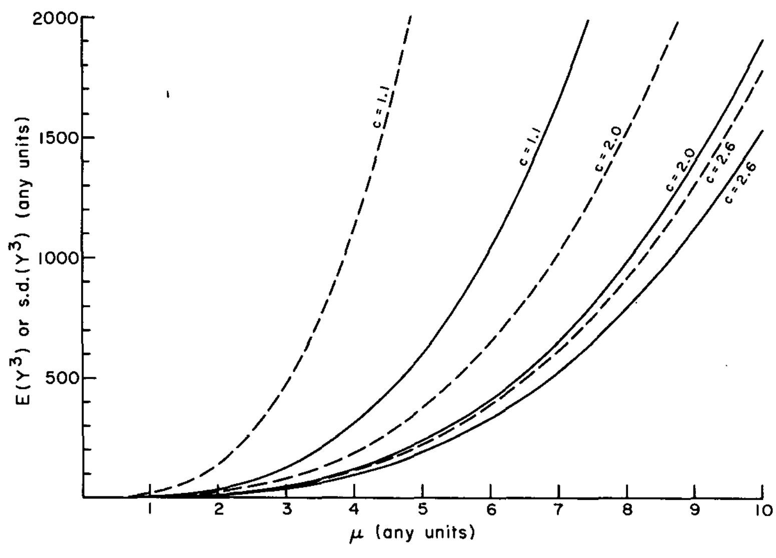

FIG. 6. The relationship between the mean wind speed and the mean (solid) and standard deviation (dashed) of the speed cubed for various values of the shape parameter.

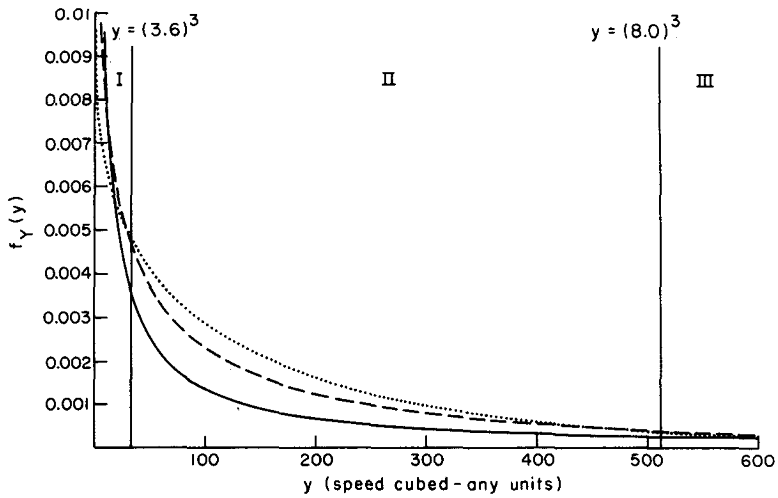

FIG. 7. The probability density functions for the cube of the wind speed when the wind speed itself follows a Weibull distribution with a mean of 5 (any units). The shape parameter c is 1.1 (solid), 2.0 (dashed) and 2.6 (dotted).

$=26.8$ m s^{-1}, so winds greater than this speed generate no power.

Statistics on what can be called usable power are important in engineering studies; however, they are more difficult to compute than total power density statistics. Justus *et al.* (1976) were the first to include the effects of the cut-in, rated and furling speeds in an extensive study. They computed mean potential output power density values based on the complete generator power output function for both NASA's 100 kW aerogenerator and a hypothetical 1 MW aerogenerator. Usable power density which takes into account only the cut-in, rated and furling speeds common to broad classes of aerogenerators is the least upper bound for potential output.

With the Weibull model for the cube of the wind speed, usable power density estimates involve finding the mean of a Weibull distribution doubly truncated between the cube of the cut-in speed (V_0) and the cube of the rated speed (V_1), i.e.,

$$f_{Y'}(y')=\frac{(ac/3)(y')^{(c/3)-1}\exp[-a(y')^{c/3}]}{\exp[-a(V_0)^c]-\exp[-a(V_1)^c]},\qquad V_0^3\leqslant y'\leqslant V_1^3. \tag{15}$$

Numerical integration techniques must be used to find the expectation of Y' for this probability density function. The standard deviation of the usable power density can also be computed. Again, this involves numerical integration to compute the expectation of the second moment of Eq. (15) about the value of the mean usable speed cubed.

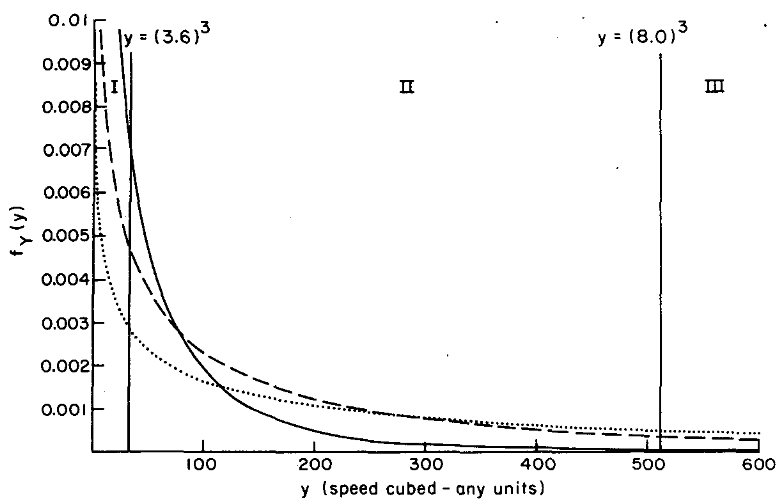

FIG. 8. The probability density functions for the cube of the wind speed when the wind speed itself follows a Weibull distribution with shape parameter equal to 2.0. The mean wind speed (any units) is 3.0 (solid), 5.0 (dashed) and 7.0 (dotted).

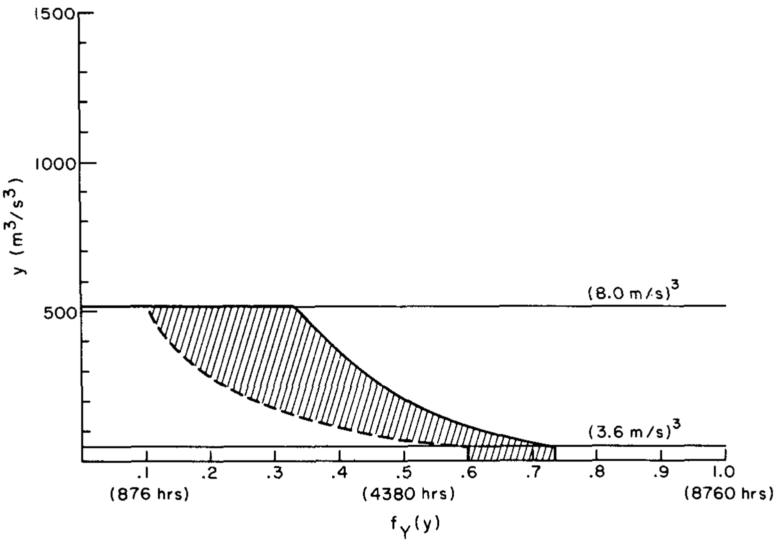

FIG. 9. Annual theoretical (Weibull) power duration curves for the Communications Station (solid) and Cannery Mountain (dashed). The hatched area is proportional to the difference in the power output for a 100 kW aerogenerator at each location.

Usable power should not be computed from the standard wind speed frequency tables not only because of the small number of frequency classes but also because the cut-in, rated and furling speeds can be expceted to fall within the available frequency classes. However, it is possible to compare the usable power at different sites using power duration curves (Golding, 1955). This technique is shown in Fig. 9.

6. The mean power density during generating hours

When an operating aerogenerator is connected to an electrical power grid, the average amount of power produced and its variability may be of some interest. For this purpose it is possible to define yet another type of power density, the power density during generating hours. The mean power density during generating hours is simply the mean of the usable power density conditioned on there being some usable power generated. Both the mean and the standard deviation for this type of power density can be computed by excluding the hours with zero contribution to the usable power density and then using the techniques described in Section 5.

The mean speed cubed during generating hours is also an indication of the amount of time that the aerogenerator will be operating since the ratio of the mean usable speed cubed to the mean speed cubed during generating hours is just the percentage of time the aerogenerator is operating.

7. Calculations

In the previous uses of the Weibull distribution on surface wind speed data, the data were generally obtained from airport locations or population centers where the terrain is relatively flat and homogeneous. Good wind power sites will often be in rugged terrain (Davidson *et al.*, 1964) where the wind distribution may be more heterogeneous.

Several wind-measuring stations have been maintained along the Oregon coast and near the Columbia Gorge. This program has been described by Hewson (1975). Data from three such stations in very different types of rough terrain are used in this study. Two of them, the Yaquina Head Communications Station and the Cannery Mountain Station, are near the coast.

TABLE 3. Comparison of the mean of the speed cubed with estimates of its value using just the skewness from the Weibull model.

Station	Mean (m s^{-1})	Standard deviation (m s^{-1})	$E(X^3)$ (m^3 s^{-3})	$\hat{c}$	$\sqrt{\beta_1}$	$\widehat{E(X^3)}$ (m^3 s^{-3})	Error (%)
Cannery Mountain	4.36	2.6	188.7	1.8	0.78	188.8	+0.05
Communication Station	6.68	4.58	842.8	1.5	1.08	822.2	−2.5
KCIV	6.47	3.2	488.3	2.2	0.51	486.3	−0.05

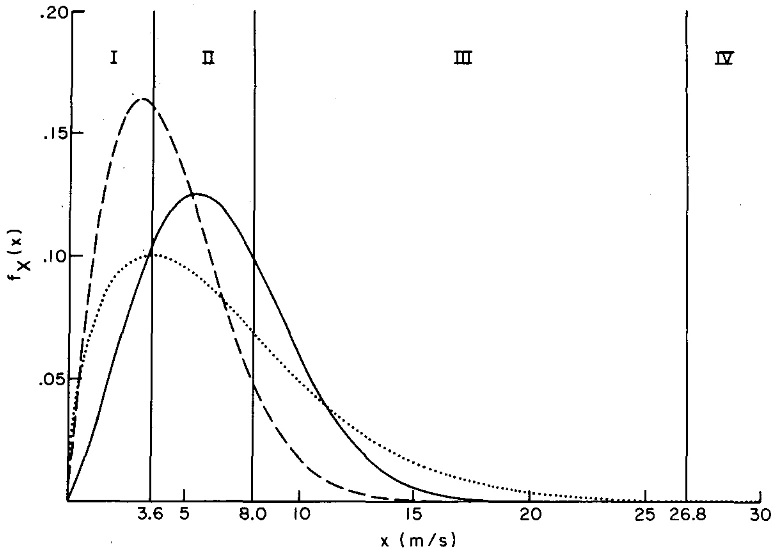

FIG. 10. Weibull model wind speed distributions at KCIV (solid), Cannery Mountain (dashed), and the Communications Station (dotted).

Yaquina Head juts out into the Pacific Ocean 5 km north of Newport, Ore., and the Communications Station is at an elevation of 114 m on the highest point of the head. The Cannery Mountain Station was located in the coastal mountains about 8 km from the coast just south of the Siletz River, at an elevation of 325 m. The third site, an inland site, is the KCIV radio station at 989 m MSL on the Columbia Hills about 11 km north of The Dalles, Ore.

The period of record was November 1971 to October 1973 at the Cannery Mountain site, January 1973 to May 1975 at the Communications Station and June 1974 to May 1975 at KCIV. The wind data were recorded on strip charts and reduced by hand to 1 h averages. Rather than utilizing all the available data as is usually done in wind power studies, this data base was reduced to sample sizes that could be handled in the statistical interactive programing system (SIPS) at Oregon State University by taking the hourly observation for every day at the Cannery Mountain site,[3] every seventh day at the Communications Station and every other day at KCIV. This resulted in 6094, 2818 and 6584 hourly observations, respectively. From these data sets, the gross statistics and frequency tables were computed for both the wind speed and the cube of the wind speed. These samples resulted in total mean power density estimates for the Communications Station and KCIV which were very close to those previously obtained by cubing all the wind speed observations.

In Table 3 the mean and standard deviations are the sample mean and standard deviations for each station. The estimate $\hat{c}$ of the shape parameter comes from Fig. 4, the estimate of the skewness $\sqrt{\beta_1}$ is from Table 1 and the estimate of the speed cubed is computed using Eq. (4). The error in the wind speed cubed estimates for these three examples is 2.5% or less. If it is assumed that $c\equiv 2.0$ ($\sqrt{\beta_1}=0.63$) as suggested by Court (1975, personal communication), then the percentage errors is -7.6% at the Communication Station, -3.3% at Cannery Mountain but less than $\frac{1}{2}\%$ at KCIV.

The Weibull distributions for these three sites are shown in Figs. 10 and 11. A 100 kW aerogenerator with a cut-in speed of 3.6 m s^{-1} and a furling speed of 26.8 m s^{-1} will not be operated if the wind speed is in either Sections I or IV in these figures. If the wind speed is in Section II, the power generated will depend on the wind speed; however, in Section III, the generated power level will be constant. A site such as the Communications Station with its high mean wind speed and low shape parameter has a large amount of wasted power density at the extremes.

It is also possible to compute the mean and the standard deviation using Eqs. (12) and (13); the results are in Table 4. The percentage error is larger but at least it is possible to estimate the dispersion about the mean. The Weibull model's lack of fit at these stations is probably at the extreme high wind speed end of the distribution. If it is again assumed that $c\equiv 2.0$ as suggested by Court (1975, personal communication), the errors in the estimates of these means are 33, 16 and -6% at the Communication Station, Cannery Mountain and KCIV, respectively.

There is a very great difference between the concepts of total mean power density, mean usable power density and mean power density during generating hours as is shown in Table 5. Judged on the basis of their total mean power density levels, the Yaquina Head Communication Station has $4\frac{1}{2}$ times more power than the Cannery Mountain site; yet, in terms of the mean usable

[3] The record at Cannery Mountain is broken in several places.

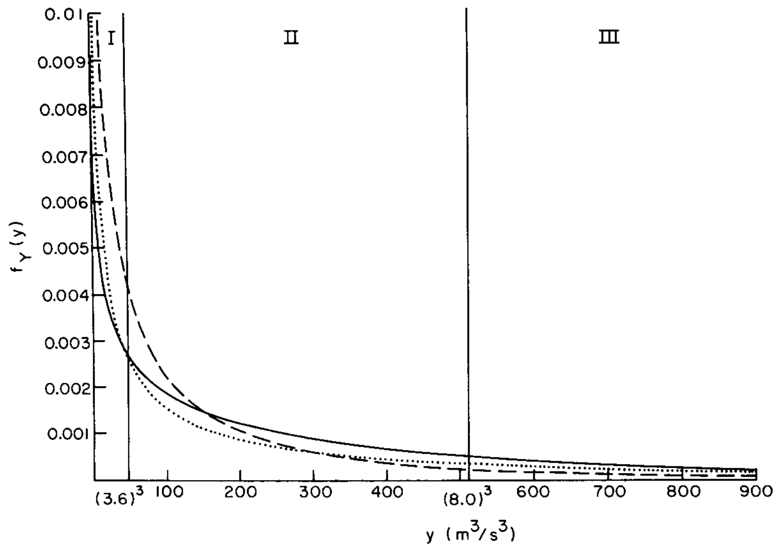

FIG. 11. As in Fig. 10 except for Weibull model wind speed cubed distributions.

power density for a 100 kW aerogenerator, the Communications Station has only twice the power. Then again, KCIV has a moderate speed cubed value, but its mean usable power density is greater than even that of the Communications Station. This is because its annual mean wind speed is about the same as the Communication Stations' but its shape parameter is much larger. The errors in this calculation of the mean and standard deviation of the usable speed cubed were each less than 2%. Had it been assumed that the shape parameter was identically 2.0 (i.e., the distribution is Rayleigh) as was recommended by Baynes (1974), then the errors in the mean usable speed cubed would have ranged from about 9% at the Communication Station to about −3.5% at KCIV.

The errors mentioned in the above analysis are not exact because the Weibull parameters have been estimated; furthermore, the difference between the mean usable power estimates for the Weibull model and the Rayleigh model may prove to be only academic. However, these examples illustrate why the Weibull model promises to be a tool which is useful even for wind power sites in rugged terrain and which provides wind power analysts with more information than they have had before.

8. Conclusions

As wind power develops into a viable alternative source of energy, the basic wind power climatology studies of Reed (1975) and Barber *et al.* (1977) will be expanded to include more than just total mean wind power density estimates. The most accurate method for computing the total mean wind power density, aside from using the wind observations themselves, is to use the general relationship between the expectation of the third noncentral moment and the mean, standard deviation and skewness.

A more complete evaluation of wind power sites will require some specification of the variance of the total wind power density and estimates of usable power densities for the different classes of aerogenerators which may be installed at a site. The mean and standard deviation of the wind speed are the minimum statistics necessary for this purpose.

The Weibull model is a very useful tool for wind power analysis with the following advantages:

1) It allows satisfactory estimates of the skewness of the wind speed distribution.

2) If the Weibull model for the wind speed distribution has a shape parameter c then the distribu-

TABLE 4. Estimates of the mean and the standard deviation of the speed cubed assuming the Weibull model is applicable.

Station	$E(X^3)$ ($m^3\,s^{-3}$)	$\widehat{E(X^3)}$ ($m^3\,s^{-3}$)	Error (%)	Standard deviation ($m^3\,s^{-3}$)	Estimated standard deviation ($m^3\,s^{-3}$)	Error (%)
Cannery Mountain	188.7	177.3	−6.0	377	312	−17
Communication Station	842.8	810.3	−3.9	2018	1812	−10
KCIV	488.3	473.1	−3.1	771	656	−15

TABLE 5. Comparison of the means and standard deviations for the total speed cubed, the usable speed cubed and the speed cubed during generating hours.

Station	Total speed cubed Mean ($m^3 s^{-3}$)	Total speed cubed Standard deviation ($m^3 s^{-3}$)	Usable speed cubed Mean ($m^3 s^{-3}$)	Usable speed cubed Standard deviation ($m^3 s^{-3}$)	Speed cubed during generating hours Mean ($m^3 s^{-3}$)	Speed cubed during generating hours Standard deviation ($m^3 s^{-3}$)
Cannery Mountain	189	312	128	168	227	145
Communication Station	843	1812	246	218	347	173
KCIV	488	656	264	203	326	167

tion of the speed cubed also follows a Weibull distribution. If it has a shape parameter of $c/3$, the second parameter is unchanged.

3) The Weibull model for the wind speed frequency distribution makes it possible to simply estimate both the total mean wind power density and the standard deviation of the total wind power density.

4) The wind power sites with the greatest total power density will have the highest mean wind speed and the lowest values of the Weibull shape parameter for the wind speed distribution.

5) In general, the standard deviation of the total power density generated from the wind will be larger than its mean, and the Weibull model makes it easy to estimate the coefficient of variation. High values of the shape parameter of the wind speed distribution result in the lowest coefficient of variation for the total power density.

6) The Weibull model facilitates the computation of the mean and standard deviation of the usable power and the wind power density during hours when the aerogenerator is operating.

Acknowledgments. The author is indebted to Dr. Arnold Court whose stimulating correspondence first interested him in this line of inquiry. The assistance of Dr. David Faulkenberry of the Department of Statistics and the faculty of the Department of Atmospheric Sciences in reviewing the manuscript is greatly appreciated. The cogent comments of the reviewer for this journal have lead to greater clarity in the final version of this paper—such a service is of inestimable value. This research was supported by Grant GZ-3649 from the National Science Foundation.

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