SOME COMMON INDEXES OF GROUP DIVERSITY: UPPER BOUNDARIES[1,2]

ANTONIO SOLANAS

Department of Behavioral Sciences Methods, Faculty of Psychology, University of Barcelona

Institute for Research in Brain, Cognition, and Behavior (IR3C), University of Barcelona

REJINA M. SELVAM

Department of Behavioral Sciences Methods, Faculty of Psychology, University of Barcelona

Institute for Research in Brain, Cognition, and Behavior (IR3C), University of Barcelona

JOSÉ NAVARRO

Department of Social Psychology, Faculty of Psychology, University of Barcelona

DAVID LEIVA

Department of Behavioral Sciences Methods, Faculty of Psychology, University of Barcelona

Institute for Research in Brain, Cognition, and Behavior (IR3C), University of Barcelona

[1] Address correspondence to Antonio Solanas, Departament de Metodologia de les Ciències del Comportament, Facultat de Psicologia, Universitat de Barcelona, Passeig de la Vall d'Hebron, 171, 08035-Barcelona, Spain. Electronic mail may be sent to Antonio Solanas at: (antonio.solanas@ub.edu)

[2] This research was supported by the Spanish Ministry of Science and Innovation, grant *PSI2009-07076* and by the Generalitat of Catalonia's Agència de Gestió d'Ajuts Universitaris i de Recerca, grant 2009SGR1492.

Summary. —Workgroup diversity can be conceptualized as variety, separation, or disparity. Thus, the proper operationalization of diversity depends on how a diversity dimension has been defined. Analytically, the minimal diversity must be obtained when there are no differences on an attribute among the members of a group, however maximal diversity has a different shape for each conceptualization of diversity. Previous work on diversity indexes indicated maximum values for variety (e.g., Blau's index and Teachman's index), separation (e.g., standard deviation and mean Euclidean distance), and disparity (e.g., coefficient of variation and the Gini coefficient of concentration), although these maximum values are not valid for all group characteristics (i.e., group size and group size parity) and attribute scales (i.e., number of categories). We demonstrate analytically appropriate upper boundaries for conditional diversity determined by some specific group characteristics, avoiding the bias related to absolute diversity. This will allow applied researchers to make better interpretations regarding the relationship between group diversity and group outcomes.

Keywords
Group diversity, diversity indexes, operationalization of diversity

Diversity has been an important concept applied in various ways across fields like ecology (Solow & Polasky, 1994), demography (Pelled, 1996), information systems (Benbasat & Weber, 1996), sociology (Lieberson, 1969), economics (Dissart, 2003), and psychology (Betz & Fitzgerald, 1993). In organizational research, diversity has been prominent in studies using work group behavior to predict performance while work groups have become increasingly diverse on various attributes (Horwitz & Horwitz, 2007). Organizations are made up of groups of individuals and it is not appropriate to do group analyses without taking into account the differences or similarities among members and the compositional effects of groups (Dansereau, Alutto, & Yammarino, 1984; Kozlowski & Klein, 2000). To explore these differences in groups, specific diversity indexes have been used in many studies (Hambrick, Cho, & Chen, 1996; Jehn, Northcraft, & Neale, 1999; Pelled, Eisenhardt, & Xin, 1999; Stewart & Johnson, 2009).

A review of past diversity research shows that most scholars choose particular indexes according to methodological priority, theory, and familiarity of the indices (Blau, 1977; Gibbs & Martin, 1962; Roberson, Sturman, & Simons, 2007). Some researchers have suggested that theoretical refinement of the conceptualization of diversity is necessary before selection of an index (Tsui & Gutek, 1999; Williams & O'Reilly, 1998). For example, Harrison and Klein (2007) proposed organizing indexes according to three types of diversity: separation, variety, and disparity. They defined diversity as the distribution of differences among the members of a unit with respect to a common attribute. Consequently, separation, variety, and disparity are respectively understood as differences in attitude or position, differences in categorical characteristics, and differences in power or status hierarchy. Quantifying diversity in these ways is expected to be useful to reflect the relationship between dissimilarity and team processes and performance. For example, as separation among group members decreases, trust and cooperation within groups is shown to improve

(Edmonson & Roloff, 2009; Ely & Thomas, 2001; Locke & Horowitz, 1990). In comparison, increased variety among group members avoids conformity and groupthink processes (Janis, 1972) and increases creativity within groups (Austin, 2003). Lastly, moderate disparity tends to lead to conformity to group norms among members (Philips & Zuckerman, 2001).

The most commonly used indexes in diversity research to operationalize these concepts are Teachman's index, Blau's index, coefficient of variation, Gini coefficient, standard deviation, and mean Euclidean distance. They meet most of the essential statistical prerequisites for analyzing within-group variance. Most of them are computationally simple and allow straightforward testing of effects related to diversity. Although these indexes have shortcomings, past research has indicated their suitability and consistency for several kinds of analyses (Barrick, Stewart, Neubert, & Mount, 1998; Conway & Schaller, 1998; Thomas, 1999). As examples of drawbacks, mainly with reference to group diversity research, the standard deviation and mean Euclidean distance are not scale invariant and are sensitive to extreme values of distributions (Allison, 1978; Lele, 1993). And Blau's and Teachman's indexes cannot be compared across different variables in cases where each variable has a distinct total number of categories.

In addition to these limitations, some studies show that biases in the operationalization for explaining group variances must be corrected, as the indexes are sensitive to differences in group size (Biemann & Kearney, 2010; Martin & Gray, 1971). Researchers have proposed simple transformations with respect to n to achieve desired bounds (Bedeian & Mossholder, 2000). However, the interpretation of index values may differ greatly with different group sizes (Allison, 1978). For instance, when estimating the sample standard deviation, the use of $n-1$ instead of n for the sample variance rectifies some but not all of the bias, as the bias depends on the particular distribution and it is not possible to estimate an unbiased value for all population distributions (Kozlowski & Hattrup, 1992).

Although some measures that produce size-dependent upper bounds are found to be desirable in demographic research, it is not always preferred in other types of diversity research (Ray & Singer, 1973).

There are evidences of other operationalization biases showing mixed findings due to inconsistency in the diversity conceptualization in relation to theories (e.g., social categorization theory and similarity-attraction theory). Recently, meta-analytical studies have shown inconclusive results concerning the relationship between variety and team performance (Bell, 2007; Bell et al., 2011). Given that it is improbable for most field studies to include several teams in which each member is from a different category (e.g., race), the authors suggest that minimum and moderate levels of variety may have mimicked the separation conceptualization of diversity. Additionally, they also proposed that future research is needed to explore the effects of sampling and range restriction on diversity conceptualizations. Note that if the amount of group members, n, divided by the number of categories, k, is not equal to an integer number, maximum variety cannot be obtained. It is likely that if meta-analytical studies combine variety measurement for different number of categories and group sizes, relationships between diversity and performance may be partially diminished or practically unseen. Hence, Byrne's (1971) similarity-attraction paradigm, which states that more similarity among team members results in higher productivity compared to more diverse teams, may be unlikely to be verified.

The current article shows that the maximum value of a group diversity measurement is a function of the group size and the distribution of members within a group across the respective attributes. The main purpose is to obtain proper upper boundaries for each of the commonly used indexes of diversity for all conditions featuring group size characteristics within the concept of the three diversity types. Following Harrison and Sin's (2006) suggestion to normalize the diversity indexes, which reduces the inflating effects on group size, a normalized range for the discrete random variables was obtained. These results are useful to applied researchers interested in comparing index values with respect to suitable maximum boundaries. The normalization is also potentially useful for the analysis of group outcomes and group diversity at group level of analysis. Complying with Harrison and Klein's (2007) conceptualization of diversity as separation, variety, and disparity, the succeeding sections discuss the specific conditions and issues related to computing the boundaries for each of the diversity indexes and show the exact values for each of the diversity types.

Conditional Maximum For Diversity Indexes

In the following paragraphs it is shown that it is essential to analyze the case where the condition of absolute diversity cannot be accomplished. Most importantly, it is considered to be useful to predict group performance and to test the similarity-attraction paradigm. In other words, we propose to distinguish between absolute and conditional diversity. All diversity indexes proposed have been referred to the absolute diversity, which can be only reached under some conditions. On the contrary, the maximum value for conditional diversity is conditioned on some of the characteristics of groups or teams (e.g., the parity of group size or the ratio between the amount of group members and the number of categories). Note that considering the conditional maximum values for the different diversity indexes could improve the results obtained when the similarity-attraction paradigm is tested (Bell et al., 2011). Moreover, some empirical support for the usefulness of these conditional indexes has been found for diversity as separation (Andrés, Salafranca, & Solanas, 2011). However, as regards variety indices, the better predictor of a dependent variable of interest (e. g., group outcome) may depend on the specific data being analyzed (Budescu & Budescu, 2012).

Variety Indexes

In research attempting to optimally quantify diversity, the conceptualization of diversity types has not been consistently satisfied in derivations of the diversity indexes (Harrison & Sin, 2006). The commonly used indexes for within-group variety have always been computed based on the proportion of differences within groups that comply with the basic axioms, for example, *diversity should be maximized when all group members' characteristics are present in equal proportions*.

Diversity as variety conceptualizes categorical differences across the relevant characteristics between group members (Carpenter, 2002; Miner, Haunschild, & Schwab, 2003). For instance, a research group may consist of members with different categories such as fellows, associates, and project leaders. Variety is commonly measured by both Blau's index, also known as the Hirschman-Herfindal index (Hirschman, 1964), and the entropy index, well-known as Teachman's index (Teachman, 1980), which are linearly correlated (McDonald & Dimmick, 2003).

Blau's index, denoted here by *B,* is defined as $1-\sum_{i=1}^{k} p_i^2,$ where p_i corresponds to the proportion of group members in *i*th category and *k* denotes the number of categories for an attribute of interest. This index quantifies the probability that two members randomly selected from a population will be in different categories if the population size is infinite or if the sampling is carried out with replacement. Hence, if *B* equals its minimum value (i.e., zero), all members of the group are classified in the same category and there is no variety. In contrast, the higher *B* is, the more dispersed group members are over the categories. The maximum value for this index is achieved in the condition where members of a group are equally distributed among all categories (i.e., $p_1 = p_2 = \ldots = p_k$), that is, if and only if $n = mk$, where group size, *n*, is equal to the number of categories multiplied by a positive integer, *m*. Thus, the maximum value is

$$B_{max} = 1 - \sum_{i=1}^{k} \left(\frac{m}{n}\right)^2 = 1 - k\left(\frac{m}{mk}\right)^2 = 1 - \frac{1}{k} = \frac{k-1}{k}.$$

Note that the maximum value of B does not depend on n. Further, as k tends to infinity the maximum value of B approaches unity. For this reason it has been suggested that Blau's values are not validly comparable if the number of categories is not identical across diversity variables (Harrison & Klein, 2007), because the maximum value is a function of k. Nevertheless, researchers have asserted that comparisons between variables with a dissimilar number of categories still make sense, as long as larger number of categories contributes to greater diversity (Agresti & Agresti, 1978). However, the index B can be normalized by dividing it by its maximum. This controls for the number of categories, and gives the Index of Qualitative Variation (IQV; Agresti & Agresti, 1978). Blau's index and IQV can be used interchangeably when comparing variables with the same number of categories because they are highly similar measures and only differ in scale.

In general, social researchers are interested in measuring variety for descriptive purposes or in measuring group heterogeneity to predict group performance (Goodwin, Burke, Wildman, & Salas, 2009). From an inferential perspective, it has been shown by Monte Carlo simulation that B underestimates the population degree of variety, at least if the population has a discrete uniform distribution (Biemann & Kearney, 2010) but not if there is random sampling. In fact, if Blau's index and IQV are multiplied by $n/(n-1)$, the estimators obtained are unbiased (Agresti & Agresti, 1978). It should be noted that social researchers may also be interested in studying the effects of group size on variables at the group level (e.g., group performance). Therefore, it will not always be possible to obtain an even distribution of group members among categories. For instance, if $k = 4$ and $n = 6$, group members cannot be uniformly distributed across categories.

Variety is dependent on the even distribution of members among categories rather than the number of individuals, that is, for $k = 4$ and $n = 4$, the same maximum value for Blau's index is obtained as for the case of $n = 8$ or $n = 12$. For a fixed number of categories, the maximum value for Blau's index will always be identical if $n = mk$. Suppose the following vectors represent the frequencies for $k = 6$ possible categories: (1, 1, 1, 1, 1, 2), (2, 2, 2, 2, 2, 3), and (3, 3, 3, 3, 3, 4). Their respective Blau's scores are .816, .828, and .831. It should be noted that individuals are distributed in such a way that the discrete uniform distribution is approximated as much as possible, given n. Hence, conditional diversity reaches its maximal representation.

This type of situation raises the issue of finding the maximum value for cases when the condition of equal distribution of members over all categories cannot be met, that is, when $n \neq mk$. Suppose $n = mk + a$, $0 \leq a \leq k{-}1$. Thus, it can be proven that the maximum value for the index B is as follows (all proofs are available from the authors on request):

$$B_{max} = \frac{(k-1)n^2 + a(a-k)}{kn^2},$$

where $a = n - k\ \text{int}\left[\frac{n}{k}\right]$. The maximum value of B depends on n and k. Also note that if n tends to infinity, B_{max} approaches $(k{-}1)/k$ and if $a = 0$, $B_{max} = (k{-}1)/k$. The analytical result obtained enables researchers to carry out suitable interpretations as it states the proper upper bound, independently of group size. Now, if $k = 3$ and $n = 4$, $B_{max} = .625$, which slightly differs from $(k{-}1)/k = .667$. Note that the index B can be normalized by dividing by B_{max}.

Continuing with diversity as variety, Teachman's index, *H*, is defined as $-\sum_{i=1}^{k} p_i \times \ln p_i$, where *k* and p_i respectively denote the number of categories and the proportion of group members in the *i*th category. The minimum value for *H* is equal to zero, meaning that there are no differences among group members for the attribute of interest. That is, apart from one of them, all proportions are equal to zero. According to Harrison and Klein (2007), the maximum value of *H* is equal to –ln(1/*k*), for simplicity here we express this maximum value as ln*k*. To obtain this maximum value it is supposed that $p_i = p_j$ for all categories, that is, $p_i = 1/k$. This assumption corresponds to the case in which entropy reaches its maximum. It is straightforward to show how this maximum value can be determined:

$$H_{max} = -\sum_{i=1}^{k} \frac{1}{k} \times \frac{\ln 1}{k} = -\frac{\ln 1}{k} = \ln k.$$

However, solving for the exact maximum value of *H* for the situation in which $n \neq mk$, $n = mk + a$, and $0 \leq a \leq k-1$:

$$H_{max} = -\frac{(kn - ka - na + a^2) \times \ln(n - a) + (na + ka - a^2) \times \ln(n - a + k) - kn \times \ln kn}{kn}.$$

It should be noted that if $a = 0$, the last expression is equal to ln*k*. Hence, the mathematical expression is valid for the condition in which data can be evenly distributed in all categories or even when it is impossible to obtain a uniform distribution of data due to the *n*/*k* ratio.

Disparity Indexes

Diversity as disparity assumes asymmetry and is defined as the difference between group members in terms of the resources each of them holds (Grusky, 1994). It reflects both the distances between group members and the dominance of those individuals that have higher amounts of the attribute. The main property of this diversity type is that the attribute at its maximum diversity will have a positively skewed distribution, with one member at the highest endpoint and others at the lowest (Harrison & Klein, 2007). The commonly-used indexes to capture these differences are the coefficient of variation and the Gini coefficient of concentration.

The coefficient of variation, V, is calculated by dividing the standard deviation by the mean and, as proposed by Bedeian and Mossholder (2000), it is an appropriate diversity index only for ratio scales. It is scale invariant but not location invariant. Allison (1978) pointed out that scale invariant indexes are appropriate as measures of inequality, specifically, diversity. V is lower and upper bounded, its minimum and maximum values being respectively equal to zero and $(n-1)^{1/2}$, where the maximum value is reached when all cases but one have zero values (Martin & Gray, 1971). As noted by Harrison and Sin (2006), the maximum V is achieved regardless of what the single non-zero value would be. Also, V is asymmetric, as its value decreases for the case where few are in the minimum level and more are in the maximum level, which is due to the mean in the denominator being larger in latter case compared to the former case. Therefore, the interpretation of V would be different from other diversity measures, as the value depends on the majority being at the bottom or top level, unlike other measures where their values remain the same for both cases.

V is often obtained for finite and discrete random variables and, moreover, for nonnegative variables in which there is not a minimum zero value. Therefore, it is required to obtain the maximum value for the coefficient V for the general case:

$$V_{max(n)} = \frac{\sqrt{n-1}(x_{max} - x_{min})}{(n-1)x_{min} + x_{max}},$$

where x_{min} and x_{max} respectively denote the minimum and maximum values for the random variables, considering all admissible values. It should be noted that, if $x_{min} = 0$, $V_{max(n)} = (n-1)^{1/2}$, which is a result previously shown (Martin & Gray, 1971). Therefore, the maximum value $(n-1)^{1/2}$ for variables having an origin at zero is not useful for discrete random variable in which x_{min} is greater than zero. The solution is to transform a discrete variable such as $x_{min} = 0$ by means of subtracting x_{min} from each admissible value of the random variable. Thus, the new boundaries will be 0 and $x_{max} - x_{min}$. However, the proper maximum value for V has been previously obtained if no such transformation is carried out. Similarly, it can be proved that, if $n-1$ is used instead of n for obtaining the standard deviation, it is then:

$$V_{max(n-1)} = \frac{\sqrt{n}(x_{max} - x_{min})}{(n-1)x_{min} + x_{max}}.$$

Note that, if $x_{min} = 0$, $V_{max(n-1)} = n^{1/2}$.

As regards the Gini coefficient, Δ, is defined as follows:

$$\Delta = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{n^2}.$$

It is a measure of inequality for quantitative variables and it is defined as the average of the absolute differences of all pairs of variate values in a sample, expressed in terms of units of the variate. This measure of mean difference is location invariant, but is not scale invariant. Consequently, it is difficult to compare variability in different variates unless the units happen to be identical. Therefore, the coefficient is not suitable for ratio scales, but can be obtained in interval scales. The Gini coefficient of concentration is defined as (Stuart & Ord, 1994)

$$G = \frac{\Delta}{2\bar{x}} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{n^2\bar{x}}.$$

Thus, Δ is converted into a scale invariant measure by dividing the coefficient of concentration by twice the arithmetic mean. Nevertheless, the coefficient G is not location invariant, which is due to the change in the arithmetic mean. Therefore, the coefficient G is only appropriate for attributes measured by ratio scales (Allison, 1978). G has lower and upper bounds, although its maximum value depends on n. Its minimum value is equal to zero and it occurs when all members of a group possess the same amount of attribute. With regards to its maximum value, it is obtained if only one individual possess the maximum level of an attribute and the others have the minimum level of that characteristic.

The practical implications need to be highlighted for the use of the coefficient G when its values are subject to limits, x_{min} and x_{max}. The literature on the Gini coefficient commonly assumes the limits of random variables that range between 0 and infinity, which satisfies the main constraint for the index to have non-negative values for the population (Cowell, 1995; Gastwirth, 1972; Krieger, 1979). It is important to note that for any other limits, the coefficient G fails to satisfy the principle of normalization. There are many occasions when economic variables do not range from 0 to infinity (e.g., salary income in an organization is based on a fixed minimum and it can never have a statutory minimum of zero). Thus, in such studies, the traditional index of G may not yield a correct value. To illustrate this, suppose the inequality of income distribution for a group of 10 individuals with an annual income differing from 25 to 60, in thousands of dollars, (where zero is an inadmissible value) is to be measured. For the case A, represented by the vector (25, 25, 25, 25, 25, 25, 25, 25, 25, 60), the Gini coefficient has a value of .11 when there is maximal inequality (i.e., all but one member possessing the minimum income). It is to be noted that the obtained value is very far from the upper bound, which is equal to $1-1/n$ (Harrison & Klein, 2007). Apart from this issue, the index G fails to depict proper values for a distribution having half of the population with the minimum value and the other half with the maximum value. Thus, suppose the case

B for which the incomes for a group of ten individuals are represented by the vector (25, 25, 25, 25, 25, 60, 60, 60, 60, 60). Note *G* equals .21, which does not fit the pattern of a disparity measure as the obtained value is higher than the previous value for case A. Therefore, it is important to address the case when the variable has a minimum value greater than zero and the population is equally distributed to both the extremes. The coefficient *G* does not behave as a measure of disparity but perhaps could be used as a measure of separation according to the definition of Harrison and Klein (2007).

Independently of the previous concerns regarding the Gini coefficient of concentration, if this index was a disparity measure, its maximum value for this index would be as follows:

$$G_{max} = \frac{x_{max}}{(n-1)x_{min} + x_{max}} - \frac{1}{n}.$$

It can be seen that the maximum value of *G* is dependent on the variate minimum and maximum values, and also depends on group size. Note that the maximum value for the Gini coefficient of concentration does not fit the previous result for the case B but for case A, which reinforces again that this index is not a disparity measure if $x_{\min} > 0$. Also note that, if $x_{\min} = 0$, the previous expression reduces to

$$G_{\max} = 1 - \frac{1}{n}.$$

If $x_{\min} > 0$, the latter upper bound may be obtained by means of a translation of the scale, guaranteeing that Gini coefficient of concentration will behave as a disparity measure.

The abovementioned upper bounds for diversity as disparity are only useful if minimum and maximum values for the scale of interest are known, as it can be the case when measuring status hierarchy (e.g., one person dominates the other members of a group, but the remaining people show equalitarian relationships) and prestige in groups. On the contrary, researchers should also consider the upper bounds for cases in which the maximum value is unknown, but the minimum value for the random variable of interest is equal to zero, to guarantee that the indexes quantify diversity as disparity.

Separation Indexes

Social researchers are increasingly focusing their attention on the relationship between psychological characteristics, such as personality attributes, and team or group performance (e.g., Tett & Burnett, 2003). Nevertheless, there is no general consensus about how to compose the lower level units (i.e., group members' characteristics) to establish the measurement of the higher level construct (i.e., composites at group level). This fact may explain why some researchers simultaneously use averages, minimum values, and variances of psychological constructs for predicting group effectiveness (e.g., Barrick, et al., 1998; Halfhill, Nielsen, & Sundstrom, 2008), at least for exploring possible relationships. Chan's (1998) typology distinguishes among five different composition models, which are defined by an explicit functional relationship specified between constructs at different levels. Thus, specifying the proper composition model is a requirement to choose adequate indexes to carry out multilevel analysis. For personality traits, the dispersion models, which correspond to diversity as separation, are appropriate compositional strategies, as supplementary and complementary group members' characteristics can be associated to team performance (Tett & Burnett, 2003).

Diversity as separation conceptualizes the member's differences in position or disagreement in opinion toward a global attribute (Barrick et al., 1998; Barsade, Ward, Turner, & Sonnenfeld, 2000). For instance, differences in interpersonal perceptions are used to measure cohesiveness in groups. The widely used measures of diversity as separation are standard deviation (*SD*) and mean Euclidean distance (*D*), which are maximized under the extreme bipolar distribution (Harrison & Sin, 2006). Hence, although the minimum value for

SD and *D* always equals zero, maximum separation is realized only when group member values are distributed such as *n*/2 respectively score the lower and upper bound of a random variable. It has been pointed out that it is difficult to determine the maximum possible value of the standard deviation statistic, because real values on the measurement scale are scale-specific, and it limits the comparison of standard deviation statistics from different empirical distributions (Roberson et al., 2007). Thus, because the lack of clear anchor points constrains proper interpretations and conclusions, it is useful to obtain the exact upper boundary for the standard deviation statistic. As diversity is obtained for descriptive purposes, the standard deviation is computed as follows (see Harrison & Klein, 2007; Harrison & Sin, 2006):

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}.$$

The maximum value for *SD* is equal to $(x_{max}-x_{min})/2$ if group size parity is even. Now suppose $x_{min} = 0$, $x_{max} = 48$, as it actually is in NEO-FFI personality inventory, and $n = 3$. Note that the maximum value for *SD* will be equal to 22.63, which is lower than the maximum value 24 for an even group size. In absolute terms, this result is coherent as maximum diversity can be only achieved for even group size. However, several limitations derive from this example. Firstly, although the maximum value of the standard deviation is the same for groups of even size, it does not remain for groups of odd size. Hence, difference metrics do not guarantee that identical values for different group sizes represent indistinguishable distributional patterns. Secondly, according to Biemann and Kearney (2009), although standard deviation's underestimation of diversity as separation is insignificant as group size increases, it is expected to have an influence on the results of studies examining samples for small groups of different sizes. Also note that mixing different group sizes while studying how standard deviation can predict group performance may distort some relationships, as a distinct metric is used for groups of odd size.

Therefore, the appropriate solution depends on group size parity, that is, whether *n* is odd or even. For even *n*, the maximum value obtained of the index is that indicated by Harrison and Klein (2007). It is to be noted that for *n* odd cases,

$$SD_{max(odd)} = \sqrt{\frac{n^2-1}{n^2}}\frac{x_{max}-x_{min}}{2},$$

where x_{min} and x_{max} respectively denote the minimum and maximum value of the random variable. Although distinguishing between even and odd group size is not critical to obtain the maximum value of the standard deviation, it is suggested to calculate the proper maximum value for $n = 3$ by the expression for odd group size. Note standard deviation is location invariant but not scale invariant (Harrison & Klein, 2007). Also note that knowing the proper maximum value for even or odd group size allows deriving a valid normalized index ranging in value between zero and unity. Additionally, from these values for the standard deviation, the range for the variance can be easily obtained.

D is defined as the square root of the mean squared differences between the *i*th member and all others in the group. Although it is not scale invariant, *D* remains unchanged by location transformations. Subsequently, this coefficient has not been recommended for use with ratio scales, but only interval scales (Harrison & Klein, 2007). The minimum value is zero when all unit members have the same value for the attribute measured, that is, there is no diversity. Maximum value depends on the group size parity, approaching 1 as the number of group members increases infinitely. As such, it is also important to calculate the maximum value based on the parity of group size. According to Harrison and Klein (2007), *D* is obtained as follows:

$$D = \frac{\sum_{i=1}^{n}\sqrt{\sum_{j=1}^{n}\left(x_i - x_j\right)^2 / n}}{n},$$

where, x_i and x_j are members' scores in a group. It is apparent that D is based on a quadratic measure of dispersion, specifically identified by the sum of squares in the numerator. After some algebraic manipulation, the index can be expressed as

$$D = \frac{1}{\sqrt{n^3}} \sum_{i=1}^{n} \sqrt{\sum_{j=1}^{n} (x_i - x_j)^2}.$$

Following D as a measure of diversity as separation, its maximum value for an even group size is:

$$D_{max(even)} = \frac{x_{max} - x_{min}}{\sqrt{2}}.$$

The above obtained maximum value is the same as shown in Harrison and Klein (2007). Nevertheless, as previously mentioned, the maximum value depends on group size parity. For n odd, the maximum value of D is given by:

$$D_{max(odd)} = \frac{\sqrt{(n^2 - 1)(n + 1)} + \sqrt{(n^2 - 1)(n - 1)}}{\sqrt{4n^3}} \frac{x_{max} - x_{min}}{\sqrt{2}}.$$

There is no general agreement about defining D. Recently, another index for measuring D has been proposed (Biemann & Kearney, 2010):

$$MED_n = \frac{2}{n^2 - n} \sum_{i=1}^{n} \sum_{j=i+1}^{n} |x_i - x_j|.$$

Note that the minimum value for MED_n is zero, as it will be obtained if $x_i = x_j$ for all i and j. Given that MED_n is an index of diversity understood as separation, its maximum value will depend on group size parity. Hence, we must obtain the maximum value for even and odd n. Regardless of whether n is even or odd, note that the problem reduces to obtain the maximum value for:

$$y = \sum_{i=1}^{n} \sum_{j=i+1}^{n} |x_i - x_j|.$$

It has been demonstrated that the maximum value for y is as follows (Andrés, Salafranca, & Solanas, 2011):

$$\max(y) = \begin{cases} \dfrac{n^2(x_{\max} - x_{\min})}{4} & \text{if n is even} \\ \\ \dfrac{(n^2 - 1)(x_{\max} - x_{\min})}{4} & \text{if n is odd} \end{cases}$$

Then, if n is even, the maximum value for MED_n equals

$$MED_{n\ max(even)} = \frac{n}{n-1}\frac{x_{max} - x_{min}}{2}.$$

And if n is odd,

$$MED_{n\ max(odd)} = \frac{n+1}{n}\frac{x_{max} - x_{min}}{2}.$$

From the previously obtained results, the normalized diversity indices can be computed by means of dividing each of the standard indices by its respective proposed maximum values (see Table 1).

INSERT TABLE 1 ABOUT HERE

Discussion

The concept of diversity has been commonly used in the analyses of work groups and organizations and mainly due to its substantial extensions for various studies in organizational research (Jackson, Joshi, & Erhardt, 2003). In order to measure how diverse the members of a group are, specific diversity indexes have been applied to examine the within-group differences. Although the increasing number of studies using these indexes have motivated researchers to explore their most important methodological and operational properties of these indexes (Roberson et al., 2007), there are still some operational issues unresolved.

Past findings on the most commonly used indices, namely, coefficient of variation, the Gini coefficient of concentration, Blau's index, Teachman's index, standard deviation, and mean Euclidean distance have presented developments and improvements in the values of these indices (Atkinson, 1970; Gibbs & Martin, 1962; McDonald & Dimmick, 2003). Some studies focus on the normalization of these indices to control for the comparison of samples with different group sizes (Champernowne, 1974). Normalization has been made starting from simple changes in the formulas, by correcting for $n-1$ for sample variance, up to introducing formulas like the index of qualitative variation (Agresti & Agresti, 1978).

Although lower and upper boundaries for most diversity indices are well-known (Harrison & Klein, 2007; Harrison & Sin, 2006), these analytical results are only useful for some specific conditions. For instance, the common upper boundaries for Blau's and Teachman's indices in the research literature assume that all data can be evenly distributed across categories. There is no doubt that the maximum uncertainty when measuring variety is only achieved if data are uniformly distributed among categories, but that specific distribution cannot be found in all studies. Similarly, we have shown that maximum values for two separation indices (i.e., standard deviation and mean Euclidean distance) depend on group size parity, while maximum values for two disparity indices rely on the range of measurement scales (i.e., minimum and maximum values of the random variable). Hence, the present study dealt with obtaining the general upper boundaries only for the most commonly used diversity indexes. The results are founded on the fact that maximal diversity is conditioned on some characteristics, as the ratio n/k and group size parity for variety and separation indexes, respectively. Additionally, indexes for measuring disparity have been shown to be properly applied only if the minimum value for the scales is equal to zero. Otherwise, these indexes could not properly work as measures of disparity.

Having derived general upper boundaries, researchers can compare the values obtained to their suitable maximum values and thus make proper conclusions for the specific conditions (e.g., group size and group size parity). That is, the corrections proposed for some common indexes of diversity enable social researchers to obtain comparable measures for several specific conditions. Thus, independently of groups characteristics (i.e., *n/k* ratio and group size parity), researchers will able to compare diversity for those groups for which the maximum absolute level of uncertainty cannot be reached. Additionally, applied researchers may find that normalized diversity indexes improve quantifications at the group level since they are expressed in the same metric. These normalized indexes allow social researchers to make proper comparisons among groups or organizations since these measures provide measurements of dispersion that can be applied to collectives of differing size (Martin & Gray, 1971). It should be noted that the present study was constrained to descriptive analysis and thus no conclusions are made about the statistical properties of the indexes as estimators. As regards to this point, a recent study has pointed out that the common diversity statistics are biased (Biemann & Kearney, 2010), although it refers to the inferential use of the indexes.

Computing the values for all presented indexes may be a tedious task. In order to facilitate this, an R package (*diversity_0.1*), which is available from the authors upon request (David Leiva at dleivaur@ub.edu), has been developed to compute both non-normalized and normalized indexes, as well as maximum values for all the statistics presented in this paper. The package can be run in the main operating systems like Windows, Linux, and MacOS. Once installed, the package can be loaded just by typing the following in the R Console: *require (diversity)*. The list of the functions included in the package as well as a brief manual can be accessed by means of the following command: *?diversity*.

Although the present study concerns with diversity as it can lead to better understanding of group dynamics and outcome, we have not dealt with some critical issues for which further research is needed. Firstly, when using diversity indices, researchers assume that identical values for the property of interest have similar consequences on group performance regardless of group members' differences on other characteristics. This drawback has been pointed out for diversity as variety (Harrison & Klein, 2007; Rushton, 2008) and several indexes have been proposed to capture possible faultlines, which are understood as subgroups based on one or more attributes, in groups (Shaw, 2004). However, the same concern can be extended to other measures of diversity. For instance, the standard deviation, an index of diversity as separation, is not useful to differentiate among distinct patterns or configurations (DeRue, Hollenbeck, Ilgen, & Feltz, 2010). Once again, it is assumed that identical values of similarity lead to analogous implications on group outcome. Secondly, although formative definitions of diversity constructs have been discouraged (Harrison & Klein, 2007; Harrison & Sin, 2006), the conditions for which reflective definitions would have theoretical sense need to be established. However, positive correlations among measures of diversity are not a sufficient criterion as it is also required that all properties have an effect on group dynamics or output in the same direction, apart from being the reflective definition founded on a diversity theory.

To sum up, Harrison and Klein (2007) have proposed to conceptualize diversity as variety, disparity, or separation and suggested using these commonly applied indexes to operationalize the concepts. Following these conceptualizations, the present article shows that it is important to consider the issue of group characteristics for discrete random distributions in order to obtain appropriate upper boundaries. Thus, we recommend using the limits proposed as the proper reference for interpreting index values.

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Table 1. Upper boundaries and normalized indexes for common diversity measures. All normalized indexes range from 0 to 1. The denominator for computing normalized indexes depends on parity of group size for separation indexes and if $n = mk$ or $n \neq mk$ for variety indexes.

Diversity Indexes	Standard Formula	Maximum value	Normalized Index
Variety indexes			
Blau's Index	$B = 1 - \sum_{i=1}^{k} p_i^2$	$B_{max} = \frac{n^2(k-1) + a(a-k)}{kn^2}$	$B_N = \frac{B}{B_{max}}$
Teachman's Index	$H = -\sum_{i=1}^{k} p_i \times \ln p_i$	$H_{max} = -\frac{1}{kn}\begin{pmatrix}(kn - ka - na + a^2) \times \ln(n-a) + \\ (na + ka - a^2) \times \ln(n - a + k) \\ -kn \times \ln kn\end{pmatrix}$	$H_N = \frac{H}{H_{max}}$
Separation indexes			
Standard deviation	$SD = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}}$	$SD_{max(even)} = \frac{x_{max} - x_{min}}{2}$ $SD_{max(odd)} = \sqrt{\frac{n^2 - 1}{n^2}} \frac{x_{max} - x_{min}}{2}$	$SD_N = \frac{SD}{SD_{max}}$
Mean Euclidean distance	$D = \frac{1}{\sqrt{n^3}} \sum_{i=1}^{n} \sqrt{\sum_{j=1}^{n} (x_i - x_j)^2}$	$D_{max(even)} = \frac{x_{max} - x_{min}}{\sqrt{2}}$ $D_{max(odd)} = \frac{\sqrt{(n^2-1)(n+1)} + \sqrt{(n^2-1)(n-1)}}{\sqrt{4n^3}} \frac{x_{max} - x_{min}}{\sqrt{2}}$	$D_N = \frac{D}{D_{max}}$
Disparity index			
Coefficient of variation	$V = \frac{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 / n}}{\overline{x}}$	$V_{max(n)} = \frac{\sqrt{n-1}(x_{max} - x_{min})}{(n-1)x_{min} + x_{max}}$	$V_N = \frac{V}{V_{max}}$