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SOME DETERMINISTIC RELIABILITY GROWTH CURVES FOR SOFTWARE ERROR DETECTION: APPROXIMATION AND MODELING ASPECTS

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Abstract: In the context of reliability engineering, the Gompertz curve (or deterministic curve model) is, for example, used to assess the reliability growth phenomenon of hardware products.

In this paper we study the one–sided Hausdorff approximation of the Heaviside step function $h_r(t)$ by deterministic curve model and find an expression for the error of the best approximation.

We propose a new transmuted deterministic software reliability model.

Some comparisons are made.

Key Words: Gompertz curve, one–sided Hausdorff approximation, Heaviside step function

1. Introduction

Software reliability is the probability of failure–free software operation for a specified period of time in a specified environment.

The Gompertz and logistic curves are still used in industry, because these curves are well fitted to the cumulative number of faults observed in existing software development processes.

Japanese software development companies prefer regression analysis based on deterministic functions such as Gompertz and Gompertz–type curves to estimate the number of residual faults (see, for instance [23]).

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For other results, see [5]–[6].

In the context of reliability engineering, the Gompertz curve is, for example, used to assess the reliability growth phenomenon of hardware products (see, [8]).

A residual–based approach for fault detection at rolling mills based on data–driven soft computing techniques, can be found in [10].

For other results, see [9].

Ohishi, Okamura and Dohi [23] formulate Gompertz software reliability model based on the following *deterministic curve model*:

$$M(t) = \omega a^{b^t}, \tag{1}$$

where $a, b \in (0, 1)$.

Satoh [6] and Satoh and Yamada [11] introduced a discrete Gompertz curve by discretizing the differential equations for the Gompertz curve and applied the discrete Gompertz curve to predict the number of detected software faults.

Yamada [5] constructed a model with the following mean value function

$$M(t) = \omega \left(a^{b^t} - a \right). \tag{2}$$

Definition 1. [24], [25] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f, g) = \max \left\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B|| \right\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t, x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

2. Main Results

We consider again the Gompertz software reliability model based on the following *deterministic curve model*, (see Ohishi, Okamura and Dohi [23]):

$$M_r(t) = a^{b^{(t-r)}}; \quad a = \frac{1}{2}; \quad M_r(r) = \frac{1}{2}. \tag{3}$$

We define the Heaviside step function $h_r(t)$ by

$$h_r(t) = \begin{cases} 0, & \text{if} \quad t < 0, \\ [0,1], & \text{if} \quad t = r, \\ 1, & \text{if} \quad t > r. \end{cases} \tag{4}$$

We study the Hausdorff approximation [24], [25] of the Heaviside step function $h_r(t)$ by deterministic curve of the form (3) and find an expression for the error of the best approximation.

The one–sided Hausdorff distance d between the function $h_r(t)$ and the function $M_r(t)$ satisfies the relation

$$M_r(r+d) = 1 - d \tag{5}$$

The following theorem gives upper and lower bounds for d

Theorem 1. The one–sided Hausdorff distance d between h_r and the curve $M_r(t)$ can be expressed in terms of the parameter b for any real $b \leq 0.35$ as follows:

$$d_l = \frac{1}{2.1(1 - 0.346574 \ln b)} < d < \frac{\ln(2.1(1 - 0.346574 \ln b))}{2.1(1 - 0.346574 \ln b)} = d_r. \tag{6}$$

Proof. We need to express d in terms of b, using (5).

Let us examine the function

$$F(d) = \left(\frac{1}{2}\right)^{b^d} - 1 + d. \tag{7}$$

From $F'(d) > 0$ we conclude that the function F is strictly monotone increasing.

Consider function

$$G(d) = -0.5 + (1 - 0.346574 \ln b)d. \tag{8}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $b \leq 0.35$ we have $G(d_l) < 0$ and $G)d_r) > 0$.

This completes the proof of the theorem.

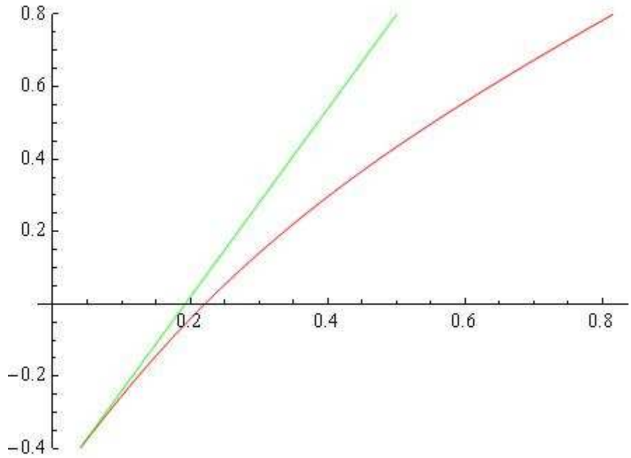

Figure 1: The functions $F(d)$ and $G(d)$ for $b = 0.01$ and $r = 0.6$.

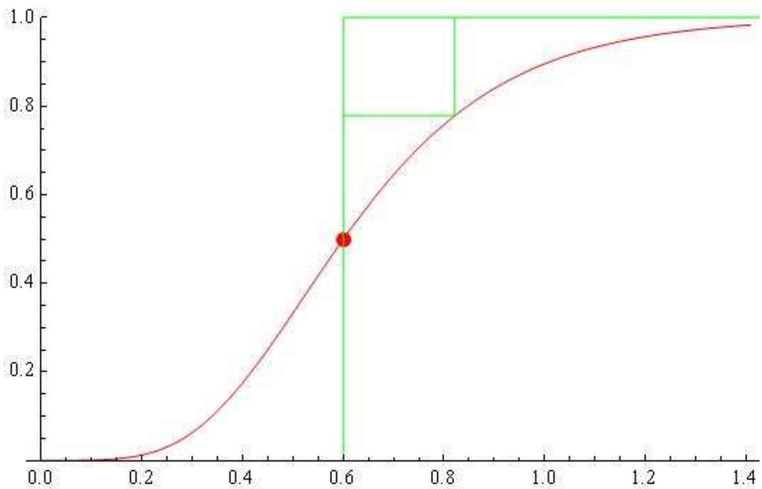

Figure 2: Approximation of the $h_r(t)$ by sigmoid (3) for $b = 0.01$, $r = 0.6$; Hausdorff distance: $d = 0.221314$; $d_l = 0.18343$, $d_r = 0.311083$.

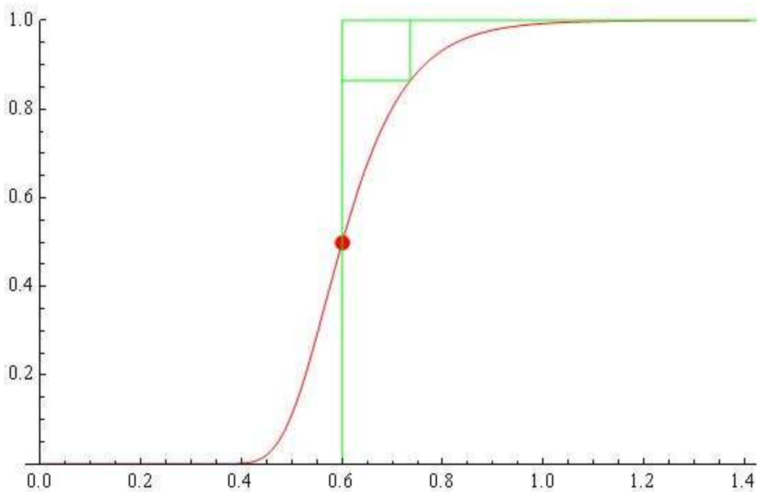

Figure 3: Approximation of the $h_r(t)$ by sigmoid (3) for $b = 0.00001$, $r = 0.6$; Hausdorff distance: $d = 0.135518$; $d_l = 0.0954274$, $d_r = 0.224196$.

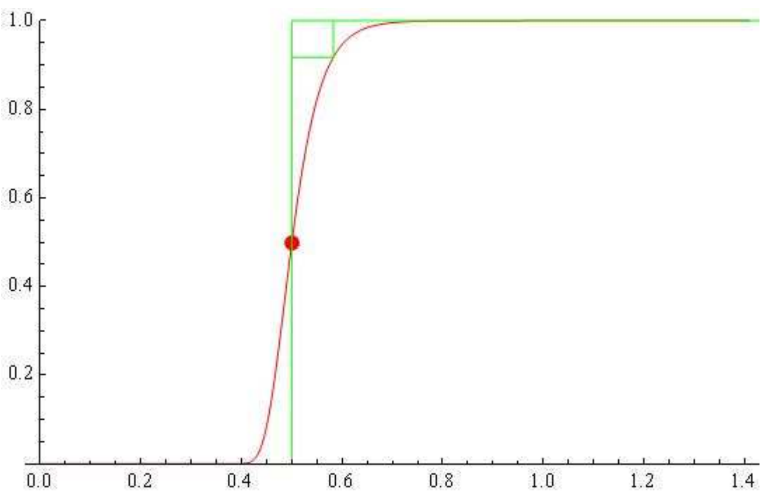

Figure 4: Approximation of the $h_r(t)$ by sigmoid (3) for $b = 0.00000000001$, $r = 0.5$; Hausdorff distance: $d = 0.0823975$; $d_l = 0.0486993$, $d_r = 0.147174$.

b	d_l	d_r	d from (5)
0.1	0.264842	0.351875	0.295836
0.01	0.18343	0.311083	0.221314
0.001	0.140302	0.275547	0.18056
0.0001	0.113593	0.24708	0.154212
0.00001	0.0954274	0.224196	0.135518
0.000001	0.0822706	0.205491	0.121441
0.0000001	0.0723022	0.189931	0.110391
0.000000001	0.0581987	0.165511	0.0940316
0.00000000001	0.0486993	0.147174	0.0823975

Table 1: Bounds for d computed by (6) for various b.

The deterministic model for various b are visualized on Fig. 2 – Fig. 4.

Some computational examples using relations (6) are presented in Table 1.

The last column of Table 1 contains the values of d computed by solving the nonlinear equation (5).

From the above table, it can be seen that the estimates for the value of the one–sided Hausdorff distance (see (6)) are precise.

Following the methodology proposed in this Section, the reader may formulate the corresponding approximation problems in terms of Theorem 1 for the 4–parametric deterministic curve

$$M_{r,\omega}(t) = \omega a^{b^{t-r}}. \tag{9}$$

We define the Heaviside step function $h_{r,\omega}(t)$ by

$$h_{r,\omega}(t) = \begin{cases} 0, & \text{if} \quad t < 0, \\ [0,\omega], & \text{if} \quad t = r, \\ \omega, & \text{if} \quad t > r. \end{cases}$$

2.1. Numerical Example

We examine the following data. (The data were reported by Musa [22] and represent the failures observed during system testing for 25 hours of CPU time).

The fitted model (9) based on the data of Table 2 and the estimated parameters is:

Hour	Number of failures	Cumulative failures
1	27	27
2	16	43
3	11	54
4	10	64
5	11	75
6	7	82
7	2	84
8	5	89
9	3	92
10	1	93
11	4	97
12	7	104
13	2	106
14	5	111
15	5	116
16	6	122
17	0	122
18	5	127
19	1	128
20	1	129
21	2	131
22	1	132
23	2	134
24	1	135
25	1	136

Table 2: Failures in 1 Hour (execution time) intervals and cumulative failures [22], [20]

$$M_{r,\omega}(t) = 137 \times 0.5^{0.84^{t-5.5}}.$$

For the Goel–Okumoto model the approximate solution (cumulative number of failures as a function of executive time) and confidence bounds are plotted on Fig. 5.

The approximate solution by our model (9) is plotted on Fig. 6.

The confidence interval (d_l, d_r) (see Theorem 1) obtained for the variable d

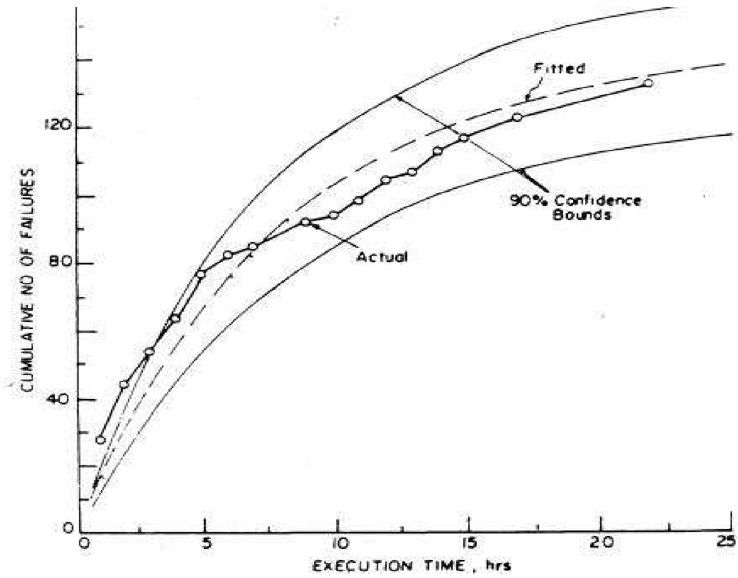

Figure 5: Approximate solution (dashed) [20].

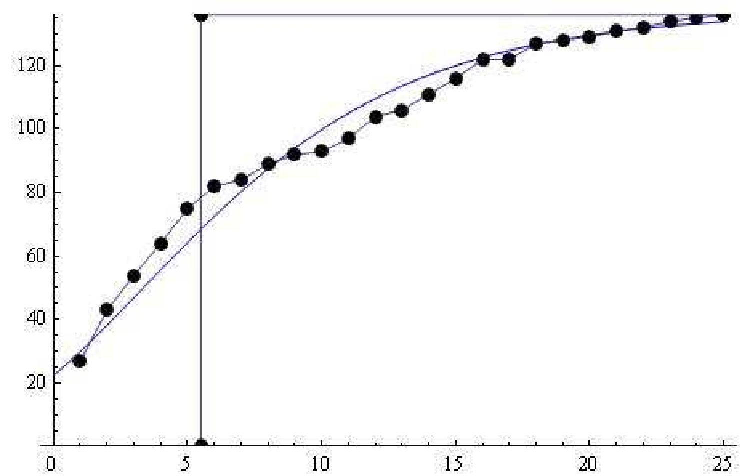

Figure 6: The deterministic model (9) (thick) with $b = 0.84$, $r = 5.5$, $a = 0.5$, $\omega = 137$.

can be used in practice as analog of ”confidence bounds” as said above.

For other results, see [26] – [34], [2] – [4].

3. The Transmuted Deterministic Model

We consider the new transmuted deterministic model:

$$M_r(t;\lambda) = (1+\lambda)a^{b^{(t-r)}} - \lambda\left(a^{b^{(t-r)}}\right)^2, \tag{10}$$

for $-1 < \lambda < 1$

We examine the special case

$$M_r(r;\lambda) = \frac{1}{2} = (1+\lambda)a - a^2, \quad 0 < \lambda < 1. \tag{11}$$

From (11) we have

$$\lambda a^2 - (1+\lambda)a + \frac{1}{2} = 0, \quad a_{1,2} = \frac{1+\lambda \pm \sqrt{1+\lambda^2}}{2\lambda}.$$

We are interested in the solution $a = \dfrac{1+\lambda-\sqrt{1+\lambda^2}}{2\lambda}$.

3.1. Special case

The special transmuted deterministic model is defined by:

$$\begin{aligned} &M_r(t;\lambda) = (1+\lambda)a^{b^{(t-r)}} - \lambda\left(a^{b^{(t-r)}}\right)^2, \\ &a = \frac{1+\lambda-\sqrt{1+\lambda^2}}{2\lambda} \\ &M_r(r;\lambda) = \tfrac{1}{2}. \end{aligned} \tag{12}$$

We study the one–sided Hausdorff approximation of the Heaviside step function $h_r(t)$ by transmuted deterministic model of the forms (11) and find an expression for the error of the best approximation.

The H–distance d between the step function $h_r(t)$ and the sigmoid (12) satisfies the relation

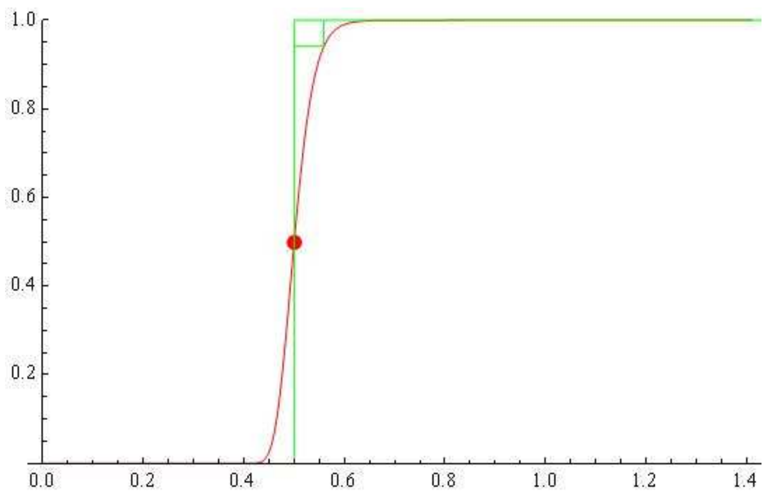

Figure 7: The model (12) with $b = 0.00000000001$, $r = 0.5$, $\lambda = 0.99$; H–distance $d = 0.05591168$.

$$M_r(r + d; \lambda) = 1 - d \tag{13}$$

or the nonlinear equation

$$(1 + \lambda)a^{b^d} - \lambda \left(a^{b^d}\right)^2 - 1 + d = 0. \tag{14}$$

The transmuted deterministic model (12) $b = 0.00000000001$, $r = 0.5$, $\lambda = 0.99$ is visualized on Fig. 7.

In some cases the approximation of Heaviside function by model (12) is better in comparison to its approximation by sigmoid (3) (see, Figure 8).

From the graphics it can be seen that the "saturation" is faster.

Following the ideas given in this section, the reader may formulate the corresponding bounds in terms of Theorem 1.

Remark. Similarly can be generated a software module within the programming environment *CAS Mathematica* for the analysis of the considered transmuted deterministic model (11).

The module offers the following possibilities:

- generation of the transmuted deterministic curve under user defined values of the parameters b, r and λ. (For the parameter a we have $a =$

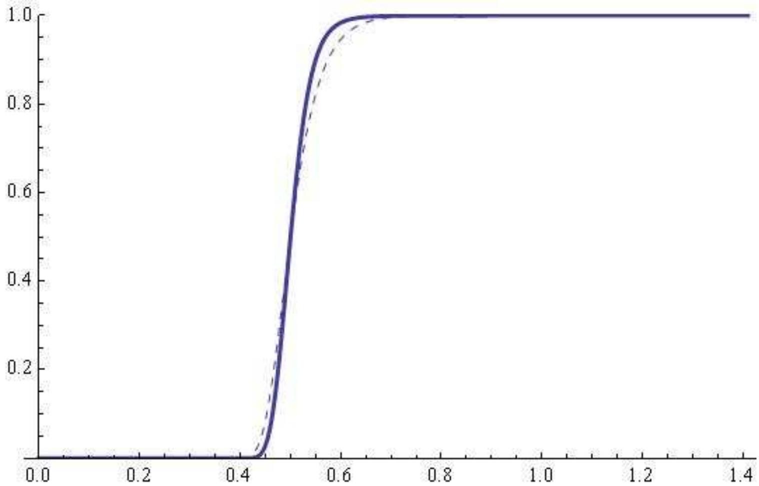

Figure 8: a) The model (3) (dashed) with $b = 0.00000000001$, $r = 0.5$; H–distance $d = 0.0823975$
b) The transmuted deterministic model (12) (thick) with $b = 0.00000000001$, $r = 0.5$, $\lambda = 0.9$; H–distance $d = 0.0626435$.

$$\frac{1+\lambda-\sqrt{1+\lambda^2}}{2\lambda});$$

- calculation of the H-distance d from the nonlinear equation (13);
- software tools for animation and visualization.

A possible architekture of a software for controlling and monitoring printers in a local network is proposed in [35].

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