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SOME EFFICIENT ESTIMATION PROCEDURES UNDER NON-RESPONSE IN TWO-OCCASION SUCCESSIVE SAMPLING

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1. Introduction

A single occasion survey provides information about the characteristics of the surveyed population for the given occasion only and unable to give information about the rate of change of characteristics over different occasions and estimates of the characteristics over all occasions or on the most recent (current) occasion. To get such information, generally sampling is done on successive occasions where a part of the sample retained while the remainder of the sample is drawn afresh for generating reliable estimates at different occasions. This kind of sampling procedure is known as successive (rotation) sampling which was initiated by Jessen (1942) in the analysis of farm data. The theory was further extended by Patterson (1950), Eckler (1955), Rao and Graham (1964), Sen (1971, 1972, 1973), Gupta (1979), Das (1982), Singh *et al.* (1991) and Singh and Singh (2001)among others.

Sometimes, information on an auxiliary variable may be readily available on the first as well as on the second occasion. Utilizing the auxiliary information on both occasions Feng and Zou (1997), Biradar and Singh (2001), Singh (2005), Singh and Priyanka (2006) , Singh and Priyanka (2008) and Singh and Karna (2009) proposed estimators of current population mean in successive (rotation) sampling.

In many sample surveys, generally the information is not obtained from all the sample units selected from the population. This incompleteness is called non-response. Hansen and Hurwitz (1946) introduced first time, sub sampling method of non-respondents to deal with the problems of non-response in mail surveys. Cochran (1977)and Fabian and Hyunshik (2000) extended the Hansen and Hurwitz

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(1946) technique for the situation when besides the information on character under study, information is also available on auxiliary character. Recently, Choudhary *et al.* (2004), Singh and Priyanka (2007) and Singh and Kumar (2011) used the Hansen and Hurwitz (1946) technique for the estimation of population mean on current occasion in two-occasion successive sampling.

In follow up of the above work, the aim of the present paper is to study the occurrence of non-response by utilizing the Hansen and Hurwitz (1946) technique, when it occurs on current occasion in two-occasion successive (rotation) sampling. Recently Bahl and Tuteja (1991), Singh and Vishwakarma (2007) and Singh and Homa (2013) suggested exponential type estimators of population mean under different practicable situations. Looking on the dominance nature of these estimators, we proposed some modified exponential type estimators of population mean on current occasion where information on a dynamic (changing over occasions) auxiliary variable has been used and are capable in reducing the negative impact of non-response to a greater extent. Properties are examined and the performances of the proposed estimators are compared with the similar estimator and natural successive estimator when the complete response is observed and with the Hansen and Hurwitz (1946) estimator under non-response. Optimum replacement strategies are suggested and empirical studies are carried out to assess the behaviors of the proposed estimation procedures.

2. Sample structures and symbols

Consider the finite population $U = (U_1, U_2, ..., U_N)$ of N units,which has been sampled over two occasions. The character under study be denoted by x (y) on the first (second) occasion respectively. It is assumed that the information on a dynamic (change over time) auxiliary variable z_k $(k = 1, 2)$ (with known population mean) is readily available on k^{th} occasion and which is positively correlated with study variable. We assume that there is non-response occurs at the current occasion, so that the population can be divided into two classes, those who will respond at the first attempt and those who will not respond. Let the sizes of these two classes be N_1 and N_2 respectively. A simple random sample (without replacement) s_n of n units is drawn on the first occasion. A random sub sample s_m of $m = n\lambda$ units is retained (matched) for its use on the second occasion under the assumption that these units will respond at the second occasion as well. Now, at the current (second) occasion a simple random sample (without replacement) s_u of $u = (n - m) = n\mu$ units is drawn afresh from the entire population so that the sample size on the current (second) occasion is also n. Here λ and μ $(\lambda + \mu = 1)$ are the fractions of matched and fresh samples respectively at the current (second) occasion. We assume that in the unmatched portion of the sample on the current (second) occasion u_1 units respond and u_2 units do not respond. Let u_{2h} denote the size of the sub sample s_{u2h} drawn from the non-responding units in the unmatched (fresh) portion of the sample (s_u) on the current (second) occasion. The following notations are considered for the further use:
$\bar{X}$, $\bar{Y}$: The population means of the study variables $x(y)$.

$\bar{Z}_k (k = 1, 2)$: The population mean of the auxiliary variable z on k^{th} occasion.

$\bar{y}_m, \bar{y}_u, \bar{y}_{u_1}, \bar{y}_{u_{2h}}, \bar{y}_n, \bar{y}_{n_1}, \bar{y}_{n_{2h}}, \bar{x}_n, \bar{x}_m, \bar{z}_{1m}, \bar{z}_{2u_1}, \bar{z}_{2u_{2h}}$: The sample means of the respective variables based on the sample sizes shown in suffices.

$\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}$: The population correlation coefficients between the variables shown in suffices.

ρ_{2yz_2}: The population correlation coefficient between the variables y and z_2 in the non-responding units of the population.

$S_x^2, S_y^2, S_{z_1}^2, S_{z_2}^2$: The population variances of the variables x, y, z_1 and z_2 respectively.

$S_{2y}^2, S_{2z_2}^2$: The population variances of the variables y and z_2 respectively in the non-responding units of the population.

$W = \frac{N_2}{N}$: The proportion of non-responding units in the population at current (second) occasion.

$\bar{y}_u^* = \frac{u_1 \bar{y}_{u_1} + u_2 \bar{y}_{u_{2h}}}{u}$ and $\bar{z}_{2u}^* = \frac{u_1 \bar{z}_{2u_1} + u_2 \bar{z}_{2u_{2h}}}{u}$: Hansen and Hurwitz estimators defined on study variable and auxiliary variable respectively for the unmatched portion of the sample on the current occasion.

$f_2 = \frac{u_2}{u_{2h}}$ and $f_2^* = \frac{n_2}{n_{2h}}$.

3. Formulation of estimators

To estimate the population mean $\bar{Y}$ on the current (second) occasion, two different sets of estimators are considered that use information on a dynamic auxiliary variable $z_1(z_2)$. One set of estimators $S_u = (T_{1u}, T_{2u})$ based on sample s_u of size u drawn afresh on the second occasion and the second set of estimator $S_m = (T_m)$ based on the sample sm of size s_m which is common to both occasions. Since the non-response occurs in the sample s_u, therefore, we have used the Hansen and Hurwitz (1946) technique to propose the estimators of set S_u. Hence, the estimators of sets S_u and S_m for estimating the current population mean $\bar{Y}$ are formulated as

$$T_{1u} = \bar{y}_u^* \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2u}}{\bar{Z}_2 + \bar{z}_{2u}}\right), T_{2u} = \bar{y}_u^* \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2u}^*}{\bar{Z}_2 + \bar{z}_{2u}^*}\right) \text{ and } T_m = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right)\left(\frac{\bar{Z}_1}{\bar{z}_{1m}}\right).$$

Combining the estimators of sets S_u and S_m, finally we have the following estimators of population mean $\bar{Y}$ at the current (second) occasion:

$$T_i = \phi_i T_{iu} + (1 - \phi_i) T_m \ ; \ (i = 1, 2) \tag{1}$$

where ϕ_i $(0 \leq \phi_i \leq 1)(i = 1, 2)$ are the unknown constants (scalars) to be determined under certain criterions.

4. Properties of the estimators T_i $(i = 1, 2)$

Since, the estimators T_{iu} $(i = 1, 2)$ and T_m are exponential type estimators, they are biased of the population mean $\bar{Y}$, therefore, the resulting estimators T_i $(i = 1, 2)$ defined in Eq. (1) are also biased estimators of $\bar{Y}$. The bias B (.) and mean square errors M (.) of the estimators T_i $(i = 1, 2)$ are derived up-to the first order of approximations using the following transformations:

$\bar{y}_m = (1+e_1)\bar{Y}, \bar{y}_u = (1+e_2)\bar{Y}, \bar{y}_u^* = (1+e_3)\bar{Y}, \bar{x}_m = (1+e_4)\bar{X}, \bar{x}_n = (1+e_5)\bar{X}, \bar{z}_{1m} = (1+e_6)\bar{Z}_1, \bar{z}_{2u} = (1+e_7)\bar{Z}_2, \bar{z}_{2u}^* = (1+e_8)\bar{Z}_2;$

where $e_i^{'}(i = 1, 2, ..., 8)$ are the relative errors of estimates while estimating the respective population parameters which satisfy the assumptions $E(e_i) = 0$ and $|e_i| \leq 1$.
Under the above transformations estimators T_{iu} $(i = 1, 2)$ and T_m take the following forms:

$$T_{1u} = \bar{Y}(1+e_3)\exp\left[-\frac{1}{2}e_7\left(1+\frac{1}{2}e_7\right)^{-1}\right], \tag{2}$$

$$T_{2u} = \bar{Y}(1+e_3)\exp\left[-\frac{1}{2}e_8\left(1+\frac{1}{2}e_8\right)^{-1}\right], \tag{3}$$

$$\textit{and } T_m = \bar{Y}(1+e_1)(1+e_6)^{-1}\exp\left[\frac{1}{2}(e_5-e_4)\left(1+\frac{1}{2}(e_5+e_4)\right)^{-1}\right] \tag{4}$$

Thus, we have the following theorems:

THEOREM 1. *Bias of the estimators T_i $(i = 1, 2)$ to the first order of approximations are obtained as*

$$B(T_i) = \phi_i B(T_{iu}) + (1-\phi_i)B(T_m); (i = 1, 2) \tag{5}$$

$$\textit{where } B(T_{1u}) = \bar{Y}\left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{3}{8}-\frac{1}{2}\rho_{yz_2}\right)C_y^2, \tag{6}$$

$$B(T_{2u}) = \bar{Y}\left[\left\{\left(\frac{1}{u}-\frac{1}{N}\right)+\frac{(f_2-1)W}{u}\right\}\left(\frac{3}{8}-\frac{1}{2}\rho_{yz_2}\right)\right]C_y^2 \tag{7}$$

$$\textit{and} B(T_m) = \bar{Y}\left\{\left(\frac{1}{m}-\frac{1}{N}\right)(1-\rho_{yz_1})+\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{3}{8}+\frac{1}{2}\rho_{xz_1}-\frac{1}{2}\rho_{yx}\right)\right\}C_y^2 \tag{8}$$

PROOF. The bias of the estimators T_i $(i = 1, 2)$ are given by

$$B(T_i) = E[T_i - \bar{Y}] = \phi_i E[T_{iu} - \bar{Y}] + (1-\phi_i)E[T_m - \bar{Y}]$$
$$= \phi_i B(T_{iu}) + (1-\phi_i)B(T_m) \tag{9}$$

where $B(T_{iu}) = E[T_{iu} - \bar{Y}]$ and $B(T_m) = E[T_m - \bar{Y}]$.

Substituting the expressions of T_{1u}, T_{2u} and T_m from Eq. (2), Eq. (3) and Eq. (4) in Eq. (9), expanding the terms binomially and exponentially, taking expectations and retaining the terms up to the first order of sample sizes, we have the expressions for the bias of the estimators T_i $(i = 1, 2)$ as described in Eq. (5).

THEOREM 2. *Mean square errors of estimators T_i $(i = 1, 2)$to the first order of approximations are obtained as*

$$M(T_i) = \phi_i^2 M(T_{iu}) + (1-\phi_i)^2 M(T_m) + 2\phi(1-\phi_i)C_i; (i = 1, 2) \tag{10}$$

$$whereM(T_{1u}) = E(T_{1u} - \bar{Y})^2 = \left[\left(\frac{1}{u} - \frac{1}{N}\right)\left(\frac{5}{4} - \rho_{yz_2}\right) + \frac{(f_2-1)W}{u}\right] S_y^2, \tag{11}$$

$$M(T_{2u}) = E(T_{2u}-\bar{Y})^2 = \left[\left(\left(\frac{1}{u} - \frac{1}{N}\right) + \frac{(f_2-1)W}{u}\right)\left(\frac{5}{4} - \rho_{yz_2}\right)\right] S_y^2, \tag{12}$$

$$M(T_m) = E(T_m - \bar{Y})^2 \tag{13}$$
$$= \left[\left\{2\left(\frac{1}{m} - \frac{1}{N}\right)(1-\rho_{yz_1})\right\} + \left\{\left(\frac{1}{m} - \frac{1}{n}\right)\left(\frac{1}{4} + \rho_{xz_1} - \rho_{yx}\right)\right\}\right] S_y^2$$

$$C_1 = E\left[(T_{1u} - \bar{Y})(T_m - \bar{Y})\right] = -\frac{S_y^2}{N}\left(1 - \frac{1}{2}\rho_{yz_2} - \rho_{yz_1} + \frac{1}{2}\rho_{z_1z_2}\right) \tag{14}$$

$$and\ C_2 = E\left[(T_{2u} - \bar{Y})(T_m - \bar{Y})\right] = -\frac{S_y^2}{N}\left(1 - \frac{1}{2}\rho_{yz_2} - \rho_{yz_1} + \frac{1}{2}\rho_{z_1z_2}\right) \tag{15}$$

PROOF. It is obvious that the mean square errors of estimators $T_i (i = 1, 2)$ are given by

$$M(T_i) = E\left[\phi_i(T_{iu} - \bar{Y}) + (1-\phi_i)(T_m - \bar{Y})\right]^2; (i = 1, 2)$$
$$= \phi_i^2 E(T_{iu} - \bar{Y})^2 + (1-\phi_i)^2 E(T_m - \bar{Y})^2 + 2\phi_i(1-\phi_i)E\left[(T_{iu} - \bar{Y})(T_m - \bar{Y})\right]$$
$$= \phi_i^2 M(T_{iu}) + (1-\phi_i)^2 M(T_m) + 2\phi(1-\phi_i)C_i. \tag{16}$$

Substituting the expressions of T_{1u}, T_{2u} and T_m from Eq. (2), Eq. (3) and Eq. (4) in Eq. (16), expanding the terms binomially and exponentially, taking expectations and retaining the terms up to the first order of sample sizes; we have the expressions of mean square errors of the estimators T_i as it is given in Eq. (10).

REMARK 3. *The expressions of bias and mean square errors in the Eq. (5) and Eq. (10) respectively are derived under the assumptions (i) that the coefficients of variation and correlation coefficients of non-response class are similar to that of the population, i.e.* $C_{2y} = C_y, C_{2z_2} = C_{z_2}, \rho_{2yz_2} = \rho_{yz_2}$ *and (ii) since, x and y are the same study variable over two occasions and* $z_k (k = 1, 2)$ *is the auxiliary variable on first and second occasion correlated to x and y, therefore, looking on the stability nature of the coefficients of variation viz. Reddy (1978), the coefficients of variation of the variables x, y and* z_k *in the population are considered equal, i.e.* $C_x = C_y = C_{z_1} = C_{z_2}$.

5. Minimum mean square errors of the estimators $T_i (i = 1, 2)$

Since, the mean square errors of the estimators $T_i (i = 1, 2)$ in Eq. (10) are functions of unknown constants $\phi_i (i = 1, 2)$, therefore, they are minimized with respect to ϕ_i and subsequently the optimum values of ϕ_i are obtained as

$$\phi_{i_{opt}} = \frac{M(T_m) - C_i}{M(T_{iu}) + M(T_m) - 2C_i}; (i = 1, 2) \tag{17}$$

Now substituting the values of $\phi_{i_{opt}}$ in Eq. (10), we get the optimum mean square errors of T_i as

$$M(T_{i_{opt}}) = \frac{M(T_{iu})M(T_m) - C_i^2}{M(T_{iu}) + M(T_m) - 2C_i}; (i = 1, 2) \tag{18}$$

Further, substituting the values from Eq. (11) − Eq. (15) in Eq. (18), we get the simplified values of $M(T_{i_{opt}})$ which are given below:

$$M(T_{1_{opt}}) = \frac{a_3 + \mu_1 a_2 + \mu_1^2 a_1}{a_6 + \mu_1 a_5 + \mu_1^2 a_4} \frac{S_y^2}{n} \tag{19}$$

$$M(T_{2_{opt}}) = \frac{a_9 + \mu_2 a_8 + \mu_2^2 a_7}{a_{12} + \mu_2 a_{11} + \mu_2^2 a_{10}} \frac{S_y^2}{n}. \tag{20}$$

where $\mu_i (i = 1, 2)$ are the fractions of fresh sample to be replaced for the estimators $T_i (i = 1, 2)$,

$$a_1 = ac + k^2 f^2, a_2 = ad + cb - k^2 f^2, a_3 = bd, a_4 = c - a + 2kf, a_5 = a - b + d - 2kf,$$

$$a_6 = b, a_7 = ac + k^2 f^2, a_8 = ad + cb_1' - k^2 f^2, a_9 = b_1' d, a_{10} = c - a + 2kf,$$

$$a_{11} = a - b_1' + d - 2kf, a_{12} = b_1', a = -f a_0, b = a_0 + (f_2 - 1)W, c = f c_1 + d_1, d = (1 - f) c_1,$$

$$b_1' = a_0 \left[1 + (f_2 - 1)W\right], a_0 = \frac{5}{4} - \rho_{yz_2}, c_1 = 2(1 - \rho_{yz_1}), d_1 = \frac{1}{4} + \rho_{xz_1} - \rho_{yx},$$

$$k = \left[-1 - \frac{1}{2}\rho_{z_1 z_2} + \frac{1}{2}\rho_{yz_2} + \rho_{yz_2}\right], f = \frac{n}{N}, f_2 = \frac{u_2}{u_{2h}}$$

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6. Optimum replacement strategies

Since, the mean square errors of the estimators $T_i (i = 1, 2)$ given in Eq. (19)and Eq. (20) are the functions of $\mu_i (i = 1, 2)$, therefore, the optimum values of μ_i are determined to estimate the population mean with maximum precision and lowest cost. To determine the optimum values of μ_i, we minimized mean square errors of the estimators T_i given in equations Eq. (19)and Eq. (20) respectively with respect to μ_i which result in quadratic equations in μ_i and the respective solutions of μ_i say $\hat{\mu}_i$ $(i = 1, 2)$ are given below:

$$p_1 \mu_1^2 + 2p_2 \mu_1 + p_3 = 0 \tag{21}$$

$$\hat{\mu}_1 = \frac{-p_2 \pm \sqrt{p_2^2 - p_1 p_3}}{p_1} \tag{22}$$

$$p_4 \mu_2^2 + 2p_5 \mu_2 + p_6 = 0 \tag{23}$$

$$\hat{\mu}_2 = \frac{-p_5 \pm \sqrt{p_5^2 - p_4 p_6}}{p_4} \tag{24}$$

where $p_1 = a_1a_5 - a_2a_4, p_2 = a_1a_6 - a_3a_4, p_3 = a_2a_6 - a_3a_5, p_4 = a_7a_{11} - a_8a_{10}, p_5 = a_7a_{12} - a_9a_{10}$ and $p_6 = a_8a_{12} - a_9a_{11}$.

From equations Eq. (22) and Eq. (24) , it is obvious that real values of μ_i $(i = 1, 2)$ exist, if, the quantities under square roots are greater than or equal to zero. For any combinations of correlations $\rho_{yx}, \rho_{xz_1}, \rho_{yz_1}, \rho_{z_1z_2}$ and ρ_{yz_2}, which satisfy the conditions of real solutions; two real values of $\hat{\mu}_i (i = 1, 2)$ are possible. Hence, while choosing the values of $\hat{\mu}_i$, it should be remembered that$0 \leq \hat{\mu}_i \leq 1$. All other values of $\hat{\mu_i}$ $(i = 1, 2)$are inadmissible. Substituting the admissible values of $\hat{\mu}_i$ say $\mu_i^{(0)}$ from equations Eq. (22)and Eq. (24) into Eq. (19)and Eq. (20) respectively, we have the optimum values of mean square errors of $T_i (i = 1, 2)$, which are shown below;

$$M(T^0_{1_{opt}}) = \frac{a_3 + \mu_1^{(0)} a_2 + \mu_1^{(0)2} a_1}{a_6 + \mu_1^{(0)} a_5 + \mu_1^{(0)2} a_4} \frac{S_y^2}{n} \tag{25}$$

$$M(T^0_{2_{opt}}) = \frac{a_9 + \mu_2^{(0)} a_8 + \mu_2^{(0)2} a_7}{a_{12} + \mu_2^{(0)} a_{11} + \mu_2^{(0)2} a_{10}} \frac{S_y^2}{n} \tag{26}$$

7. Efficiencies comparison

7.1. Comparison with estimators under complete response:

The percent relative loss in efficiencies of the estimators $T_i (i = 1, 2)$ are obtained with respect to the similar estimator and natural successive sampling estimator when the non-response not observed on any occasion. The estimator ξ_1 is defined under the similar circumstances as the estimators T_i but under complete response, where as the estimator ξ_2 is the natural successive sampling estimator and they are given as

$$\xi_j = \Psi_j \xi_{ju} + (1 - \Psi_j) T_{jm}; (j = 1, 2) \tag{27}$$

where $\xi_{1u} = \bar{y}_u \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2u}}{\bar{Z}_2 + \bar{z}_{2u}}\right)$,$\xi_{2u} = \bar{y}_u$, $T_{1m} = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right)\left(\frac{\bar{Z}_1}{\bar{z}_{1m}}\right)$.
and $T_{2m} = \bar{y}_m + b_{yx}(m)(\bar{x}_n - \bar{x}_m)$,$T_{1m}$ is same as T_m defined in section 3 and $b_{yx}(m)$ is the sample regression coefficient between the variables shown in suffix.

Proceeding on the similar line as discussed for the estimators $T_i (i = 1, 2)$, the optimum mean square errors of the estimators $\xi_j (j = 1, 2)$ are derived as

$$M(\xi^0_{1_{opt}}) = \frac{b_3 + \mu_1^{*(0)} b_2 + \mu_1^{*(0)2} b_1}{b_6 + \mu_1^{*(0)} b_5 + \mu_1^{*(0)2} b_4} \frac{S_y^2}{n} \tag{28}$$

$$M(\xi^0_{2_{opt}}) = \left[\frac{1}{2}\left[1+\sqrt{(1-\rho^2_{xy})}\right]-f\right]\frac{S^2_y}{n}. \tag{29}$$

$\mu_1^{*(0)} = \frac{-q_2 \pm \sqrt{q_2^2 - q_1 q_3}}{q_1}$(fraction of the fresh sample for the estimator), $b_1 = ac + k^2f^2, b_2 = ad + ca_0 - k^2f^2, b_3 = a_0 d, b_4 = c - a + 2kf, b_5 = a - a_0 + d - 2kf, b_6 = a_0, q_1 = b_1b_5 - b_2b_4, q_2 = b_1b_6 - b_3b_4$ *and* $q_3 = b_2b_6 - b_3b_5$.

Remark 4. *To compare the performance of the estimators* $T_i(i = 1, 2)$ *with the estimators* $\xi_j(j = 1, 2)$*, we introduce an assumption* $\rho_{yz_2} = \rho_{xz_1} = \rho_{yz_1}$ *, which is an intuitive assumption and also considered by Cochran (1977) and Feng and Zou (1997).*

The percent relative losses in precision of estimators $T_i(i = 1, 2)$ with respect to $\xi_j(j = 1, 2)$ under their respective optimality conditions are given by

$$L_{ij} = \frac{M(T^0_{i_{opt}}) - M(\xi^0_{j_{opt}})}{M(T^0_{i_{opt}})}100; (i, j = 1, 2) \tag{30}$$

For $N = 5000, n = 500, f_2 = 1.5$ and different choices of $\rho_{yx}, \rho_{z_1z_2} and \rho_{yz_2}$, Tables 1 − 4 give the optimum values of $\mu_i^{(0)}$ and percent relative losses in precision $L_{ij}(i, j = 1, 2)$ of estimators $T_i(i = 1, 2)$ with respect to estimators $\xi_j(j = 1, 2)$.

7.2. Comparison with Hansen and Hurwitz (1946) estimator under non-response:

In this section, the percent relative loss in efficiencies of the estimators $T_i(i = 1, 2)$ are obtained with respect to the Hansen and Hurwitz (1946) estimator ($\bar{y}^*_n$), when non-response occurs at current occasion and when there is no matching from the previous occasion;

$$\bar{y}^*_n = \frac{n_1\bar{y}_{n_1} + n_2\bar{y}_{n_{2h}}}{n} \tag{31}$$

Since, $\bar{y}^*_n$ is an unbiased estimator of $\bar{Y}$, therefore, following Sukhatme *et al.* (1984) the variance of $\bar{y}^*_n$ is given as:

$$V(\bar{y}^*_n) = [(1-f) + (f^*_2 - 1)W]\frac{S^2_y}{n} \tag{32}$$

The expression of the variance in Eq. (32) is written under the assumption $S^2_y = S^2_{2y}$.

Remark 5. *To compare the performances of the estimators* $T_i(i = 1, 2)$ *with the estimator* $(\bar{y}^*_n)$*, we introduce one more assumption* $f_2 = f^*_2$*.*

The percent relative losses in precision of estimators $T_i(i = 1, 2)$ with respect to $(\bar{y}^*_n)$ under their respective optimality conditions are given by

$$L_i = \frac{M(T^0_{i_{opt}}) - V(\bar{y}^*_n)}{M(T^0_{i_{opt}})}100; (i = 1, 2) \tag{33}$$

For $N = 5000, n = 500, f_2 = 1.5$ and different choices of $\rho_{yx}, \rho_{z_1z_2} and \rho_{yz_2}$, Tables 5 and 6 give the optimum values of $\mu_i^{(0)}$ and percent relative losses in precision $L_i (i = 1, 2)$ of estimators $T_i (i = 1, 2)$ with respect to estimator $(\bar{y}_n^*)$.

8. Interpretations of results

The following interpretations may be read out from Tables 1-6:
(1) From Table 1 it is clear that:
(a) For the fixed values of W and ρ_{yx}, no pattern is visible for the fix value 0.5 of $\rho_{z_1z_2}$ but for the fix value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_1^{(0)}$ and L_{11} with the increasing values of ρ_{yz_2}.
(b) For the fixed values of W and ρ_{yz_2}, no pattern is observed for fix value 0.5 of $\rho_{z_1z_2}$but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_1^{(0)}$ and L_{11} with increasing values of ρ_{yx} . This behavior is in agreement with Sukhatme *et al.* (1984) results, which explains that more the value of ρ_{yx} , more the fraction of fresh sample is required on the current occasion.
(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_1^{(0)}$ and L_{11} increase with the increasing values of $\rho_{z_1z_2}$. If less correlation is observed between auxiliary variables from first and second occasion we get less amount of loss.
(d) For the fixed values of $\rho_{z_1z_2}$, ρ_{yx} and ρ_{yz_2}, the values of $\mu_1^{(0)}$ and L_{11} increase with the increasing values of W. This pattern showed that the more the non-response rate more loss is observed.

(2) From Table 2 it is visible that:
(a) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yx}, no definite trends are noticed for fix value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_2^{(0)}$ and L_{21} with the increasing values of ρ_{yz_2}. Though, the amount of loss is not as much appreciable as one may see in Table 1 for the similar situation.
(b) For the fixed values of $W$and ρ_{yz_2}, no pattern is observed for value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_2^{(0)}$ and L_{21} with the increasing values of ρ_{yx} .This indicates that loss is reduces when the study variables are remotely correlated.
(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_2^{(0)}$ and L_{21} increase with the increasing values of $\rho_{z_1z_2}$.
(d) For the fixed values of $\rho_{z_1z_2}$, $\rho_{yx}$and ρ_{yz_2}, the values of $\mu_2^{(0)}$ and L_{21} increase with the increasing values of W. Thus, the higher the non-response rate on the current occasion, the larger the fresh sample is desirable on the current occasion which enhances the precision of the estimates.

(3) From Table 3 it is indicated that:
(a) For the fixed values of W, $\rho_{z_1z_2}$and ρ_{yx}, we did not get any pattern for the value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_1^{(0)}$ and decreasing values of L_{12} with the increasing values of ρ_{yz_2}.
(b) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yz_2}, the values of L_{12} increase with the

TABLE 1

Percent relative loss in precision L_{11} of the estimator T_1 with respect to ξ_1

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_1^{(0)}$	L_{11}	$\mu_1^{(0)}$	L_{11}	$\mu_1^{(0)}$	L_{11}	$\mu_1^{(0)}$	L_{11}
0.05	0.6	0.5	0.2903	1.3057	0.2613	1.1180	0.1985	0.7626	*	-
	0.7		0.2823	1.2764	0.2480	1.0612	0.1714	0.6326	*	-
	0.8		0.4039	2.1549	0.4117	2.1241	0.4183	2.0728	0.4205	1.9732
	0.9		0.4029	2.1650	0.4103	2.1310	0.4159	2.0728	0.4157	1.9544
	0.6	0.7	0.4781	3.0971	0.5022	3.1495	0.5327	3.2248	0.5746	3.3443
	0.7		0.4806	3.1352	0.5055	3.1925	0.5376	3.2758	0.5826	3.4106
	0.8		0.5277	4.3504	0.5598	4.4627	0.6003	4.6173	0.6540	4.8457
	0.9		0.5321	4.4186	0.5655	4.5398	0.6079	4.7074	0.6648	4.9578
0.10	0.6	0.5	0.3281	2.6903	0.3084	2.3668	0.2630	1.7526	*	-
	0.7		0.3220	2.6431	0.2983	2.2727	0.2423	1.5332	*	-
	0.8		0.4358	4.2957	0.4492	4.2616	0.4648	4.2034	0.4837	4.0879
	0.9		0.4361	4.3205	0.4496	4.2831	0.4653	4.2172	0.4848	4.0798
	0.6	0.7	0.5065	6.0572	0.5341	6.1744	0.5700	6.3437	0.6203	6.6137
	0.7		0.5098	6.1323	0.5386	6.2601	0.5765	6.4468	0.6309	6.7504
	0.8		0.5540	8.3552	0.5887	8.5787	0.6325	8.8865	0.6910	9.3421
	0.9		0.5591	8.4827	0.5951	8.7230	0.6411	9.0560	0.7032	9.5537
0.15	0.6	0.5	0.3653	4.1372	0.3549	3.7231	0.3266	2.9349	*	-
	0.7		0.3611	4.0816	0.3478	3.6079	0.3122	2.6602	*	-
	0.8		0.4671	6.4143	0.4861	6.4000	0.5105	6.3722	0.5459	6.3118
	0.9		0.4686	6.4573	0.4882	6.4418	0.5139	6.4103	0.5528	6.3377
	0.6	0.7	0.5342	8.8876	0.5654	9.0793	0.6065	9.3569	0.6651	9.8014
	0.7		0.5383	8.9983	0.5710	9.2067	0.6146	9.5120	0.6783	10.0104
	0.8		0.5796	12.0582	0.6167	12.3909	0.6639	12.8493	0.7270	13.5287
	0.9		0.5852	12.2374	0.6238	12.5941	0.6733	13.0886	0.7405	13.8288
0.20	0.6	0.5	0.4020	5.6327	0.4006	5.1674	0.3895	4.2796	*	-
	0.7		0.3995	5.5764	0.3966	5.0446	0.3812	3.9782	*	-
	0.8		0.4979	8.5048	0.5223	8.5296	0.5554	8.5637	0.6073	8.6184
	0.9		0.5005	8.5685	0.5261	8.5961	0.5616	8.6342	0.6198	8.6965
	0.6	0.7	0.5613	11.5958	0.5961	11.8691	0.6422	12.2661	0.7091	12.9038
	0.7		0.5661	11.7404	0.6027	12.0368	0.6518	12.4723	0.7247	13.1854
	0.8		0.6045	15.4957	0.6440	15.9350	0.6943	16.5408	0.7621	17.4394
	0.9		0.6106	15.7200	0.6518	16.1898	0.7047	16.8418	0.7768	17.8183

Note: $*$ indicate $\mu_1^{(0)}$ do not exist.

TABLE 2

Percent relative loss in precision L_{21} of the estimator T_2 with respect to ξ_1

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_2^{(0)}$	L_{21}	$\mu_2^{(0)}$	L_{21}	$\mu_2^{(0)}$	L_{21}	$\mu_2^{(0)}$	L_{21}
0.05	0.6	0.5	0.2770	0.8388	0.2447	0.7105	0.1758	0.4678	*	-
	0.7		0.2682	0.8183	0.2302	0.6711	0.1464	0.3783	*	-
	0.8		0.3893	1.1864	0.3946	1.1657	0.3971	1.1315	0.3917	1.0655
	0.9		0.3878	1.1913	0.3923	1.1684	0.3934	1.1296	0.3842	1.0511
	0.6	0.7	0.4623	1.4111	0.4842	1.4329	0.5118	1.4642	0.5491	1.5136
	0.7		0.4643	1.4284	0.4870	1.4523	0.5158	1.4869	0.5555	1.5428
	0.8		0.5101	1.5650	0.5406	1.6044	0.5788	1.6585	0.6293	1.7385
	0.9		0.5142	1.5901	0.5458	1.6326	0.5857	1.6915	0.6392	1.7792
0.10	0.6	0.5	0.3017	1.7136	0.2755	1.4801	0.2180	1.0374	*	-
	0.7		0.2943	1.6778	0.2632	1.4101	0.1928	0.8766	*	-
	0.8		0.4071	2.3698	0.4155	2.3375	0.4230	2.2836	0.4268	2.1789
	0.9		0.4063	2.3811	0.4142	2.3455	0.4209	2.2844	0.4227	2.1599
	0.6	0.7	0.4753	2.7937	0.4989	2.8402	0.5289	2.9070	0.5700	3.0131
	0.7		0.4777	2.8280	0.5022	2.8789	0.5337	2.9529	0.5777	3.0726
	0.8		0.5196	3.0838	0.5510	3.1625	0.5904	3.2707	0.6426	3.4307
	0.9		0.5239	3.1326	0.5565	3.2175	0.5977	3.3351	0.6530	3.5105
0.15	0.6	0.5	0.3262	2.6194	0.3061	2.3016	0.2598	1.6982	*	-
	0.7		0.3200	2.5729	0.2958	2.2090	0.2388	1.4825	*	-
	0.8		0.4247	3.5486	0.4362	3.5128	0.4486	3.4524	0.4616	3.3338
	0.9		0.4245	3.5678	0.4359	3.5284	0.4482	3.4600	0.4608	3.3190
	0.6	0.7	0.4881	4.1484	0.5134	4.2222	0.5459	4.3285	0.5907	4.4975
	0.7		0.4909	4.1996	0.5172	4.2802	0.5513	4.3977	0.5996	4.5882
	0.8		0.5290	4.5584	0.5613	4.6764	0.6019	4.8386	0.6558	5.0785
	0.9		0.5335	4.6299	0.5670	4.7570	0.6096	4.9330	0.6667	5.1958
0.20	0.6	0.5	0.3505	3.5519	0.3364	3.1690	0.3013	2.4409	*	-
	0.7		0.3455	3.4987	0.3281	3.0605	0.2844	2.1846	*	-
	0.8		0.4421	4.7214	0.4567	4.6896	0.4740	4.6346	0.4962	4.5249
	0.9		0.4426	4.7496	0.4574	4.7147	0.4751	4.6525	0.4985	4.5219
	0.6	0.7	0.5008	5.4758	0.5278	5.5792	0.5626	5.7284	0.6113	5.9662
	0.7		0.5040	5.5436	0.5321	5.6564	0.5688	5.8211	0.6213	6.0886
	0.8		0.5383	5.9911	0.5715	6.1482	0.6133	6.3642	0.6689	6.6838
	0.9		0.5430	6.0841	0.5774	6.2532	0.6213	6.4873	0.6803	6.8371

Note: $*$ indicate $\mu_2^{(0)}$ do not exist.

TABLE 3

Percent relative loss in precision L_{12} of the estimator T_1 with respect to ξ_2

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_1^{(0)}$	L_{12}	$\mu_1^{(0)}$	L_{12}	$\mu_1^{(0)}$	L_{12}	$\mu_1^{(0)}$	L_{12}
0.05	0.6	0.5	0.2903	-9.0503	0.2613	-7.1528	0.1985	-3.6519	0.0191	2.7849
	0.7		0.2823	-9.6982	0.2480	-7.7578	0.1714	-4.1290	*	-
	0.8		0.4039	-29.4047	0.4117	-28.0393	0.4183	-25.1588	0.4205	-19.9006
	0.9		0.4029	-30.4156	0.4103	-29.0691	0.4159	-26.1969	0.4157	-20.9262
	0.6	0.7	0.4781	-56.2496	0.5022	-55.2468	0.5327	-52.4679	0.5746	-46.8240
	0.7		0.4806	-57.7031	0.5055	-56.7325	0.5376	-53.9667	0.5826	-48.2952
	0.8		0.5277	-94.8741	0.5598	-94.3728	0.6003	-91.6707	0.6540	-85.3084
	0.9		0.5321	-97.0247	0.5655	-96.5590	0.6079	-93.8488	0.6648	-87.3882
0.10	0.6	0.5	0.3281	-7.5204	0.3084	-5.7996	0.2630	-2.6179	0.1278	3.0839
	0.7		0.3220	-8.1795	0.2983	-6.4383	0.2423	-3.1852	0.0594	2.8337
	0.8		0.4358	-26.5734	0.4492	-25.2431	0.4648	-22.4357	0.4837	-17.3140
	0.9		0.4361	-27.5423	0.4496	-26.2310	0.4653	-23.4335	0.4848	-18.3048
	0.6	0.7	0.5065	-51.4766	0.5341	-50.3980	0.5700	-47.5541	0.6203	-41.8576
	0.7		0.5098	-52.8236	0.5386	-51.7660	0.5765	-48.9191	0.6309	-43.1675
	0.8		0.5540	-86.7147	0.5887	-85.9987	0.6325	-83.0916	0.6910	-76.5519
	0.9		0.5591	-88.6473	0.5951	-87.9454	0.6411	-85.0028	0.7032	-78.3268
0.15	0.6	0.5	0.3653	-5.9217	0.3549	-4.3298	0.3266	-1.3830	0.2354	3.7809
	0.7		0.3611	-6.5811	0.3478	-4.9840	0.3122	-2.0042	0.1877	3.3156
	0.8		0.4671	-23.7715	0.4861	-22.4456	0.5105	-19.6637	0.5459	-14.5939
	0.9		0.4686	-24.6939	0.4882	-23.3840	0.5139	-20.6072	0.5528	-15.5200
	0.6	0.7	0.5342	-46.9128	0.5654	-45.7416	0.6065	-42.8069	0.6651	-37.0153
	0.7		0.5383	-48.1576	0.5710	-46.9955	0.6146	-44.0399	0.6783	-38.1624
	0.8		0.5796	-79.1703	0.6167	-78.2428	0.6639	-75.1284	0.7270	-68.3987
	0.9		0.5852	-80.9077	0.6238	-79.9747	0.6733	-76.7993	0.7405	-69.8978
0.20	0.6	0.5	0.4020	-4.2693	0.4006	-2.7647	0.3895	0.0216	0.3422	4.8242
	0.7		0.3995	-4.9201	0.3966	-3.4192	0.3812	-0.6230	0.3146	4.2374
	0.8		0.4979	-21.0067	0.5223	-19.6597	0.5554	-16.8629	0.6073	-11.7726
	0.9		0.5005	-21.8796	0.5261	-20.5430	0.5616	-17.7414	0.6198	-12.6107
	0.6	0.7	0.5613	-42.5461	0.5961	-41.2697	0.6422	-38.2234	0.7091	-32.3026
	0.7		0.5661	-43.6933	0.6027	-42.4135	0.6518	-39.3276	0.7247	-33.2878
	0.8		0.6045	-72.1668	0.6440	-71.0323	0.6943	-67.7105	0.7621	-60.7828
	0.9		0.6106	-73.7288	0.6518	-72.5708	0.7047	-69.1646	0.7768	-62.0319

Note: $*$ indicate $\mu_1^{(0)}$ do not exist.

TABLE 4

Percent relative loss in precision L_{22} of the estimator T_2 with respect to ξ_2

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_2^{(0)}$	L_{22}	$\mu_2^{(0)}$	L_{22}	$\mu_2^{(0)}$	L_{22}	$\mu_2^{(0)}$	L_{22}
0.05	0.6	0.5	0.2770	-9.5662	0.2447	-7.5944	0.1758	-3.9597	*	-
	0.7		0.2682	-10.2072	0.2302	-8.1826	0.1464	-4.3954	*	-
	0.8		0.3893	-30.6856	0.3946	-29.2931	0.3971	-26.3618	0.3917	-21.0108
	0.9		0.3878	-31.7136	0.3923	-30.3386	0.3934	-27.4124	0.3842	-22.0403
	0.6	0.7	0.4623	-58.9681	0.4842	-57.9984	0.5118	-55.2417	0.5491	-49.6048
	0.7		0.4643	-60.4820	0.4870	-59.5500	0.5158	-56.8143	0.5555	-51.1629
	0.8		0.5101	-100.5489	0.5406	-100.1881	0.5788	-97.6162	0.6293	-91.3595
	0.9		0.5142	-102.8554	0.5458	-102.5451	0.5857	-99.9841	0.6392	-93.6551
0.10	0.6	0.5	0.3017	-8.5996	0.2755	-6.7605	0.2180	-3.3648	0.0518	2.8295
	0.7		0.2943	-9.2521	0.2632	-7.3777	0.1928	-3.8732	*	-
	0.8		0.4071	-29.1205	0.4155	-27.7602	0.4230	-24.8894	0.4268	-19.6490
	0.9		0.4063	-30.1274	0.4142	-28.7862	0.4209	-25.9242	0.4227	-20.6727
	0.6	0.7	0.4753	-56.7388	0.4989	-55.7426	0.5289	-52.9685	0.5700	-47.3271
	0.7		0.4777	-58.2033	0.5022	-57.2402	0.5337	-54.4807	0.5777	-48.8142
	0.8		0.5196	-97.4546	0.5510	-97.0182	0.5904	-94.3765	0.6426	-88.0641
	0.9		0.5239	-99.6758	0.5565	-99.2817	0.5977	-96.6406	0.6530	-90.2417
0.15	0.6	0.5	0.3262	-7.5987	0.3061	-5.8702	0.2598	-2.6746	0.1224	3.0590
	0.7		0.3200	-8.2575	0.2958	-6.5077	0.2388	-3.2384	0.0530	2.8223
	0.8		0.4247	-27.5615	0.4362	-26.2226	0.4486	-23.3955	0.4616	-18.2364
	0.9		0.4245	-28.5456	0.4359	-27.2262	0.4482	-24.4092	0.4608	-19.2431
	0.6	0.7	0.4881	-54.5545	0.5134	-53.5274	0.5459	-50.7291	0.5907	-45.0722
	0.7		0.4909	-55.9703	0.5172	-54.9715	0.5513	-52.1809	0.5996	-46.4873
	0.8		0.5290	-94.4502	0.5613	-93.9382	0.6019	-91.2259	0.6558	-84.8551
	0.9		0.5335	-96.5893	0.5670	-96.1117	0.6096	-93.3900	0.6667	-86.9189
0.20	0.6	0.5	0.3505	-6.5684	0.3364	-4.9302	0.3013	-1.8989	0.1925	3.4578
	0.7		0.3455	-7.2289	0.3281	-5.5802	0.2844	-2.5026	0.1365	3.0665
	0.8		0.4421	-26.0104	0.4567	-24.6832	0.4740	-21.8845	0.4962	-16.7796
	0.9		0.4426	-26.9703	0.4574	-25.6618	0.4751	-22.8725	0.4985	-17.7596
	0.6	0.7	0.5008	-52.4141	0.5278	-51.3521	0.5626	-48.5235	0.6113	-42.8412
	0.7		0.5040	-53.7821	0.5321	-52.7434	0.5688	-49.9151	0.6213	-44.1836
	0.8		0.5383	-91.5312	0.5715	-90.9438	0.6133	-88.1601	0.6689	-81.7288
	0.9		0.5430	-93.5916	0.5774	-93.0310	0.6213	-90.2280	0.6803	-83.6828

Note: $*$ indicate $\mu_2^{(0)}$ do not exist.

TABLE 5

Percent relative loss in precision L_1 of the estimator T_1 with respect to $\bar{y}_n^*$

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_1^{(0)}$	L_1	$\mu_1^{(0)}$	L_1	$\mu_1^{(0)}$	L_1	$\mu_1^{(0)}$	L_1
0.05	0.6	0.5	0.2903	-21.0924	0.2613	-23.8954	0.1985	-26.6432	0.0191	-28.4628
	0.7		0.2823	-21.8119	0.2480	-24.5949	0.1714	-27.2262	*	-
	0.8		0.4039	-43.6945	0.4117	-48.0455	0.4183	-52.9207	0.4205	-58.4401
	0.9		0.4029	-44.8170	0.4103	-49.2362	0.4159	-54.1891	0.4157	-59.7954
	0.6	0.7	0.4781	-73.5038	0.5022	-79.5041	0.5327	-86.2874	0.5746	-94.0174
	0.7		0.4806	-75.1178	0.5055	-81.2219	0.5376	-88.1186	0.5826	-95.9615
	0.8		0.5277	-116.3935	0.5598	-124.7436	0.6003	-134.1858	0.6540	-144.8718
	0.9		0.5321	-118.7816	0.5655	-127.2714	0.6079	-136.8471	0.6648	-147.6201
0.10	0.6	0.5	0.3281	-22.6205	0.3084	-25.6370	0.2630	-28.7685	0.1278	-31.5290
	0.7		0.3220	-23.3721	0.2983	-26.3954	0.2423	-29.4805	0.0594	-31.8686
	0.8		0.4358	-44.3492	0.4492	-48.7261	0.4648	-53.6367	0.4837	-59.2119
	0.9		0.4361	-45.4541	0.4496	-49.8993	0.4653	-54.8887	0.4848	-60.5565
	0.6	0.7	0.5065	-72.7498	0.5341	-78.5976	0.5700	-85.1562	0.6203	-92.5210
	0.7		0.5098	-74.2860	0.5386	-80.2221	0.5765	-86.8689	0.6309	-94.2988
	0.8		0.5540	-112.9367	0.5887	-120.8735	0.6325	-129.7499	0.6910	-139.6062
	0.9		0.5591	-115.1407	0.5951	-123.1852	0.6411	-132.1481	0.7032	-142.0149
0.15	0.6	0.5	0.3653	-23.9761	0.3549	-27.1519	0.3266	-30.5668	0.2354	-34.0195
	0.7		0.3611	-24.7479	0.3478	-27.9492	0.3122	-31.3669	0.1877	-34.6676
	0.8		0.4671	-44.8684	0.4861	-49.2306	0.5105	-54.1098	0.5459	-59.6130
	0.9		0.4686	-45.9480	0.4882	-50.3743	0.5139	-55.3249	0.5528	-60.9029
	0.6	0.7	0.5342	-71.9541	0.5654	-77.6226	0.6065	-83.9149	0.6651	-90.8427
	0.7		0.5383	-73.4111	0.5710	-79.1507	0.6146	-85.5028	0.6783	-92.4405
	0.8		0.5796	-109.7100	0.6167	-117.2334	0.6639	-125.5404	0.7270	-134.5553
	0.9		0.5852	-111.7435	0.6238	-119.3442	0.6733	-127.6923	0.7405	-136.6433
0.20	0.6	0.5	0.4020	-25.1713	0.4006	-28.4559	0.3895	-32.0594	0.3422	-35.9655
	0.7		0.3995	-25.9526	0.3966	-29.2741	0.3812	-32.9109	0.3146	-36.8037
	0.8		0.4979	-45.2639	0.5223	-49.5746	0.5554	-54.3618	0.6073	-59.6752
	0.9		0.5005	-46.3119	0.5261	-50.6787	0.5616	-55.5222	0.6198	-60.8724
	0.6	0.7	0.5613	-71.1212	0.5961	-76.5871	0.6422	-82.5764	0.7091	-89.0038
	0.7		0.5661	-72.4983	0.6027	-78.0168	0.6518	-84.0350	0.7247	-90.4112
	0.8		0.6045	-106.6797	0.6440	-113.7903	0.6943	-121.5254	0.7621	-129.6898
	0.9		0.6106	-108.5548	0.6518	-115.7135	0.7047	-123.4460	0.7768	-131.4742

Note: $*$ indicate $\mu_1^{(0)}$ do not exist.

TABLE 6

Percent relative loss in precision L_2 of the estimator T_2 with respect to $\bar{y}_n^*$

ρ_{yx}			0.5		0.6		0.7		0.8	
W	ρ_{yz_2}	$\rho_{z_1z_2}$	$\mu_2^{(0)}$	L_2	$\mu_2^{(0)}$	L_2	$\mu_2^{(0)}$	L_2	$\mu_2^{(0)}$	L_2
0.05	0.6	0.5	0.2770	-21.6653	0.2447	-24.4060	0.1758	-27.0194	*	-
	0.7		0.2682	-22.3771	0.2302	-25.0861	0.1464	-27.5517	*	-
	0.8		0.3893	-45.1168	0.3946	-49.4951	0.3971	-54.3905	0.3917	-59.9071
	0.9		0.3878	-46.2583	0.3923	-50.7040	0.3934	-55.6742	0.3842	-61.2675
	0.6	0.7	0.4623	-76.5226	0.4842	-82.6856	0.5118	-89.6764	0.5491	-97.6921
	0.7		0.4643	-78.2036	0.4870	-84.4796	0.5158	-91.5978	0.5555	-99.7509
	0.8		0.5101	-122.6949	0.5406	-131.4675	0.5788	-141.4501	0.6293	-152.8680
	0.9		0.5142	-125.2562	0.5458	-134.1928	0.5857	-144.3432	0.6392	-155.9014
0.10	0.6	0.5	0.3017	-23.8512	0.2755	-26.7781	0.2180	-29.7058	0.0518	-31.8743
	0.7		0.2943	-24.5954	0.2632	-27.5110	0.1928	-30.3438	*	-
	0.8		0.4071	-47.2540	0.4155	-51.7152	0.4230	-56.7156	0.4268	-62.3808
	0.9		0.4063	-48.4024	0.4142	-52.9336	0.4209	-58.0141	0.4227	-63.7701
	0.6	0.7	0.4753	-78.7510	0.4989	-84.9443	0.5289	-91.9503	0.5700	-99.9439
	0.7		0.4777	-80.4211	0.5022	-86.7227	0.5337	-93.8478	0.5777	-101.9621
	0.8		0.5196	-125.1849	0.5510	-133.9591	0.5904	-143.9106	0.6426	-155.2299
	0.9		0.5239	-127.7180	0.5565	-136.6470	0.5977	-146.7516	0.6530	-158.1852
0.15	0.6	0.5	0.3262	-25.9390	0.3061	-29.0293	0.2598	-32.2303	0.1224	-35.0249
	0.7		0.3200	-26.7100	0.2958	-29.8062	0.2388	-32.9563	0.0530	-35.3547
	0.8		0.4247	-49.3044	0.4362	-53.8338	0.4486	-58.9158	0.4616	-64.6864
	0.9		0.4245	-50.4563	0.4359	-55.0569	0.4482	-60.2213	0.4608	-66.0886
	0.6	0.7	0.4881	-80.8984	0.5134	-87.1115	0.5459	-94.1176	0.5907	-102.0648
	0.7		0.4909	-82.5555	0.5172	-88.8715	0.5513	-95.9873	0.5996	-104.0359
	0.8		0.5290	-127.5943	0.5613	-136.3621	0.6019	-146.2716	0.6558	-157.4767
	0.9		0.5335	-130.0981	0.5670	-139.0112	0.6096	-149.0587	0.6667	-160.3513
0.20	0.6	0.5	0.3505	-27.9313	0.3364	-31.1628	0.3013	-34.5962	0.1925	-37.9175
	0.7		0.3455	-28.7242	0.3281	-31.9753	0.2844	-35.3936	0.1365	-38.4764
	0.8		0.4421	-51.2707	0.4567	-55.8540	0.4740	-60.9947	0.4962	-66.8280
	0.9		0.4426	-52.4230	0.4574	-57.0772	0.4751	-62.2997	0.4985	-68.2279
	0.6	0.7	0.5008	-82.9673	0.5278	-89.1901	0.5626	-96.1817	0.6113	-104.0589
	0.7		0.5040	-84.6095	0.5321	-90.9292	0.5688	-98.0198	0.6213	-105.9766
	0.8		0.5383	-129.9259	0.5715	-138.6797	0.6133	-148.5368	0.6689	-159.6125
	0.9		0.5430	-132.3994	0.5774	-141.2887	0.6213	-151.2683	0.6803	-162.4041

Note: $*$ indicate $\mu_2^{(0)}$ do not exist.

increasing values of ρ_{yx} and we also get increasing values of $\mu_1^{(0)}$ for the value 0.7 of $\rho_{z_1z_2}$. This implies that if one uses the information on low correlated study variables there is an appreciable gain in the precision of estimates.
(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_1^{(0)}$ increase and L_{12} decrease with the increasing values of $\rho_{z_1z_2}$. These behaviors are on the expected lines as the efficiencies of the proposed estimators will increase when one utilizes the information on highly correlated auxiliary variable over both occasions.
(d) For the fixed values of $\rho_{z_1z_2}$, $\rho_{yx}$and ρ_{yz_2}, the values of $\mu_1^{(0)}$ and L_{12} increase with the increasing values of W. This behavior is similar to that discussed in 2 (d).

(4) From Table 4 it is seen that:
(a) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yx}, no pattern is seen for the value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_2^{(0)}$ and decreasing values of L_{22} with the increasing values of ρ_{yz_2}.
(b) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yz_2}, the values of L_{22} increase with the increasing values of ρ_{yx} and we get increasing values of $\mu_2^{(0)}$ for the value 0.7 of $\rho_{z_1z_2}$.

(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_2^{(0)}$ increase and L_{22} decrease with the increasing values of $\rho_{z_1z_2}$. These results indicate that precision of estimates will increase if auxiliary variables over both occasions are highly correlated.
(d) For the fixed values of $\rho_{z_1z_2}$, ρ_{yx} and ρ_{yz_2}, the values of $\mu_2^{(0)}$ and L_{22} increase with the increasing values of W. This behavior shows that the larger the non-response rate, larger the fresh sample is required at the current occasion.

(5) From Table 5 it is visible that:
(a) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yx}, we did not get any pattern for the value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_1^{(0)}$ and decreasing values of L_1 with the increasing values of ρ_{yz_2}. Thus, more the correlation between study variables and auxiliary variable more gain is observed.
(b) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yz_2}, the values of L_1 decrease with the increasing values of $\rho_{yx}$and we get increasing values of $\mu_1^{(0)}$ for the value 0.7 of $\rho_{z_1z_2}$.
(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_1^{(0)}$ increase and L_1 decrease with the increasing values of $\rho_{z_1z_2}$.
(d) For the fixed values of $\rho_{z_1z_2}$, $\rho_{yx}$and ρ_{yz_2}, the values of $\mu_1^{(0)}$ increase and L_1 decrease with the increasing values of W.

(6) From Table 6 it is observed that:
(a) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yx}, no pattern is seen for the value 0.5 of $\rho_{z_1z_2}$ but for the value 0.7 of $\rho_{z_1z_2}$ we get increasing values of $\mu_2^{(0)}$ and decreasing values of L_2 with the increasing values of ρ_{yz_2}.
(b) For the fixed values of W, $\rho_{z_1z_2}$ and ρ_{yz_2}, the values of L_2 decrease with the increasing values of ρ_{yx} and we also get increasing values of $\mu_2^{(0)}$ for the value 0.7 of $\rho_{z_1z_2}$.

(c) For the fixed values of W, ρ_{yx} and ρ_{yz_2}, the values of $\mu_2^{(0)}$ increase and L_2 decrease with the increasing values of $\rho_{z_1 z_2}$.
(d) For the fixed values of $\rho_{z_1 z_2}$, ρ_{yx} and ρ_{yz_2}, the values of $\mu_2^{(0)}$ increase and L_2 decrease with the increasing values of W.

9. Conclusions and recommendations

From above tables, it may be concluded that for all cases the percent relative losses in precisions are observed wherever the optimum values of μ exist when non-response occurs at the current occasion. From the Tables 1- 2, it is obvious that loss is observed due to the presence of non-response, but the negative impact of non-response is not appreciable due to the utilization of the sub-sampling technique of non respondents. From the Tables 3-6, when the proposed estimators compared with the natural successive sampling estimator and Hansen and Hurwitz (1946) estimator, negative losses (gain) are observed, because of the presence of dynamic auxiliary information in the form of exponential type estimators. Further, it is also being noticed that for all parametric combinations, the estimator T_2 is performing much better than the estimator T_1. Hence, if one has to make a choice between T_1 and T_2 then T_2 is always preferable over T_1. Finally, looking on the nice behaviors of the proposed estimators one may recommend them to the survey statisticians and practitioners for their practical applications.

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SUMMARY

Some Efficient Estimation Procedures under Non-Response in Two-Occasion Successive Sampling

This paper deals with the estimation of current population mean under non-response in two-occasion successive sampling. Information on a dynamic auxiliary variable has been used and efficient estimation procedures have been suggested which are capable in minimizing the negative impact of non-response when it occurred on current occasion in two-occasion successive sampling. Properties of the proposed estimation procedures have been studied and suitable recommendations are made.

Keywords: Non-response; successive sampling; dynamic auxiliary variable; bias; mean square error