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# SOME HOMOLOGICAL INVARIANTS OF THE MAPPING CLASS GROUP OF A THREE-DIMENSIONAL HANDLEBODY

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**Abstract.** We show that, if $g \geq 2$, the virtual cohomological dimension of the mapping class group of a three-dimensional handlebody of genus $g$ is equal to $4g-5$ and its Euler number is equal to 0.

**1. Introduction.** A genus $g$ *handlebody* $H_g$ is an oriented 3-manifold which is constructed from 3-ball by attaching $g$ 1-handles. The *mapping class group* $\mathcal{H}_g$ of $H_g$ is defined to be the group of isotopy classes of orientation-preserving diffeomorphisms of $H_g$. This group $\mathcal{H}_g$ is a subgroup of the mapping class group $\mathcal{M}_g$ of a surface $\partial H_g$, that is, $\mathcal{M}_g = \pi_0(\mathrm{Diff}^+(\partial H_g))$, where $\mathrm{Diff}^+(\partial H_g)$ is the group of orientation preserving diffeomorphisms of $\partial H_g$. Throughout this paper, we assume $g \geq 2$.

The *cohomological dimension* of a group $G$, $\mathrm{cd}(G)$, is defined to be the largest number $n$ for which there exists a $G$-module $M$ with $H^n(G, M)$ nonzero. We remark that if $G_1 \subset G_2$, then $\mathrm{cd}(G_1) \leq \mathrm{cd}(G_2)$. Also, when $G$ has torsion, $\mathrm{cd}(G)$ is infinite. However, if $G$ has finite index torsion-free subgroups (we call $G$ *virtually torsion-free*), we define the *virtual cohomological dimension* of $G$, $\mathrm{vcd}(G)$, to be the cohomological dimension of a finite index torsion-free subgroup $\hat{G}$. A theorem of Serre [13] states that this number is independent of the choice of $\hat{G}$. For the virtual cohomological dimensions of $\mathcal{M}_g$ and $\mathcal{H}_g$, Harer [5] showed that $\mathrm{vcd}(\mathcal{M}_g) = 4g-5$, and McCullough [11] showed that $\mathrm{vcd}(\mathcal{H}_2) = 3$ and, if $g \geq 3$, $3g-2 \leq \mathrm{vcd}(\mathcal{H}_g) \leq 4g-5$. In this paper, we prove the following result.

THEOREM 1.1. *If $g \geq 2$, the virtual cohomological dimension of $\mathcal{H}_g$ is equal to $4g-5$.*

McCullough [12] informed the author that Hatcher has obtained (not published) this result by investigating the action of $\mathcal{H}_g$ on the disk complex defined by McCullough [11]. In this paper, by making an essential use of the construction of Mess given in [9], we prove the result and give an explicit description of a subgroup of $\mathcal{H}_g$ that attains $\mathrm{vcd}(\mathcal{H}_g)$.

We also give some remarks on the relationship between $\mathcal{H}_g$ and the outer automorphism group of the free group of rank $g$. We denote by $F_g$ the free group of rank $g$ and by $\mathrm{Out}(F_g)$ its outer automorphism group. There is a natural homomorphism from $\mathcal{H}_g$ to $\mathrm{Out}(F_g)$ defined

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by the action of diffeomorphisms on the fundamental group of $H_g$, which is a surjection [4]. Culler and Vogtmann [3] showed that $\mathrm{vcd}(\mathrm{Out}(F_g)) = 2g - 3$. This fact indicates that the kernel of the above surjection is, in some sense, big. In fact, McCullough [10] showed that the kernel of the above surjection is not finitely generated.

For any finitely generated abelian group $A$, we define the *rank* of $A$ by $\mathrm{rk}_{\mathbf{Z}}(A) = \dim_{\mathbf{Q}}(\mathbf{Q} \otimes_{\mathbf{Z}} A)$. For a torsion-free group $G$ of finite homological type, we define the *Euler characteristic* $\chi(G)$ (see [2]) by

$$\chi(G) = \sum_i (-1)^i \mathrm{rk}_{\mathbf{Z}}(H_i(G)) .$$

For a group $G$ of finite homological type which may have torsion, we choose a torsion-free subgroup $\hat{G}$ of finite index, and define $\chi(G)$ by

$$\chi(G) = \frac{\chi(\hat{G})}{(G : \hat{G})} ,$$

where $(G : \hat{G})$ denotes the index of $\hat{G}$ in $G$. Since, $\mathcal{H}_g$ is of type VFL [11], we can define $\chi(\mathcal{H}_g)$. Then we show the following result.

THEOREM 1.2. $\chi(\mathcal{H}_g) = 0$.

Harer and Zagier [6] calculated $\chi(\mathcal{M}_g)$, which turned out to be quite different from $\chi(\mathcal{H}_g)$. This result indicates considerable difference between $\mathcal{M}_g$ and $\mathcal{H}_g$.

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**2. Proof of Theorem 1.1.** In general, for an oriented $C^\infty$-manifold $A$ and its subset $B$, by $\mathrm{Diff}^+(A)$ we denote the group of all orientation preserving diffeomorphisms of $A$, by $\mathrm{Diff}^+(A,\ \mathrm{fix}\ B)$ the group of elements of $\mathrm{Diff}^+(A)$ whose restriction to $B$ are the identity map, and by $\mathrm{Diff}^+(A,\ B)$ the group of elements of $\mathrm{Diff}^+(A)$ which preserve $B$ as a set. For a disk $D$ in $\partial H_g$, we define $\mathcal{H}_{g,1} = \pi_0(\mathrm{Diff}^+(H_g, \mathrm{fix}\ D))$, and $\mathcal{M}_{g,1} = \pi_0(\mathrm{Diff}^+(\partial H_g, \mathrm{fix}\ D))$. For the center $p$ of the disk $D$, we define $\mathcal{H}_g^1 = \pi_0(\mathrm{Diff}^+(H_g, \mathrm{fix}\ \{p\}))$, and $\mathcal{M}_g^1 = \pi_0(\mathrm{Diff}^+(\partial H_g, \mathrm{fix}\ \{p\}))$. Let $D_1, D_2, \dots, D_g$ be the cocores of 1-handles which are used to construct $H_g$. These disks $D_1, D_2, \dots, D_g$ are properly embedded disks in $H_g$. Let $E_1, \dots, E_{g-1}$ and $C$ be properly embedded disks as indicated in Figure 1.

We introduce some specific elements of $\mathcal{H}_g$. For a disk $D$ properly embedded in $H_g$, let $N$ be a regular neighborhood of $D$ in $H_g$. We parametrize $N$ by $\phi : [-1, 1] \times D^2 \to N$ such that $\phi(\{0\} \times D^2) = D$ and $\phi([-1, 1] \times \partial D^2)$ is an annulus in $\partial H_g$. Let $\psi$ be a diffeomorphism of $[-1, 1] \times D^2$ defined by $\psi(t, r, \theta) = (t, r, \theta + (1 - t)\pi)$, where $(r, \theta)$ is a polar coordinate of $D^2$. The map $\delta_D : H_g \to H_g$, defined by $\delta_D(x) = \phi \circ \psi \circ \phi^{-1}(x)$ if

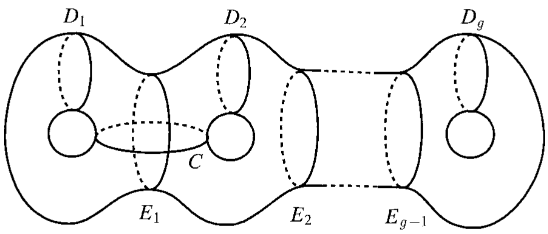


FIGURE 1.

$x \in N$, $= x$ if $x \notin N$, is an orientation preserving diffeomorphism of $H_g$, which we call *a disk twist* about $D$. The isotopy class of $\delta_D$, denoted by $d_D$, is an element of $\mathcal{H}_g$, which we call *a disk twist* about $D$. For an annulus $A$ properly embedded in $H_g$, let $\phi : [-1, 1] \times S^1 \times [0, 1] \to N$ be a parametrization of a regular neighborhood $N$ of $A$ in $H_g$ such that $\phi|_{\{0\}\times S^1\times[0,1]}$ is a parametrization of $A$, and let $\psi$ be the diffeomorphism on $[-1, 1] \times S^1 \times [0, 1]$ defined by $\psi(t, \theta, s) = (t, \theta + (1 - t)\pi, s)$, where $\theta$ is a polar coordinate of $S^1$. We define $\alpha_A \in \mathrm{Diff}^+(H_g)$, which we call *an annulus twist* about $A$, in the same manner as the definition of $\delta_D$. The isotopy class of $\alpha_A$, denoted by $a_A$, is an element of $\mathcal{H}_g$, which is called *an annulus twist* about $A$.

We now introduce the following terminologies for later use. Let $N$ be a regular neighborhood of $\partial H_g$ in $H_g$, and $A$ be an annulus in $\partial H_g$. We parametrize $N$ as $\phi : [0, 1] \times \partial H_g \to N$ such that $\phi(\{0\} \times \partial H_g) = \partial H_g$ and $\phi|_{\{0\}\times\partial H_g}$ is an identity map. The set $A' = \phi(\partial A \times [0, 1] \cup A \times \{1\})$ is an annulus properly embedded in $H_g$. We say that "we *push* $A$ *into* $H_g$" if we obtain $A'$ from $A$. Similarly, for a disk $D$ in $\partial H_g$, we say that "we *push* $D$ *into* $H_g$" if we obtain a disk $D'$ from $D$ in the same manner as above.

Mess [9] discovered certain subgroups $B_g$, $B_{g,1}$, called *Mess subgroups*, of the mapping class groups $\mathcal{M}_g$, $\mathcal{M}_{g,1}$, respectively, which are defined in a recursive manner as follows (this definition is quoted from §6.3 of [8]):

*Step* 0: Let $B_2$ be the subgroup of $\mathcal{M}_2$ generated by Dehn twists about any three pairwise disjoint and pairwise nonisotopic simple closed curves $C_0, C_1, C_2$ in $\partial H_2$.

*Step* $1_g$: We assume that $B_g$ $(g \geq 2)$ is already defined. There is a surjection from $\mathrm{Diff}^+(\partial H_g, \text{fix } D)$ to $\mathrm{Diff}^+(\partial H_g)$ defined by forgetting the disk $D$, which induces a surjection $f : \mathcal{M}_{g,1} \to \mathcal{M}_g$. Let $B_{g,1}$ be the preimage of $B_g$ under $f$.

*Step* $2_g$: By restricting each diffeomorphism, we obtain a homomorphism $\rho : \mathrm{Diff}^+(\partial H_g, \text{fix } D) \to \mathrm{Diff}^+(\partial H_g \setminus \text{int}\, D, \text{fix } \partial D)$. We consider an embedding $\partial H_g \setminus \text{int}\, D$ into $\partial H_{g+1}$ and identify $\partial H_g \setminus \text{int}\, D$ with its image. By extending each diffeomorphism of $\partial H_g \setminus \text{int}\, D$, whose restriction on $\partial D$ is the identity, across the complement of $\partial H_g \setminus \text{int}\, D$ in $\partial H_{g+1}$, we obtain a homomorphism $\iota : \mathrm{Diff}^+(\partial H_g \setminus \text{int}\, D, \text{fix } \partial D) \to \mathrm{Diff}^+(\partial H_{g+1})$. The composition $\iota \circ \rho$ induces a homomorphism $i : \mathcal{M}_{g,1} \to \mathcal{M}_{g+1}$. In the complement of $\partial H_g \setminus \text{int}\, D$ in $\partial H_{g+1}$, we choose a nontrivial simple closed curve $C$ that is not isotopic into $\partial H_g \setminus \text{int}\, D$ and consider the Dehn twist $t \in \mathcal{M}_{g+1}$ about this curve. Let $T$ be the infinite cyclic group generated by $t$. We define $B_{g+1}$ as the group generated by $i(B_{g,1})$ and $T$.

Mess [9] showed the following result (see also Corollary 6.3B of [8]).

PROPOSITION 2.1. *The cohomological dimension of $B_g$ is equal to* $4g - 5$.

We will first show the following lemma, which is remarked by Mess [9, p. 4] without a proof.

LEMMA 2.2. *$\mathcal{H}_g$ contains a subgroup isomorphic to $B_g$.*

REMARK 2.3. The definition of $B_g$ involves some choices. This lemma means that, with some good choices, $B_g$ is realized as a subgroup of $\mathcal{H}_g$.

PROOF. Along the steps of the definition of $B_g$, we will check that $B_2$, $B_{g,1}$, $B_{g+1}$ can be constructed as subgroups of $\mathcal{H}_2$, $\mathcal{H}_{g,1}$, $\mathcal{H}_{g+1}$, respectively. In each step, we use the same notation as used in definitions of $B_g$ and $B_{g,1}$.

*Step* 0: We choose $C_0 = \partial D_1$, $C_1 = \partial C$, $C_2 = \partial D_2$. Then $B_2 \subset \mathcal{H}_2$.

*Step* $1_g$: We assume that $B_g \subset \mathcal{H}_g$. Let $g_1, \ldots, g_n$ be generators of $B_g$. For each $g_i$, we can choose an element $\tilde{g}_i$ of $\mathcal{H}_{g,1}$ such that $f(\tilde{g}_i) = g_i$. By the definition, $B_{g,1}$ is generated by the kernel of $f$ and $\tilde{g}_1, \ldots, \tilde{g}_n$. In order to obtain generators for the kernel of $f$, we consider the following two short exact sequences:

$$1 \longrightarrow \mathbf{Z} \longrightarrow \mathcal{M}_{g,1} \xrightarrow{\alpha} \mathcal{M}_g^1 \longrightarrow 1\,, \tag{S1}$$

$$1 \longrightarrow \pi_1(\partial H_g, p) \xrightarrow{\beta} \mathcal{M}_g^1 \xrightarrow{\gamma} \mathcal{M}_g \longrightarrow 1\,. \tag{S2}$$

The group $\mathbf{Z}$ in (S1) is an infinite cyclic group generated by the Dehn twist $d$ about $\partial D$. The homomorphism $\alpha$ is induced by the homomorphism from $\mathrm{Diff}^+(\partial H_g, \mathrm{fix}\ D)$ to $\mathrm{Diff}^+(\partial H_g, \mathrm{fix}\{p\})$ defined by collapsing $D$ into a point $p$. The sequence (S2) is introduced by Birman [1]. The homomorphism $\gamma$ is induced by the homomorphism from $\mathrm{Diff}^+(\partial H_g,$ $\mathrm{fix}\ \{p\})$ to $\mathrm{Diff}^+(\partial H_g)$ defined by forgetting the point $p$. The group $\pi_1(\partial H_g, p)$ is generated by simple loops in $\partial H_g$ with base point $p$. Let $l_1, \ldots, l_{2g}$ be simple loops in $\partial H_g$, whose homotopy classes generate $\pi_1(\partial H_g, p)$. For each $l_i$, let $L_i$ be an annulus in $\partial H_g$, which is a regular neighborhood of $l_i$ such that $L_i \supset D \ni p$. $\partial L_i$ consists of two simple closed curves $l_i^1$ and $l_i^2$ in $\partial H_g$. The homomorphism $\beta$ is defined so that it maps a homotopy class of $l_i$ (denote by $[l_i]$ for short) to that of $\lambda_i =$ (+Dehn twist about $l_i^1$) $\times$ ($-$Dehn twist about $l_i^2$), which is also an element of $\mathrm{Diff}^+(\partial H_g, \mathrm{fix}\ D)$, and $\alpha$ (an element of $\mathcal{M}_{g,1}$ represented by $\lambda_i) = \beta([l_i])$. Let $\tilde{l}_i$ be an element of $\mathcal{M}_{g,1}$ represented by $\lambda_i$. The kernel of $f$ is generated by $d$ and $\tilde{l}_1, \ldots, \tilde{l}_{2g}$, since it is equal to $\alpha^{-1}$ (the kernel of $\gamma$) $= \alpha^{-1}$ (the image of $\beta$). Let $D'$ be a disk in $H_g$ obtained by pushing $D$ into $H_g$, and $\delta_{D'}$ be the disk twist about $D'$. Let $L_i'$ be an annulus obtained by pushing $L_i$ into $H_g$, and $\alpha_{L_i'}$ be the annulus twist about $L_i'$. The diffeomorphisms $\delta_{D'}$ and $\alpha_{L_i'}$ are elements of $\mathrm{Diff}^+(H_g, \mathrm{fix}\ D)$, whose restrictions to $\partial H_g$ represent $d$ and $\tilde{l}_i$, respectively. This fact shows that the kernel of $f$ is included in $\mathcal{H}_{g,1}$. Hence, $B_{g,1} \subset \mathcal{H}_{g,1}$.

*Step* $2_g$: It is easy to see that $i(B_{g,1}) \subset \mathcal{H}_{g+1}$. If we choose $C = \partial D_{g+1}$, then $t \in \mathcal{H}_{g+1}$. Therefore, $B_{g+1} \subset \mathcal{H}_{g+1}$. □

Along the line of the proof of Theorem 6.4.A in [8], we will prove Theorem 1.1.

PROOF OF THEOREM 1.1. There is a natural homomorphism $\mathcal{M}_g \to \mathrm{Aut}\,(H_1(\partial H_g, \mathbf{Z}/3\mathbf{Z}))$ defined by the action of diffeomorphisms on $H_1(\partial H_g, \mathbf{Z}/3\mathbf{Z})$. Let $\Gamma$ be the kernel of this homomorphism. It is a classical result that $\Gamma$ is torsion-free (see, e.g., Ivanov [7, Corollary 1.5]). Therefore, $\Gamma$, $\mathcal{H}_g \cap \Gamma$ and $B_g \cap \Gamma$ are finite index torsion-free subgroups of $\mathcal{M}_g$, $\mathcal{H}_g$ and $B_g$, respectively. By the definition of virtual cohomological dimension, $\mathrm{vcd}(\mathcal{M}_g) = \mathrm{cd}(\Gamma)$, $\mathrm{vcd}(\mathcal{H}_g) = \mathrm{cd}(\mathcal{H}_g \cap \Gamma)$ and $\mathrm{vcd}(B_g) = \mathrm{cd}(B_g \cap \Gamma)$. By Harer [5, Theorem 4.1], $\mathrm{vcd}(\mathcal{M}_g) = 4g - 5$, and hence, $\mathrm{cd}(\Gamma) = 4g - 5$. By Proposition 2.1, $\mathrm{vcd}(B_g) = \mathrm{cd}(B_g) = 4g - 5$, and hence, $\mathrm{cd}(B_g \cap \Gamma) = 4g - 5$. By Lemma 2.2, $B_g \cap \Gamma \subset \mathcal{H}_g \cap \Gamma \subset \Gamma$. Therefore, $\mathrm{cd}(B_g \cap \Gamma) \leq \mathrm{cd}(\mathcal{H}_g \cap \Gamma) \leq \mathrm{cd}(\Gamma)$. These facts show the theorem.

**3. Proof of Theorem 1.2.** McCullough defined a *disk complex* L in [11] and used it to estimate $\mathrm{vcd}(\mathcal{H}_g)$. We here review the definition of $L$. By a *disk* in $H_g$, we mean a properly embedded 2-disk in $H_g$. A disk $D$ is called *essential* when $\partial D$ does not bound a 2-disk in $\partial H_g$. The disk complex $L$ of $H_g$ is a simplicial complex whose vertices are the isotopy classes of essential disks in $H_g$, and whose simplices are defined by the rule that a collection of $n+1$ distinct vertices spans an $n$-simplex if and only if it admits a collection of representatives which are pairwise disjoint. McCullough showed the following Theorem in [11, Theorem 5.3].

THEOREM 3.1 ([11]). *The disk complex $L$ of $H_g$ is contractible.*

We use the following Propositions regarding Euler characteristics of groups (see, e.g., [2, Proposition §IX 7.3]) .

PROPOSITION 3.2. *Let $1 \to G' \to G \to G'' \to 1$ be a short exact sequence of groups, where $G'$ and $G''$ are of finite homology type. If $G$ is virtually torsion-free, then $G$ is of finite homological type and $\chi(G) = \chi(G')\chi(G'')$.*

PROPOSITION 3.3. *Let $X$ be a contractible simplicial complex on which $G$ act simplicially. For each simplex $\sigma$ of $X$, let $G_\sigma = \{g \in G \mid g\sigma = \sigma\}$. If $X$ has only finitely many cells* mod $G$, *and, for each simplex $\sigma$ of $X$, $G_\sigma$ is of finite homological type, then*

$$\chi(G) = \sum_{\sigma \in \mathcal{E}} (-1)^{\dim \sigma} \chi(G_\sigma)\,,$$

*where $\mathcal{E}$ is a set of representatives for the cells of $X$* mod $G$.

For each simplex $\sigma = \langle D_0, \ldots, D_n \rangle$ of $L$, Proposition 6.5 of [11] shows that $G_\sigma = \pi_0\mathrm{Diff}^+(H_g,\ D_0 \cup \cdots \cup D_n)$. For the same simplex $\sigma$, let $\Gamma_\sigma$ be the graph defined as follows. The vertices of $\Gamma_\sigma$ correspond to the components of $H_g \setminus D_0 \cup \cdots \cup D_n$. Each edge corresponds to one of $D_0, \ldots, D_n$ and connects the vertices corresponding to the components attached along this disk. Let $H_g/D_0 \cup \cdots \cup D_n$ be the space obtained from $H_g$ by collapsing each $D_i$ to one point, and $\delta$ be a homomorphism from $G_\sigma$ to $\pi_0\mathrm{Diff}^+(H_g/D_0 \cup \cdots \cup D_n, D_0/D_0 \cup \cdots \cup D_n/D_n)$ which is induced by collapsing each $D_i$ to one point. Let $\mathbf{Z}^{n+1}$ denote the free abelian group generated by disk twists about $D_0, \ldots, D_n$. Then we have the following exact sequence:

(S3)

$$1 \longrightarrow \boldsymbol{Z}^{n+1} \longrightarrow G_\sigma \xrightarrow{\delta} \pi_0\mathrm{Diff}^+(H_g/D_0 \cup \cdots \cup D_n,\, D_0/D_0 \cup \cdots \cup D_n/D_n) \longrightarrow 1\,.$$

Let $\varepsilon$ be the natural homomorphism from $\pi_0\mathrm{Diff}^+(H_g/D_0\cup\cdots\cup D_n,\, D_0/D_0\cup\cdots\cup D_n/D_n)$ to the group of automorphisms of $\Gamma_\sigma$. Let $A_\sigma$ be the image of $\varepsilon$, which is a finite group, since the group of automorphisms of $\Gamma_\sigma$ is a finite group. By $H'_g$ we denote the 3-manifold obtained by cutting $H_g$ along $D_0, \dots, D_n$, and by $D_0^1, D_0^2, \dots, D_n^1, D_n^2$ the disks on $\partial H'_g$ obtained as a result of cutting, and by $H'_g/D_0^1 \cup D_0^2 \cup \cdots \cup D_n^1 \cup D_n^2$ the space obtained from $H'_g$ by collapsing each $D_j^i$ ($i = 1, 2,\ j = 0, \dots, n$) to one point. Then we obtain the following exact sequence:

(S4)

$$\begin{aligned} 1 \longrightarrow \pi_0\mathrm{Diff}^+(&H'_g/D_0^1 \cup D_0^2 \cup \cdots \cup D_n^1 \cup D_n^2,\\ &\mathrm{fix}\ D_0^1/D_0^1 \cup D_0^2/D_0^2 \cup \cdots \cup D_n^1/D_n^1 \cup D_n^2/D_n^2)\\ \longrightarrow \pi_0\mathrm{Diff}^+(&H_g/D_0 \cup \cdots \cup D_n,\, D_0/D_0 \cup \cdots \cup D_n/D_n) \xrightarrow{\varepsilon} A_\sigma \longrightarrow 1\,. \end{aligned}$$

Since $\chi(\boldsymbol{Z}^{n+1}) = \chi((S^1)^{n+1}) = 0$, by applying Proposition 3.2 to (S3) and (S4), we obtain $\chi(G_\sigma) = 0$. Theorem 1.2 now follows from the above observation together with Theorem 3.1 and Proposition 3.3.

## References


[1] J. S. Birman, Mapping class groups and their relationship to braid groups, Comm. Pure and Appl. Math. 22 (1969), 213–238.

[2] K. S. Brown, Cohomology of groups, Grad. Texts in Math. 87, Springer-Verlag, New York-Berlin, 1982.

[3] M. Culler and K. Vogtmann, Moduli of graphs and automorphisms of free groups, Invent. Math. 84 (1986), 91–119.

[4] H. B. Griffiths, Automorphisms of a 3-dimensional handlebody, Abh. Math. Sem. Univ. Hamburg 26 (1964), 191–210.

[5] J. L. Harer, The virtual cohomological dimension of the mapping class group of an orientable surface, Invent. Math. 84 (1986), 157–176.

[6] J. L. Harer and D. Zagier, The Euler characteristic of the moduli space of curves, Invent. Math. 85 (1986), 457–485.

[7] N. V. Ivanov, Subgroups of Teichmüller modular groups, Transl. Math. Monogr. 115, American Mathematical Society, Providence, RI, 1992.

[8] N. V. Ivanov, Mapping class groups, Handbook of geometric topology, 523–633, North-Holland, Amsterdam, 2002.

[9] G. Mess, Unit tangent bundle subgroups of the mapping class groups, preprint (1990).

[10] D. McCullough, Twist groups of compact 3-manifolds, Topology 24 (1985), 461–474.

[11] D. McCullough, Virtually geometrically finite mapping class groups of 3-manifolds. J. Differential Geom. 33 (1991), 1–65.

[12] D. McCullough, private communication.

[13] J.-P. Serre, Cohomology des groupes discrets, Prospects in Mathematics (Proc. Sympos., Princeton Univ., Princeton, N. J., 1970), 77–169, Ann. Math. Studies 70, Princeton Univ. Press, Princeton, N. J., 1971.

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