

## SOME INEQUALITIES FOR HAUSDORFF OPERATORS

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*Abstract.* In this paper, we give the sufficient and necessary conditions for the boundedness of Hausdorff operators on various function spaces. Moreover, we consider Lipschitz estimates for the commutator of Hausdorff operators. We extend some known results.

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