Mathematical
Inequalities
& Applications
Volume 18, Number 2 (2015), 555–568 doi:10.7153/mia-18-41

SOME INEQUALITIES FOR QUANTUM TSALLIS ENTROPY RELATED TO THE STRONG SUBADDITIVITY

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Abstract. In this paper we investigate the inequality $S_q(\rho_{123}) + S_q(\rho_2) \leqslant S_q(\rho_{12}) + S_q(\rho_{23})$ $(*)$ where ρ_{123} is a state on a finite dimensional Hilbert space $\mathscr{H}_1 \otimes \mathscr{H}_2 \otimes \mathscr{H}_3$, and S_q is the Tsallis entropy. It is well-known that the strong subadditivity of the von Neumann entropy can be derived from the monotonicity of the Umegaki relative entropy. Now, we present an equivalent form of (*), which is an inequality of relative quasi-entropies. We derive an inequality of the form $S_q(\rho_{123}) + S_q(\rho_2) \leqslant S_q(\rho_{12}) + S_q(\rho_{23}) + f_q(\rho_{123})$, where $f_1(\rho_{123}) = 0$. Such a result can be considered as a generalization of the strong subadditivity of the von Neumann entropy. One can see that (*) does not hold in general (a picturesque example is included in this paper), but we give a sufficient condition for this inequality, as well.

Mathematics subject classification (2010): 46N50, 94A15, 46L30, 47L90.
Keywords and phrases: Tsallis entropy, strong subadditivity.

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