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Some Inequalities of Hermite-Hadamard Type for Functions Whose 3rd Derivatives Are *P*-Convex

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ABSTRACT

In the paper, the authors establish some new Hermite-Hadamard type inequalities for functions whose 3rd derivatives are *P*-convex.

Keywords: Integral Inequality; Hermite-Hadamard's Integral Inequality; *P*-Convex Function; Derivative

1. Introduction

The following definition is well known in the literature.

Definition 1.1. A function $f: I \subset \mathbb{R} \to \mathbb{R}$ is said to be convex if

$$f(\lambda x + (1-\lambda) y) \le \lambda f(x) + (1-\lambda) f(y) \tag{1.1}$$

holds for all $x, y \in I$ and $\lambda \in [0,1]$.

In [1], the concept of the so-called P-convex functions was introduced as follows.

Definition 1.2. ([1]) We say that a map $f: I \subset \mathbb{R} \to \mathbb{R}$ belongs to the class $P(I)$ if it is nonnegative and satisfies

$$f(\lambda x + (1-\lambda) y) \le f(x) + f(y) \tag{1.2}$$

for all $x, y \in I$ and $\lambda \in [0,1]$.

In [2], S. S. Dragomir proved the following theorems.

Theorem 1.1. ([2]) Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I° and $a, b \in I^\circ$ with $a < b$. If $|f'(x)|$ is convex on $[a,b]$, then

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x \right| \le \frac{(b-a)\left(|f'(a)| + |f'(b)|\right)}{8}, \tag{1.3}$$

Theorem 1.2. ([2]) Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I° and $a, b \in I^\circ$ with $a < b$. If $|f'(x)|^q$ is convex on $[a,b]$ for $q \ge 1$, then

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x \right| \le \frac{b-a}{4}\left(\frac{|f'(a)|^q + |f'(b)|^q}{2}\right)^{1/q}. \tag{1.4}$$

Theorem 1.3. ([3], Theorems 2) Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be an absolutely continuous function on I° such that $f''' \in L([a,b])$ for $a, b \in I^\circ$ with $a < b$. If $|f'''(x)|$ is quasi-convex on $[a,b]$, then

$$\left| \int_a^b f(x)\,\mathrm{d}x - \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \right| \le \frac{(b-a)^4}{1152}\left[\max\left\{ |f'''(a)|, \left| f'''\left(\frac{a+b}{2}\right) \right| \right\} + \max\left\{ \left| f'''\left(\frac{a+b}{2}\right) \right|, |f'''(b)| \right\} \right].$$

For more information and recent developments on this topic, please refer to [4-14] and closely related references therein.

The concepts of various convex functions have indeed found important places in contemporary mathematics as can be seen in a large number of research articles and books devoted to the field these days.

In this paper, we will establish some new Hermite-Hadamard type inequalities for functions whose 3rd derivatives are *P*-convex.

2. A Lemma

In this section, we establish an integral identity.

Lemma 2.1. Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be a three times differentiable mapping on I° and $a,b \in I^{\circ}$ with $a<b$. If $f' \in L[a,b]$, then

$$\begin{aligned}&\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24} f''\left(\frac{a+b}{2}\right)\\ &= \frac{(b-a)^3}{96}\left\{\int_0^1 (1-t)^3 f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\mathrm{d}t - \int_0^1 (1-t)^3 f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\mathrm{d}t\right\}.\end{aligned} \tag{2.1}$$

Proof. Integrating by part and changing variable of definite integral yield

$$\begin{aligned}&\int_0^1 (1-t)^3 f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\mathrm{d}t = -\frac{2}{b-a}\int_0^1 (1-t)^3 \mathrm{d}f''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\\ &= -\frac{2}{b-a}\left\{-f''\left(\frac{a+b}{2}\right) + 3\int_0^1 (1-t)^2 f''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\mathrm{d}t\right\}\\ &= \frac{2}{b-a}f''\left(\frac{a+b}{2}\right) + \frac{12}{(b-a)^2}\int_0^1 (1-t)^2 \mathrm{d}f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\\ &= \frac{2}{b-a}f''\left(\frac{a+b}{2}\right) - \frac{12}{(b-a)^2}f'\left(\frac{a+b}{2}\right) - \frac{48}{(b-a)^3}\int_0^1 (1-t)\mathrm{d}f\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\\ &= \frac{2}{b-a}f''\left(\frac{a+b}{2}\right) - \frac{12}{(b-a)^2}f'\left(\frac{a+b}{2}\right)\\ &+ \frac{48}{(b-a)^3}\left\{f\left(\frac{a+b}{2}\right) - \int_0^1 f\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\mathrm{d}t\right\}\end{aligned}$$

and

$$\begin{aligned}&\int_0^1 (1-t)^3 f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\mathrm{d}t = \frac{2}{b-a}(1-t)\int_0^1 (1-t)^3 \mathrm{d}f''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\\ &= -\frac{2}{b-a}f''\left(\frac{a+b}{2}\right) + \frac{12}{(b-a)^2}\int_0^1 (1-t)^2 \mathrm{d}f'\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\\ &= -\frac{2}{b-a}f''\left(\frac{a+b}{2}\right) - \frac{12}{(b-a)^2}f'\left(\frac{a+b}{2}\right) + \frac{48}{(b-a)^3}\int_0^1 (1-t)\mathrm{d}f\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\\ &= -\frac{2}{b-a}f''\left(\frac{a+b}{2}\right) - \frac{12}{(b-a)^2}f'\left(\frac{a+b}{2}\right) - \frac{48}{(b-a)^3}\left\{f\left(\frac{a+b}{2}\right) - \int_0^1 f\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\mathrm{d}t\right\}.\end{aligned}$$

The proof of Lemma 2.1 is complete.

3. Hermite-Hadamard's Type Inequalities for *P*-Convex Functions

Theorem 3.1. Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be differentiable on I°, $a,b \in I^{\circ}$ with $a<b$, and $f''' \in L([a,b])$. If $|f'''|^q$ is P-convex on $[a,b]$ for $q \ge 1$, then

$$\begin{aligned}&\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24} f''\left(\frac{a+b}{2}\right)\right|\\ &\le \frac{(b-a)^3}{192}\left[\left|f'''(a)\right|^q + \left|f'''(b)\right|^q\right]^{1/q}.\end{aligned} \tag{3.1}$$

Proof. Since $|f'''|^q$ is a P-convex function on $[a,b]$, by Lemma 2.1 and Hölder's inequality, we obtain

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x-f\left(\frac{a+b}{2}\right)-\frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|\le\frac{(b-a)^3}{96}\left\{\int_0^1(1-t)^3\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|\mathrm{d}t\right.$$

$$\left.+\int_0^1(1-t)^3\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\right\}\le\frac{(b-a)^3}{96}\left(\int_0^1(1-t)^3\,\mathrm{d}t\right)^{1-1/q}$$

$$\times\left\{\left(\int_0^1(1-t)^3\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}+\left(\int_0^1(1-t)^3\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}\right\}$$

$$\le\frac{(b-a)^3}{48}\left(\int_0^1(1-t)^3\,\mathrm{d}t\right)^{1-1/q}\left(\int_0^1(1-t)^3\,\mathrm{d}t\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}$$

$$=\frac{(b-a)^3}{48}\left(\int_0^1(1-t)^3\,\mathrm{d}t\right)\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}=\frac{(b-a)^3}{192}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}.$$

The proof of Theorem 3.1 is complete.

Corollary 3.1.1. Under the conditions of Theorem 3.1, if $q=1$, we have

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x-f\left(\frac{a+b}{2}\right)-\frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|\le\frac{(b-a)^3}{192}\left[\left|f'''(a)\right|+\left|f'''(b)\right|\right].$$

Theorem 3.2. Let $f:I\subset\mathbb{R}\to\mathbb{R}$ be differentiable on I°, $a,b\in I^\circ$ with $a<b$, and $f'''\in L([a,b])$. If $\left|f'''\right|^q$ is P-convex on $[a,b]$ for $q>1$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x-f\left(\frac{a+b}{2}\right)-\frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|\le\frac{(b-a)^3}{48}\left(\frac{1}{3q+1}\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}. \tag{2.2}$$

Proof. From Lemma 2.1, Hölder's inequality, and the P-convexity of $\left|f'''\right|^q$ on $[a,b]$, we drive

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x-f\left(\frac{a+b}{2}\right)-\frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|\le\frac{(b-a)^3}{96}\left\{\int_0^1(1-t)^3\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|\mathrm{d}t\right.$$

$$\left.+\int_0^1(1-t)^3\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\right\}\le\frac{(b-a)^3}{96}\left(\int_0^1 1\mathrm{d}t\right)^{1-1/q}$$

$$\times\left\{\left(\int_0^1(1-t)^{3q}\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}+\left(\int_0^1(1-t)^{3q}\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}\right\}$$

$$\le\frac{(b-a)^3}{48}\left(\int_0^1(1-t)^{3q}\,\mathrm{d}t\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}=\frac{(b-a)^3}{48}\left(\frac{1}{3q+1}\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}.$$

Theorem 3.2 is proved.

Theorem 3.3. Let $f:I\subset\mathbb{R}\to\mathbb{R}$ be differentiable on I°, $a,b\in I^\circ$ with $a<b$, and $f'''\in L([a,b])$. If $\left|f'''\right|^q$ is P-convex on $[a,b]$ for $q>1$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x-f\left(\frac{a+b}{2}\right)-\frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|\le\frac{(b-a)^3}{48}\left(\frac{q-1}{4q-1}\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}. \tag{2.3}$$

Proof. From Lemma 2.1, Hölder's inequality, and the P-convexity of $\left|f'''\right|^q$ on $[a,b]$, we have

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^3}{96}\left\{\int_0^1 (1-t)^3\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|\mathrm{d}t\right.$$
$$\left.+\int_0^1 (1-t)^3\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\right\} \le \frac{(b-a)^3}{96}\left(\int_0^1 (1-t)^{3q/(q-1)}\mathrm{d}t\right)^{1-1/q}$$
$$\times\left\{\left(\int_0^1\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q} + \left(\int_0^1\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}\right\}$$
$$\le \frac{(b-a)^3}{48}\left(\int_0^1 (1-t)^{3q/(q-1)}\mathrm{d}t\right)^{1-1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q} = \frac{(b-a)^3}{48}\left(\frac{q-1}{4q-1}\right)^{1/q}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}.$$

Theorem 3.3 is thus proved.

Theorem 3.4. Let $f: I\subset\mathbb{R}\to\mathbb{R}$ be differentiable on I°, $a,b\in I^\circ$ with $a<b$, and $f'''\in L([a,b])$. If $|f'''|^q$ for $q>1$ is P-convex on $[a,b]$ and $0<r,s<3$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right|$$
$$\le \frac{(b-a)^3}{96}\left\{\left(\frac{q-1}{(4-s)q-1}\right)^{1-1/q}\left(\frac{1}{sq+1}\right)^{1/q} + \left(\frac{q-1}{(4-r)q-1}\right)^{1-1/q}\left(\frac{1}{rq+1}\right)^{1/q}\right\}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}. \quad (2.5)$$

Proof. Using Lemma 2.1, Hölder's inequality, and the P-convexity of $|f'''|^q$ on $[a,b]$ yields

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^3}{96}\left\{\int_0^1 (1-t)^3\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|\mathrm{d}t\right.$$
$$\left.+\int_0^1 (1-t)^3\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\right\} \le \frac{(b-a)^3}{96}\left\{\left(\int_0^1 (1-t)^{(3-s)q/(q-1)}\mathrm{d}t\right)^{1-1/q}\right.$$
$$\times\left(\int_0^1 (1-t)^{sq}\left|f'''\left(\frac{1-t}{2}a+\frac{1+t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q} + \left(\int_0^1 (1-t)^{(3-r)q/(q-1)}\mathrm{d}t\right)^{1-1/q}\left.\left(\int_0^1 (1-t)^{rq}\left|f'''\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{1/q}\right\}$$
$$\le \frac{(b-a)^3}{96}\left\{\left(\int_0^1 (1-t)^{(3-s)q/(q-1)}\mathrm{d}t\right)^{1-1/q}\left(\int_0^1 (1-t)^{sq}\mathrm{d}t\right)^{1/q} + \left(\int_0^1 (1-t)^{(3-r)q/(q-1)}\mathrm{d}t\right)^{1-1/q}\left(\int_0^1 (1-t)^{rq}\mathrm{d}t\right)^{1/q}\right\}$$
$$\times\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}$$
$$= \frac{(b-a)^3}{96}\left\{\left(\frac{q-1}{(4-s)q-1}\right)^{1-1/q}\left(\frac{1}{sq+1}\right)^{1/q} + \left(\frac{q-1}{(4-r)q-1}\right)^{1-1/q}\left(\frac{1}{rq+1}\right)^{1/q}\right\}\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q}.$$

The proof of Theorem 3.4 is complete.

Corollary 3.3.1. Under the conditions of Theorem 3.4,

(1) if $r=s$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24}f''\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^3}{48}\left(\frac{q-1}{(4-r)q-1}\right)^{1-1/q}\left(\frac{1}{rq+1}\right)^{1/q}\times\left[\left|f'''(a)\right|^q+\left|f'''(b)\right|^q\right]^{1/q};$$

(2) if $r=s=1$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24} f''\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^3}{48}\left(\frac{q-1}{3q-1}\right)^{1-1/q}\left(\frac{1}{q+1}\right)^{1/q}\left[\left|f'''(a)\right|^q + \left|f'''(b)\right|^q\right]^{1/q};$$

(3) if $r = s = 2$, then

$$\left|\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x - f\left(\frac{a+b}{2}\right) - \frac{(b-a)^2}{24} f''\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^3}{48}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\left(\frac{1}{2q+1}\right)^{1/q}\left[\left|f'''(a)\right|^q + \left|f'''(b)\right|^q\right]^{1/q}.$$

Finally we would like to note that these Hermite-Hadamard type inequalities obtained in this paper can be applied to the fields of integral inequalities, approximation theory, special means theory, optimization theory, information theory, and numerical analysis, as done before by a number of mathematicians.

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