

SOME INEQUALITIES ON GENERAL L_p -CENTROID BODIES

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Abstract. In this article, we define the general L_p -centroid bodies, which extend the notion of L_p -centroid bodies by Lutwak and Zhang. Further, we generalize the two monotone inequalities by Wang, Lu and Leng, and establish the Brunn-Minkowski type inequalities of dual quermass-integrals for this new notion. In particular, the extremal values of dual quermassintegrals of the polars of general L_p -centroid bodies are also provided.

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