

A CORRECTION TO  
“SOME MEAN CONVERGENCE AND COMPLETE CONVERGENCE  
THEOREMS FOR SEQUENCES OF  $m$ -LINEARLY NEGATIVE  
QUADRANT DEPENDENT RANDOM VARIABLES”

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*Abstract.* The authors provide a correction to “Some mean convergence and complete convergence theorems for sequences of  $m$ -linearly negative quadrant dependent random variables”.

*Keywords:*  $m$ -linearly negative quadrant dependence; mean convergence; complete convergence

*MSC 2010:* 60F15, 60F25

Professor Xuejun Wang (School of Mathematical Sciences, Anhui University, Hefei, People’s Republic of China) has so kindly pointed out to us that there is an error in the formulation of the main results of our paper [1]. Specifically, Professor Wang pointed out apropos of Theorem 2.1 that it does not follow from Lemma 3.1 and  $\{X_n, n \geq 1\}$  being  $m$ -LNQD that the sequences  $\{Y_k, k \geq 1\}$  and  $\{Z_k, k \geq 1\}$  are  $m$ -LNQD. These sequences of random variables are  $m$ -LNQD in the corrected formulation given below although  $\{Z_k, k \geq 1\}$  being  $m$ -LNQD is not used in the proof. A similar comment pertains to Theorems 2.2 and 2.3. The proofs presented in [1] are valid with the corrected formulations. It is an open problem as to whether in general the  $m$ -LNQD property is preserved by non-decreasing functions.

Corrections will now be given.

1. Replace the first sentence of the statement of Theorems 2.1 and 2.2 by the following sentence: Let  $\{X_n, n \geq 1\}$  be a sequence of random variables such that

$\{f(X_n), n \geq 1\}$  is a sequence of  $m$ -LNQD random variables for every non-decreasing function  $f$ .

2. Replace the first sentence of the statement of Corollary 2.1 by the following sentence: Let  $\{X_n, n \geq 1\}$  be a sequence of identically distributed random variables such that  $\{f(X_n), n \geq 1\}$  is a sequence of  $m$ -LNQD random variables for every non-decreasing function  $f$  and suppose that  $E|X_1|^p < \infty$  for some  $1 \leq p < 2$ .

3. Replace the first sentence of the statement of Theorem 2.3 by the following sentence: Let  $\{X_n, n \geq 1\}$  be a sequence of random variables such that  $\{f(X_n), n \geq 1\}$  is a sequence of  $m$ -LNQD random variables for every non-decreasing function  $f$  and let  $1 \leq p < 2$ .

We now give an example of a sequence of random variables  $\{X_n, n \geq 1\}$  such that  $\{f(X_n), n \geq 1\}$  is a sequence of  $m$ -LNQD random variables for every non-decreasing function  $f$ .

*Example.* Let  $\{X_n, n \geq 1\}$  be a sequence of negatively associated random variables; that is, for every pair of nonempty disjoint subsets  $A_1$  and  $A_2$  of positive integers and every choice of functions  $f_1: \mathbb{R}^{A_1} \rightarrow \mathbb{R}$  and  $f_2: \mathbb{R}^{A_2} \rightarrow \mathbb{R}$  which are coordinatewise non-decreasing,




$$\text{Cov}(f_1(X_i: i \in A_1), f_2(X_j: j \in A_2)) \leq 0.$$

It is well known that NA sequences are LNQD. It is clear that for a sequence  $\{X_n, n \geq 1\}$  of NA random variables, the sequence  $\{f(X_n), n \geq 1\}$  is NA (hence is LNQD) for every non-decreasing function  $f$ . Thus  $\{f(X_n), n \geq 1\}$  is  $m$ -LNQD for all  $m \geq 2$ .

In view of the above reformulations of the main results, we point out that Lemma 3.1 is not needed in their proofs. We also point out that in Remark 1.1, the constant  $a$  needs to be non-negative.

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### References

- [1] *Y. Wu, A. Rosalsky, A. Volodin*: Some mean convergence and complete convergence theorems for sequences of  $m$ -linearly negative quadrant dependent random variables. *Appl. Math., Praha* 58 (2013), 511–529.   

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