Some new distance-4 constant weight codes

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Abstract

Improved binary constant weight codes with minimum distance 4 and length at most 28 are constructed. A table with bounds on the chromatic number of small Johnson graphs is given.

1 Introduction

A binary constant weight code of word length n and weight w and minimum distance d is a collection of (0,1)-vectors of length n, all having w ones and $n-w$ zeros, such that any two of these vectors differ in at least d places. The maximum size of such a code is denoted by $A(n,d,w)$. In this note we give improved lower bounds for $A(n,d,w)$ for $d=4$ and smallish n.

The standard reference for constructions of binary constant weight codes of length at most 28 is [3]. One of the constructions discussed there depends on the existence of partitions of all words of a given length and weight into codes with minimum distance at least 4 (that is, on proper colorings of the Johnson graph). Such partitions are typically found using some form of heuristic search, and today it is easy to improve on the results of [3]. For example, [3] says that $A(22,4,11) \geq 39688$, while we find $A(22,4,11) \geq 40624$ here. Earlier improvements have been given in [9] and [10]. The bounds here improve all but one of the bounds from [9] and all from [10]. For example, [3] gives $A(26,4,13) \geq 424868$, [10] gives $A(26,4,13) \geq 425950$, and we find $A(26,4,13) \geq 431672$.

Motivated by an application to frequency hopping lists in radio networks, the authors of [18] extended the tables of constant weight codes to word length 63. For the case of $d=4$ they give bounds on $A(n,4,5)$. Table 2 below gives improvements.

Apart from codes obtained via this construction using partitions, we also give five direct constructions, showing that $A(15,4,6) \geq 399$, $A(16,4,5) \geq 322$, $A(16,4,6) \geq 616$, $A(18,4,5) \geq 544$, and $A(21,4,5) \geq 1113$.

Finally, this note contains (in §6) a discussion of the chromatic number of Johnson graphs and determines this number in a few new cases.

2 Direct constructions

2.1 $A(15,4,6) \geq 399$

We show $A(15,4,6) \geq 399$ using the group of order 21 (that permutes the 15 coordinate positions, numbered right-to-left 0–14) that fixes position 14, and acts on positions 0–13 with the two generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13) and (0,2,4)(1,3,5)(7,9,11)(8,10,12). The 23 base blocks:

000000010011111	000001111100100	100001110000101
100000011011001	000001110110001	000101111000010
000000111011010	100000110101100	100101010001001
100000010111010	000011010001101	100011010010001
100000010100111	000001110101010	100001111010000
000001110010110	000101010010101	000111110010000
000101010001110	000011010011010	100011110000010
000001111001001	000011011110000	

2.2 $A(16,4,5) \geq 322$

We show $A(16,4,5) \geq 322$ using the group of order 21 that fixes the first two coordinate positions, and acts on positions 0–13 with the two generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13) and (0,2,4)(1,3,5)(7,9,11)(8,10,12). The 20 base blocks:

0000000011011001	0000101010000011	0000001111001000
0000000010111010	0000001110100010	0100101011000000
0000000010100111	1000000110000101	1100000111000000
0100000010011100	0000011010011000	1000011010000010
1000000010010110	0000001110010001	0100011110000000
1100000000001101	1000000110001010	1000111100000000
0100000110000011	0100000110110000	

2.3 $A(16,4,6) \geq 616$

We show $A(16,4,6) \geq 616$ using a group of order 32 isomorphic to the direct product $C_2 \times D_{16}$, generated by the three permutations

(0,1,2,3,4,5,6,7)(8,9,10,11,12,13,14,15),
(0,1)(2,7)(3,6)(4,5)(8,9)(10,15)(11,14)(12,13),
(0,8)(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15).

The 27 base blocks:

0000000111000111	0000011101001010	0000101101010100
0000000100101111	0000011101110000	0000101101101000
0000000101110011	0000100110110001	0001001110100010
0000000110111010	0000100110010110	0001001101010010
0000010101011100	0000001110110100	0001001101100100
0001000100111001	0000101100010011	0001001111001000
0000011100000111	0000101100001101	0001000101010101
0000011100011001	0000101110001010	0001010110100100
0000011100101100	0000101100100110	0010010101001001

2.4 $A(18,4,5) \geq 544$

We show $A(18,4,5) \geq 544$ using a cyclic group of order 17 that fixes the first coordinate. The 32 base blocks:

001111000000000001	011001000000010100	010100101000001000
011101010000000000	011000110001000000	111100000010000000
011100100000100000	011000101000000100	111010000000000100
011100001100000000	011000100110000000	111000100000000010
011100000001001000	011000010010000010	111000010000001000
011010010000100000	011000001000010010	111000001001000000
011010001000001000	011000000101000010	110101000000010000
011010000100010000	011000000100100100	110100100000000100
011001001010000000	011000000010101000	110100010000100000
011001000100001000	010101001000000100	110010000010001000
011001000000100010	010101000000101000	

2.5 $A(21, 4, 5) \geq 1113$

We show $A(21, 4, 5) \geq 1113$ using a group of order 63 that acts on positions 0–20 with the three generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13)(14,16,15,18,19,17,20), (0,2,4)(1,3,5)(7,9,11)(8,10,12)(14,16,18)(15,17,19), and (0,7,14)(1,8,15)(2,9,16) (3,10,17)(4,11,18)(5,12,19)(6,13,20). The 19 base blocks:

000000000011110000010	000000000001111010000	000000100011001001000
000000000000010111010	000000000011010010001	000000100001100000101
000000000111100000100	000000000101010000011	000000100101000100100
000000100000100110010	000000100010100001010	000000100101000010010
000000100010000111000	000000100110000001001	000000100010100100100
000000100000010001011	000000100011000010100	000000100010101010000
000000000000110101100		

3 Tables with lower bounds from partitioning

Table 1 below gives lower bounds on $A(n, d, w)$, the maximum size of a binary constant weight code of word length n, minimum distance d, and constant weight w, where $d = 4$ and $2w \leq n$. These lower bounds are obtained using the partitioning construction discussed below.

Table 1: Lower bounds for $A(n, 4, w)$

$n \backslash w$	6	7	8	9	10	11	12	13	14
18	1260^p	2042	3186^s	3540^s					
19	1620^n	3172^p	4667^p	6726^s					
20	2304^n	4213^p	7730^p	10048	13452^g				
21	2856^s	6161	10767	17177	20654				
22	3927^s	8338	16527	25902	37127	40624			
23	$5313.^s$	11696	23467	41413	58659	76233			
24	$7084.^S$	15656^{eb}	34914^g	59904	98852	118422	151484		
25	7787	21220	47265	89736	142372	198386	231530		
26	10010^p	27050	66352	129682	222723	320512	401937	431672	
27	12012^s	35874	88604	188561	334834	517989	686152	791449	
28	15288^g	44915	122685	262980	508952	818897	1167909	1420892	1535756

Legenda: ^{s}: shortened code. ^{S}: Steiner system $S(5, 6, 24)$. ^{g}: a group code from [3]. ^{n}: idem from [17]. ^{p}: product construction from [3]. ^{eb}: idem from [9]. Unmarked entries are from this paper.

Next we give a table with lower bounds for $A(n,4,5)$ for $29 \le n \le 64$, to be compared with the table in [18]. It improves all bounds from that paper except for the three values for $n = 45, 46, 47$. The values marked ^{s} are derived from Steiner systems $S(5,6,36)$ and $S(5,6,48)$ ([1, 5]). The values marked with a dot are exact.

Table 2: Lower bounds for $A(n,4,5)$

n	29	30	31	32	33	34	35	36	37
bd	4423	4901	5697	6582	7656^s	$8976.^s$	$10472.^s$	10948	12473
n	38	39	40	41	42	43	44	45	46
bd	13471	15010	17119	19258	20671	22728	25564	28413^s	$31878.^s$
n	47	48	49	50	51	52	53	54	55
bd	$35673.^s$	36809	40560	42920	46612	51420	56251	59293	63973
n	56	57	58	59	60	61	62	63	64
bd	69931	75550	79330	85728	93206	100527	105472	112457	121902

4 The Partitioning Construction

A *partition* $\Pi(n,w) = (C_1, ..., C_m)$ is a partition of the set of all $\binom{n}{w}$ binary vectors of length n and weight w into codes C_i that all have minimum distance at least 4. By definition, $C_j = \emptyset$ for $j > m$.

The *direct product* $\Pi(n_1, w_1) \times \Pi(n_2, w_2)$ of two partitions $(C_1, ..., C_{m_1})$ and $(D_1, ..., D_{m_2})$ is the code $\bigcup_i C_i * D_i$ (of word length $n_1 + n_2$ and weight $w_1 + w_2$ and size $\sum |C_i|.|D_i|$), where for two codes C and D the code $C * D$ is the code consisting of all possible concatenations $c * d$ with $c \in C$ and $d \in D$.

The *partitioning construction* for codes of length n, weight w and minimum distance 4 constructs the code $C = \bigcup_i \Pi(n_1, 2i+\epsilon) \times \Pi(n_2, w-2i-\epsilon)$ where $n = n_1 + n_2$ and $\epsilon \in \{0,1\}$ and the union is over all i with $i \ge 0$ and $2i + \epsilon \le w$.

It is usually nontrivial to construct the required ingredients $\Pi(n,w)$. However, for $w \le 1$ the partition is trivial, namely the partition into singletons, and for $w = 2$ the optimal partition is that of the $n(n-1)/2$ pairs into $n-1$ parts of size $n/2$ if n is even, and into n parts of size $(n-1)/2$ if n is odd. Partitions $\Pi(n,w)$ and $\Pi(n,n-w)$ are related by complementation. It is always possible to find a $\Pi(n,w)$ with at most n parts, cf. [12].

Example We show $A(18,4,7) \ge 2042$. Take $n_1 = 8$, $n_2 = 10$, $\epsilon = 1$, using direct products $\Pi(8,1) \times \Pi(10,6)$, $\Pi(8,3) \times \Pi(10,4)$, $\Pi(8,5) \times \Pi(10,2)$, $\Pi(8,7) \times \Pi(10,0)$. From a $\Pi(10,4)$ with sizes $(30,30,30,28,26,23,22,20,1)$ we find $\binom{10}{4} - 1 = 209$ for the first product, from $\Pi(8,3)$ with 7 parts of size 8 and a $\Pi(10,4)$ with sizes $(30,30,30,30,30,22,22,12,2,2)$ we find $8(\binom{10}{4} - 16) = 1552$ for the second, then $5\binom{8}{5} = 280$ for the third, and 1 for the last, 2042 altogether.

4.1 Improvements by Etzion & Bitan

The code C that results from the partitioning construction is not always maximal. Etzion & Bitan [9] gave a handful of examples of improvements. Let us redo two of their examples here (using improved ingredients).

Example We show $A(21,4,7) \geq 6161$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 0$. The products $\Pi(10,0) \times \Pi(11,7)$, $\Pi(10,2) \times \Pi(11,5)$, $\Pi(10,4) \times \Pi(11,3)$, $\Pi(10,6) \times \Pi(11,1)$ contribute $A(11,4,4) + 5\binom{11}{5} + 17\binom{10}{4} + \binom{10}{4} = 6125$. For $\Pi(11,5)$ and $\Pi(10,4)$ we used partitions with 9 parts, for $\Pi(11,3)$ the Etzion-Bitan partition with 9 parts of size 17, 1 part of size 3, and 9 parts of size 1, where the 9 parts of size 1 are the triples covering the pair $0^9 1^2$. Only the first 9 parts were used, and of $\Pi(11,1)$ also only the first 9 parts were used, so that the vector $0^9 1^2$ has distance at least 3 to all second halves used so far, and $A(10,4,5) = 36$ vectors $u * 0^9 1^2$ can be added.

Example We show $A(21,4,8) \geq 10767$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 1$. The products $\Pi(10,1) \times \Pi(11,7)$, $\Pi(10,3) \times \Pi(11,5)$, $\Pi(10,5) \times \Pi(11,3)$, $\Pi(10,7) \times \Pi(11,1)$ contribute $\binom{11}{4} + 13\binom{11}{5} + 17\binom{10}{5} + 9.13 = 10737$. For $\Pi(11,7)$ we used a partition with 10 parts. For $\Pi(10,7)$ one with 9 parts of size 13 and one part of size 3, that is not used to leave room for $A(10,4,6) = 30$ vectors $u * 0^9 1^2$.

4.2 Varying the split

Instead of keeping n_1 and n_2 fixed in the partitioning construction, one can use a varying split. For example, one can show that $A(23,4,8) \geq 23467$ using the union of $\Pi(12,0) \times \Pi(11,8)$, $\Pi(12,2) \times \Pi(11,6)$, $\Pi(12,4) \times \Pi(11,4)$, $\Pi(12,6) \times \{0\} \times \Pi(10,2)$, $\Pi(13,8) \times \Pi(10,0)$. Because $\Pi(12,6)$ can be taken to have 9 parts, nothing is lost by taking $\Pi(10,2)$ instead of $\Pi(11,2)$, but something is gained taking $\Pi(13,8)$ instead of $\Pi(12,8)$.

5 Partitions used

Partitions $\Pi(n,0)$ and $\Pi(n,1)$ are trivial, and it is easy to see what the best partitions $\Pi(n,2)$ are (cf. [3]). Nowadays also optimal partitions $\Pi(n,3)$ are known. If $n \equiv 1,3 \pmod 6$, $n \neq 7$, then a partition of all triples on n points into Steiner triple systems exists ([15, 19, 13]), so that we have a $\Pi(n,3)$ consisting of $n-2$ parts, each of size $n(n-1)/6$. Shortening these we find that for $n \equiv 0,2 \pmod 6$, $n \neq 6$, there is a partition $\Pi(n,3)$ consisting of $n-1$ parts, each of size $n(n-2)/6$. In [6, 7] partitions $\Pi(n,3)$ are constructed for $n \equiv 4 \pmod 6$, consisting of n parts, $n-1$ of size $(n^2-2n-2)/6$ and 1 of size $(n-1)/3$. Finally, [14] constructs partitions $\Pi(n,3)$ for $n \equiv 5 \pmod 6$, $n \neq 5$, with $n-1$ parts, $n-2$ of size $(n^2-n-8)/6$ and 1 of size $4(n-2)/3$. All of these are optimal.

For Table 2 we used only the obvious partitions: for $w \leq 3$ the above ones, for $w = 4$ the Graham-Sloane partitions ([3], Theorem 14), and finally for $w = 5$ the partition with one part as large as possible (the best lower bound known for $A(n,4,w)$) and all other parts arbitrary, for example of size 1. It will be easy to improve these bounds a little.

For Table 1 we spent some effort to find good partitions. In Table 3 below we give the vector of part sizes for the partitions used. The actual partitions can be found near [2].

Table 3: Partitions used

n	w	#	part sizes														
8	4	6	14	14	12	12	10	8									
9	4	8	18	18	18	18	16	15	15	8							
9	4	9	18	18	18	17	17	17	13	7	1						
10	4	10	30	30	30	30	30	22	22	12	2	2					
10	4	9	30	30	30	28	26	23	22	20	1						
10	4	9	30	30	30	30	26	25	22	15	2						
10	5	8	36	36	34	34	29	29	27	27							
10	5	9	36	33	33	33	31	31	31	12	12						
11	4	10	35	35	35	35	34	33	32	32	32	27					
11	4	10	35	35	35	35	35	34	31	31	31	28					
11	4	12	35	35	35	35	35	35	35	32	29	16	7	1			
11	4	13	35	35	35	34	34	34	34	34	34	12	3	3	3		
11	5	10	66	66	59	59	55	48	47	41	18	3					
11	5	9	66	66	60	55	55	55	54	39	12						
12	4	11	51	51	51	50	49	49	48	48	47	27	24				
12	4	11	51	51	51	51	49	47	47	46	45	43	14				
12	4	11	51	51	51	51	51	49	47	47	45	34	18				
12	4	11	51	51	51	51	51	50	50	47	41	27	25				
12	4	12	51	51	51	51	50	49	48	46	44	39	11	4			
12	4	12	51	51	51	51	51	50	50	44	41	38	16	1			
12	4	12	51	51	51	51	51	50	50	47	40	31	20	2			
12	4	12	51	51	51	51	51	50	50	47	45	24	19	5			
12	4	13	51	51	51	51	51	50	50	46	46	26	13	8	1		
12	5	11	72	72	72	72	72	72	72	72	72	72	72				
12	5	13	80	80	80	80	76	72	70	68	67	62	45	11	1		
12	5	13	80	80	80	80	77	73	73	71	64	50	45	18	1		
12	6	10	132	132	116	116	100	100	95	63	61	9					
12	6	9	132	132	120	110	110	110	108	78	24						
13	4	13	65	65	65	65	65	60	59	58	55	55	54	44	5		
13	4	13	65	65	65	65	65	60	59	58	58	56	55	33	11		
13	4	13	65	65	65	65	65	61	60	59	58	55	49	42	6		
13	4	13	65	65	65	65	65	62	60	59	58	56	50	32	13		
13	4	13	65	65	65	65	65	62	61	61	57	49	47	43	10		
13	5	13	123	123	122	122	113	111	107	106	101	94	85	62	18		
13	5	13	123	123	123	122	114	112	109	102	95	92	84	76	12		
13	5	13	123	123	123	122	114	112	109	102	97	92	85	72	13		
13	5	13	123	123	123	122	114	113	111	104	100	89	77	64	24		
13	5	14	123	123	122	122	114	111	107	107	101	95	85	56	19	2	
13	6	13	166	166	164	159	151	146	139	136	127	124	109	89	40		
13	6	13	166	166	166	157	149	149	140	134	130	118	116	84	41		
13	6	13	166	166	166	159	150	146	139	136	125	120	109	99	35		
13	6	13	166	166	166	159	151	149	139	136	129	119	105	92	39		
13	6	13	166	166	166	159	151	149	141	136	130	115	105	89	43		
13	6	14	166	166	166	159	150	149	140	137	129	118	111	79	44	2	
13	6	14	166	166	166	159	151	146	139	136	124	121	109	98	33	2	
13	6	14	166	166	166	159	151	149	140	135	129	114	106	99	35	1	
14	4	13	91	91	91	90	88	81	81	78	77	73	70	61	29		
14	4	13	91	91	91	90	88	82	81	77	77	73	68	64	28		
14	4	14	91	91	91	90	88	82	79	78	78	73	68	65	26	1	
14	4	14	91	91	91	90	88	82	80	79	78	72	67	65	25	2	
14	4	14	91	91	91	90	88	82	81	78	76	71	69	66	26	1	
14	4	14	91	91	91	90	88	82	81	78	78	72	71	52	35	1	
14	4	14	91	91	91	90	88	82	81	78	78	74	69	52	34	2	
14	5	15	169	169	169	169	157	157	149	148	144	143	130	125	108	59	6
14	5	15	169	169	169	169	157	157	149	148	144	143	131	125	106	62	4
14	5	15	169	169	169	169	157	157	150	148	144	143	131	127	101	59	9

n	w	#	part sizes
14	6	14	253 252 243 243 243 243 212 212 212 212 212 212 212 42
14	6	15	278 275 272 258 248 228 227 218 213 206 195 166 137 81 1
14	6	15	278 276 272 259 247 229 227 220 210 207 186 180 139 68 5
14	6	15	278 276 272 259 247 229 227 220 212 206 186 180 135 71 5
14	6	15	278 276 272 259 248 229 226 218 209 205 186 178 153 64 2
14	6	15	278 276 273 258 248 231 229 219 208 207 189 168 138 77 4
14	6	15	278 276 273 259 248 231 228 222 209 199 184 173 141 81 1
14	7	14	282 282 280 280 274 274 272 271 271 269 269 268 126 14
14	7	15	325 325 317 304 282 268 262 246 238 221 206 195 160 80 3

We give a partition $\Pi(11,4)$ with 10 parts 35 35 35 35 33 32 32 32 31 30 explicitly (in the notation of [3]).

4135296314652A8398582A147716A295143712A9437568A253437916544 83A219257686793579681
24719583274A2516973A3A846A1536721849468A739251586968791 4A2445718362713624965 93A8
7396A2154A8615921A873413A248A253761A985491248257368576794893153 98AA274A472313586
8425923964818524A176916A73321A244A656593116582787A93244A21A52317376256894A175439
8259768431

We also give a partition $\Pi(11,5)$ with 9 parts 66 66 60 55 55 55 54 39 12. No part has two disjoint 5-sets, so extending by a point and adding complements yields a partition $\Pi(12,6)$ with 9 parts 132 132 120 110 110 110 108 78 24.

263715581364371627524315677124952483674247523121386531671687424375412367283312 47
515665274314658283312365721167552114798464783287453154226431726175417827952431 38
652713281644366387512184396623558214932613562167318587244685142673321758124535 46
721493735216753418253176642192426385753217648132416834725641357238135761316247 49
253562427163714918325764538151822374617923465185826482653141356122763277615938 87
6428345161452763582417312546251783251847734545621172698368 1274

6 Chromatic numbers

The Johnson graph $J(n,w)$ is the graph on the binary vectors of length n and weight w, adjacent when they have Hamming distance 2. The graphs $J(n,w)$ and $J(n,n-w)$ are isomorphic. The chromatic number $\chi = \chi(n,w) = \chi(J(n,w))$ is the smallest number of distance-at-least-4 codes its vertex set can be partioned into. One has $\max(w+1, n-w+1) \le \chi(n,w) \le n$, where the lower bound is the size of a maximal clique, and the upper bound is due to [12]. One also has the monotonicity inequalities $\chi(n,w) \ge \chi(n-1,w-1)$ and $\chi(n,w) \ge \chi(n-1,w)$. Table 4 below gives lower and upper bounds for χ.

Table 4: Bounds on the chromatic number of Johnson graphs

$n\backslash w$	1	2	3	4	5	6	7	8
5	5	5						
6	6	5	6					
7	7	7	6					
8	8	7	7	6				
9	9	9	7	8				
10	10	9	10	8–9	8			
11	11	11	10	10	8–9			
12	12	11	11	10–11	10–11	8–9		
13	13	13	11	11–13	10–13	10–13		
14	14	13	13	11–13	11–14	10–14	10–14	
15	15	15	13	13–14	12–15	11–15	10–15	
16	16	15	16	13–14	13–15	12–15	11–16	10–15

Discussion The cases $w = 1$ and $w = 2$ are trivial. For $w \geq 3$ and $n \leq 14$ the upper bounds were already given in [3] (Table VI), except that $\chi(11,4) \leq 10$ and $\chi(11,5) \leq \chi(12,6) \leq 9$ were given above, and $\chi(12,5) \leq 11$ follows from a partition $\Pi(12,5)$ with parts 72^{11} given in [9]. That $\chi(15,3) = 13$ follows from the existence of a large set of STS(15) ([4]). Optimal partitions $\Pi(11,3)$ with parts 17^9 12 and $\Pi(17,3)$ with parts 44^{15} 20 were given in [14]. An optimal partition $\Pi(16,3)$ with parts 37^{15} 5 was constructed by Doron Cohen and given in [6]. In particular, $\chi(16,3) = 16$, in spite of the claim in [9] that $\chi(16,w) \leq 15$ for $2 \leq w \leq 6$. A partition $\Pi(16,4)$ with parts 140^6 136^6 116 48 was given in [8]. That $\Pi(16,w) \leq 15$ for $w = 6, 8$ follows from [11]. A partition $\Pi(16,5)$ with parts 302^{14} 140 is found by the product construction from [11], since one can take the union of the last two parts of size 70 each.

Concerning the lower bounds for $w \geq 4$, all except two follow from monotonicity. We verified explicitly that $\chi(9,4) > 7$—there are five, not seven mutually disjoint codes of word length $n = 9$, constant weight $w = 4$ and size 18. That $\chi(15,5) > 11$ follows since there is no Steiner system $S(4,5,15)$ ([16]), let alone eleven mutually disjoint ones.

In the case of J(10,4) there exists a coloring with 9 colors where the last color is used only once. So J(10,4) minus a vertex has chromatic number 8.

It would be interesting to give more general constructions for colorings of $J(n,w)$ with fewer than n colors.

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References

[1] Anton Betten, Reinhard Laue, Alfred Wassermann, *A Steiner 5-Design on 36 points*, Designs, Codes and Cryptography **17** (1999) 181–186.

[2] `http://www.win.tue.nl/~aeb/codes/andw.html`.

[3] A. E. Brouwer, James B. Shearer, N. J. A. Sloane & Warren D. Smith, *A new table of constant weight codes*, IEEE Trans. Inf. Th. **36** (1990) 1334–1380.

[4] R. H. F. Denniston, *Sylvester's problem of the 15 school-girls*, Discr. Math. **9** (1974) 229–233.

[5] R. H. F. Denniston, *Some new 5-designs*, Bull. London Math. Soc. **8** (1976) 263–267.

[6] T. Etzion, *Optimal partitions for triples*, J. Combin. Th. (A) **59** (1992) 161–176.

[7] T. Etzion, *Partitions of triples into optimal packings*, J. Combin. Th. (A) **59** (1992) 269–284.

[8] T. Etzion, *Partitions for Quadruples*, Ars Combinatoria **36** (1993) 296–308.

[9] Tuvi Etzion & Sara Bitan, *On the chromatic number, colorings, and codes of the Johnson graph*, Discr. Applied Math. **70** (1996) 163–175.

[10] Tuvi Etzion & Patric R. J. Östergård, *Greedy and heuristic algorithms for codes and colorings*, IEEE Trans. Inf. Th. **44** (1998) 382–388.

[11] T. Etzion & C. L. M. van Pul, *New lower bounds for constant weight codes*, IEEE Trans. Inf. Th. **35** (1989) 1324–1329.

[12] R. L. Graham & N. J. A. Sloane, *Lower bounds for constant weight codes*, IEEE Trans. Inf. Th. **26** (1980) 37–43.

[13] Lijun Ji, *A new existence proof for large sets of disjoint Steiner triple systems*, J. Combin. Th. (A) **112** (2005)308–327.

[14] Lijun Ji, *Partition of triples of order* $6k + 5$ *into* $6k + 3$ *optimal packings and one packing of size* $8k + 4$, Graphs Combin. **22** (2006) 251–260.

[15] J. X. Lu, *On large sets of disjoint Steiner triple systems I, II, III*, J. Combin. Th. (A) **34** (1983) 140–146, 147–155, 156–183, and *IV, V, VI*, ibid. **37** (1984) 136–163, 164–188, 189–192.

[16] N. S. Mendelsohn & S. H. Y. Hung, *On the Steiner systems* $S(3, 4, 14)$ *and* $S(4, 5, 15)$, Utilitas Math. **1** (1972) 5-95.

[17] Kari J. Nurmela, Markku K. Kaikkonen & Patric R. J. Östergård, *New constant weight codes from linear permutation groups*, IEEE Trans. Inf. Th. **43** (1997) 1623–1630,

[18] D. H. Smith, L. A. Hughes & S. Perkins, *A new table of constant weight codes of length greater than 28*, Electronic J. Combin. **13** (2006) #A2.

[19] L. Teirlinck, *A completion of Lu's determination of the spectrum of large sets of disjoint Steiner Triple systems*, J. Combin. Th. (A) **57** (1991) 302–305.