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Some Nonlinear Model for Solving the Nonlinear Partial Differential Equations in Mathematical Physics

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Abstract: The solitary waves are derived from the traveling waves. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and the rational functions.

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INTRODUCTION

The investigation of the exact solutions for nonlinear evolution equations plays an important role in the study of soliton theory. In recent years, searching for explicit solutions of nonlinear evolution equations by using various methods has become the main goal for many authors and many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the tanh-function expansion and its various extension [8, 9] the Jacobi elliptic function expansion [10, 11]. Very recently, Wang *et al*. [1] introduced a new method called the (G′/G)-expansion method to look for travelling wave solutions of nonlinear evolution equations [2, 7]. The (G′/G)-expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in (G′/G) and that G = G(ξ) satisfies a second order linear ordinary differential equation(ODE). Recently modified (G′/G)-expansion method is presented to derive traveling wave solutions for a class of nonlinear partial differential equations called Whitham-Broer-Kaup-Like equations. The paper is arranged as follows. In Section 2, we describe briefly the (G′/G)-expansion method. In Sections 3-4, we apply the method to thecombined Kdv-MKdv equation,the Shorma-Tasso-Olver equation, respectively. In Section 5 we apply the method for the (2+1)-dimensional Konopelchenko-Dubrovsky equation. In section 6 some conclusions are given.

DESCRIPTION OF THE (G′/G)-EXPANSION METHOD

Considering the nonlinear partial differential equation in the form

$$P(u,u_x,u_t,u_{tt},u_{xt},u_{xx},....)=0 \tag{1}$$

where u = u(x,t) is an unknown function, P is a polynomial in u = u(x,t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the (G′/G)-expansion method.

Step 1: Combining the independent variables x and t into one variable ξ = x-vt, we suppose that

$$u(x,t)=u(\xi),\ \ \xi=x-vt \tag{2}$$

The travelling wave variable (2) permits us to reduce Eq(1) to an ODE for G = G(ξ), namely

$$P(u,-vu',u',v^2u'',-vu'',u'',\ldots)=0 \tag{3}$$

Step 2: Suppose that the solution of ODE (3) can be expressed by a polynomial in (G′/G) as follows

$$u(\xi)=\alpha_m\left(\frac{G'}{G}\right)+.... \tag{4}$$

where G = G(ξ) satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0 \tag{5}$$

$\alpha_m,\ldots,\lambda$ and μ are constants to be determined later $\alpha_m\neq 0$, the unwritten part in 4 is also a polynomial in (G′/G), but the degree of which is generally equal to or less than m-1, the positive integer m can be determined by considering the homogeneous balance between the

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highest order derivatives and nonlinear terms appearing in ODE (3).

Step 3: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order (G′/G) together, the left-hand side of Eq. (3) is converted into another polynomial in (G′/G). Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_m,\ldots,\lambda$ and μ.

Step 4: Assuming that the constants $\alpha_m,\ldots,\lambda$ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known for us, then substituting $\alpha_m,\ldots,v$ and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution equation (1).

COMBINED Kdv-MKdv EQUATION

In order we consider the combined Kdv-MKdv equation in the form

$$u_t + puu_x + qu^2u_{xx} + u_{xxx} = 0 \tag{6}$$

The travelling wave variable below

$$u(x,t) = u(\xi),\ \ \xi = x - vt \tag{7}$$

Permits us converting Eq.(7) into an ODE for

$$G = G(\xi) - vu' + \frac{1}{2}p(u^2)' + \frac{1}{3}q(u^3)' + u''' = 0$$

Integrating it with respect to ξ once yields

$$c - vu + \frac{1}{2}p(u^2) + \frac{1}{3}q(u^3) + u'' = 0 \tag{8}$$

where c is an integration constant that is to be determined later. Suppose that the solution of ODE (8) can be expressed by a polynomial in (G′/G) as follows:

$$u(\xi) = \alpha_m(\frac{G'}{G}) + \ldots. \tag{9}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{10}$$

α_1, α_0, v and μ are to be determined later.

By using (9) and (10) and considering the homogeneous balance between u'' and u^3 in Eq. (8) we required that $3m = m+2$ then $m = 1$. So we can write (9) as:

$$u(\xi) = \alpha_1(\frac{G'}{G}) + \alpha_0 \tag{11}$$

Therefore

$$u^3 = \alpha_1^3(\frac{G'}{G})^3 + 3\alpha_1^2\alpha_0(\frac{G'}{G})^2 + 3\alpha_1\alpha_0^2(\frac{G'}{G}) + \alpha_0^3 \tag{12}$$

$$u^2 = \alpha_1^2(\frac{G'}{G})^2 + 2\alpha_1\alpha_0(\frac{G'}{G}) + \alpha_0 \tag{13}$$

By using (11) and (10) it is derived that

$$\begin{aligned} u'' &= 2\alpha_1(\frac{G'}{G})^3 + 3\alpha_1\lambda(\frac{G'}{G})^2 \\ &+ (\alpha_1\lambda^2 + 2\alpha_1\mu)(\frac{G'}{G}) + \alpha_1\lambda\mu \end{aligned} \tag{14}$$

By substituting (11)-(14) into Eq. (8) and collecting all terms with the same power of (G′/G) together, the left-hand side of Eq. (8) is converted into another polynomial in (G′/G). Equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for α_1, α_0, v, λ, μ and c as follows:

$$\begin{gathered} c - v\alpha_0 + \frac{1}{2}p\alpha_0^2 + \frac{1}{3}q\alpha_0^3 + \alpha_1\lambda\mu = 0 \\ -v\alpha_1 + p\alpha_1\alpha_0 + q\alpha_1\alpha_0^2 + \alpha_1\lambda^2 + 2\alpha_1\mu = 0 \\ \frac{1}{2}p\alpha_1^2 + q\alpha_1^2\alpha_0 + 3\alpha_1\lambda = 0 \\ \frac{1}{3}q\alpha_1^3 + 2\alpha_1 = 0 \end{gathered} \tag{15}$$

By solving the algebraic equations above, yields

$$\alpha_1 = \pm\sqrt{\frac{6}{q}}i,\ \ \alpha_0 = \frac{-p\sqrt{\frac{6}{q}}i \pm 6\lambda}{2q\sqrt{\frac{6}{q}}i} \tag{16}$$

$$v = \frac{-p^2}{4q} - \frac{\lambda^2}{2} + 2\mu$$

$$c = \frac{p^3}{24q^2} + \frac{\lambda^2 p}{4q} \pm \frac{\sqrt{6}\lambda^3}{4q\sqrt{\frac{1}{q}}i} - \frac{\mu p}{q} \pm \frac{\sqrt{6}\mu\lambda}{q\sqrt{\frac{1}{q}}i} + \frac{\sqrt{6}\lambda^3 i}{4q\sqrt{\frac{1}{q}}i} + \frac{i\sqrt{6}\mu\lambda}{q\sqrt{\frac{1}{q}}} \tag{17}$$

By using (16), expression (11) can be written as

$$u(\xi)= \pm\sqrt{\frac{6}{q}}i(\frac{G'}{G})+\frac{-p\sqrt{\frac{6}{q}}i\pm 6\lambda}{2q\sqrt{\frac{6}{q}}i} \tag{18}$$

And

$$\xi = x-(\frac{-p^2}{4q}-\frac{\lambda^2}{2}+2\mu)t$$

Eq (18) is the formula of a solution of Eq (8), provided that the integration constant c in Eq. (8) is taken as that in (17). Substituting the general solutions of Eq. (10) into (18) we have three types of travelling wave solutions of the Kdv-MKdv equation (6) as follows:

When $\lambda^2-4\mu \succ 0$

$$u(\xi)= \pm\sqrt{\frac{6(\lambda^2-4\mu)}{q}}i(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi})+\frac{-\sqrt{\frac{6}{q}}i\pm 6\lambda}{2q\sqrt{\frac{6}{q}}i}-\frac{\lambda}{2}$$

Where

$$\xi = x-(\frac{-p^2}{4q}-\frac{\lambda^2}{2}+2\mu)t$$

C_1 and C_2, are arbitrary constants. When $\lambda^2-4\mu \prec 0$

$$u(\xi)= \pm\sqrt{\frac{6(4\mu-\lambda^2)}{q}}i(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})+\frac{-p\sqrt{\frac{6}{q}}i\pm 6\lambda}{2q\sqrt{\frac{6}{q}}i}-\frac{\lambda}{2}$$

When $\lambda^2-4\mu=0$

$$u(\xi)=\frac{\pm\sqrt{\frac{6}{q}}iC_2}{C_1+C_2\xi}$$

Where C_1 and C_2 are arbitrary constants.

SHORMA-TASSO-OLVER EQUATION

Now we consider the Shorma-Tasso-Olver Eq in the form

$$u_t+\alpha(u^3)_x+\frac{3}{2}\alpha(u^2)_{xx}+\alpha u_{xxx}=0 \tag{19}$$

and look for the travelling wave solution of Eq. (19) in the form

$$u(x,t)=u(\xi),\ \ \xi=x-vt \tag{20}$$

where the speed v of the travelling waves is to be determined later.

By using the travelling wave variable (20), Eq. (19) is converted into an ODE for $u=u(\xi)$

$$-vu'+\alpha(u^3)'+\frac{3}{2}\alpha(u^2)''+\alpha u'''=0 \tag{21}$$

Integrating it with respect to ξ once yields

$$c-vu+\alpha(u^3)+\frac{3}{2}\alpha(u^2)'+\alpha u''=0 \tag{22}$$

where c is an integration constant that is to be determined later.Considering the homogeneous balance between u'' and u^3 in Eq. (22) $3m = m+2 \rightarrow m = 1$ we can suppose that the solution of Eq. (22) is of the form

$$u(\xi)= \alpha_1(\frac{G'}{G})+\alpha_0 \tag{23}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0 \tag{24}$$

α_1, α_0, λ and μ are to be determined later. Therefore

$$u^3 = \alpha_1^3 (\frac{G'}{G})^3 + 3\alpha_1^2\alpha_0 (\frac{G'}{G})^2 + 3\alpha_1 \alpha_0^2 (\frac{G'}{G}) + \alpha_0^3 \quad (25)$$

By using (23) and (24) it is derived that

$$u'' = 2\alpha_1 (\frac{G'}{G})^3 + 3\alpha_1\lambda (\frac{G'}{G})^2 + (\alpha_1\lambda^2 + 2\alpha_1\mu)(\frac{G'}{G}) + \alpha_1\lambda\mu \quad (26)$$

$$(u^2)' = -2\alpha_1 (\frac{G'}{G})^3 + (-2\alpha_1\lambda - 2\alpha_1\mu_0)(\frac{G'}{G})^2 + (-2\alpha_1^2\mu - 2\alpha_1\alpha_0\lambda)(\frac{G'}{G}) - 2\alpha_1\alpha_0\mu \quad (27)$$

Substituting the expressions (23) and (25)-(27) into Eq. (22) and collecting all terms with the same power of (G′/G) together, the left hand sides of Eq.(22) are which is the solitary wave solution of the Shorma-Tasso-Olver Equation.

When $\lambda^2 - 4\mu \prec 0$

$$u(\xi) = 2\sqrt{\lambda^2 - 4\mu}\left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right) + \frac{\lambda}{2}$$

When $\lambda^2 - 4\mu = 0$

$$u(\xi) = \frac{2C_2}{C_1 + C_2\xi}$$

$$\xi = x - \frac{\alpha}{2}(29\lambda^2 - 8\mu - 9\lambda)t$$

Where C_1 and C_2 are arbitrary constants.

(2+1)-DIMENSIONAL KONOPELCHENKO-DUBROVSKY EQUATION

In this section we consider the (2 + 1)-dimensional Konopelchenko-Dubrovsky equation. in the form

$$u_t - u_{xxx} - 6buu_x + \frac{3a^2}{2}u^2u_x - 3v_y + 3avu_x = 0,\ u_y = v_x \quad (30)$$

Using the wave solutions

$$u(x,t) = u(\xi),\ \xi = kx + ly + vt \quad (31)$$

and after integration with respect to ξ, we obtaine the second order differential equation

$$c + (v - \frac{3l^2}{k})u - k^3u'' + (\frac{3al}{2} - 3bk)u^2 + \frac{a^2}{2}ku^3 = 0 \quad (32)$$

where c is an integration constant that is to be determined later. Considering the homogeneous balance between u'' and u^3 in Eq. (32) 3m = m+2→m = 1 we can suppose that the solution of Eq. (32) is of the form

$$u(\xi) = \alpha_1(\frac{G'}{G}) + \alpha_0 \quad (33)$$

where G = G(ξ) satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (34)$$

α_1, α_0, λ and μ are to be determined later. On substituting (33) into (32), collecting all terms with the same powers of (G′/G) and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$c + (v - \frac{3l^2}{k})\alpha_0 - k^2\lambda\mu\alpha_1 + (\frac{3al}{2} - 3bk)\alpha_0^2 + \frac{ka^2\alpha_1^3}{2} = 0 \quad (35)$$

$$(v - \frac{3l^2}{k})\alpha_1 - k^3(\alpha_1\lambda^2 + 2\alpha_1\mu) + 2(\frac{3al}{2} - 3bk)\alpha_0\alpha_1 + \frac{3ka^2\alpha_1\alpha_0^0}{2} = 0$$

$$-3k^3\alpha_1\lambda + (\frac{3al}{2} - 3bk)\alpha_1^2 + \frac{3ka^2\alpha_1^2\alpha_0}{2} = 0$$

$$-2k^3\alpha_1 + \frac{ka^2\alpha_1^3}{2} = 0$$

On solving the above algebraic by using the Maple, we get

$$\alpha_1 = \pm\frac{2k}{a},\ \alpha_0 = \pm\frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2} \quad (36)$$

And for

$$\alpha_1 = \frac{2k}{a}, \alpha_0 = \frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2}$$

we obtain

$$v = \frac{9l^2}{k} + k^2\lambda^2 + 2k^2\mu - 3al(\frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2}) + \frac{12k\lambda b}{a} - \frac{12bl}{a} + \frac{18kb^2}{a^2} + \frac{3k\lambda^2}{2} - 3\lambda l \quad (37)$$

Substituting Eqs. (36), (37) into the Eq.(35), we obtain the integration constant c. Substituting (36) into (33) yields:

$$u(\xi) = \pm\frac{2k}{a}(\frac{G'}{G}) \pm \frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2}$$

On substituting the general solutions of the second order LODE (34) into formulae, we deduce the following traveling wave solutions

Case 1: If $\lambda^2 - 4\mu \succ 0$ then we have

$$u(\xi) = \pm\frac{2k}{a}\sqrt{\lambda^2-4\mu}\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) \pm \frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2} - \frac{\lambda}{2}$$

Case 2: If $\lambda^2 - 4\mu \prec 0$ then we have

$$u(\xi) = \pm\frac{2k}{a}\sqrt{4\mu-\lambda^2}\left(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right) \pm \frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2} - \frac{\lambda}{2}$$

Case 3: When $\lambda^2 - 4\mu = 0$

$$u(\xi) = \frac{\pm\frac{2k}{a}C_2}{C_1 + C_2\xi}$$

$$\xi = x - \left(\frac{9l^2}{k} + k^2\lambda^2 + 2k^2\mu - 3al\left(\frac{\lambda}{a} - \frac{1}{ka} + \frac{2b}{a^2}\right) + \frac{12k\lambda b}{a} - \frac{12bl}{a} + \frac{18kb^2}{a^2} + \frac{3k\lambda^2}{2} - 3\lambda l\right)t$$

Where C_1 and C_2 are arbitrary constants.

CONCLUSIONS

The (G′/G)-expansion method has its own advantages: direct, concise, elementary that the general solutions of the second order LODE have been well known for many researchers and effective that it can be used for many other nonlinear evolution equations, for instance the Burgers equation [2], the Kdv equation [3], the MKdv equation [3], the Boussinesq equation [3], the Kdv-Burgers equation [6], the Gardner equation [4] and various variant Boussinesq equations [5, 7] and so on. The researching results of these equations mentioned will be appeared elsewhere. We have noted that the (G′/G)-expansion method changes the given difficult problems into simple problems which can be solved easily.

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