Some Physical Consequences of Vacuum Polarization*

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The phenomenon of vacuum polarization can be studied apart from other higher-order electrodynamic effects through its modification of the electrostatic interaction of heavy charged particles. In particular the principal deviations of the $2p-1s$ level separations from the Bohr formula in light mu-mesonic atoms ($Z\leq 6$) are expected to arise from vacuum polarization effects rather than relativistic effects (ordinary hydrogenic fine structure) or finite nuclear size. The vacuum polarization contribution to the electrostatic interaction of two protons requires slight changes in the usual analysis of proton-proton scattering data to obtain information about nuclear forces. The effect of vacuum polarization on the electrostatic energies of nuclei is also briefly discussed.

ONE of the most interesting of the phenomena predicted by contemporary quantum electrodynamics is the polarization of the vacuum arising from the existence of the electron-positron field.[1–4] It is in a very real sense a new physical phenomenon which may be largely distinguished from other higher order electrodynamic effects (radiation reaction) and deserves study in its own right. An important consequence of the polarization of the vacuum is a modification of the electrostatic interaction between two electrically charged particles at spatial separations of the order of or smaller than an electron Compton wavelength. Thus to first order in the fine structure constant, $\alpha=e^2/\hbar c=1/137.0$, the potential energy of two particles at rest with charges e_1 and e_2 and situated a distance r apart is predicted to be:[3–5]

$$V(r)=\frac{e_1e_2}{r}\left[1+\frac{2\alpha}{3\pi}\int_1^\infty e^{-2\kappa\xi r}\left(1+\frac{1}{2\xi^2}\right)\frac{(\xi^2-1)^{\frac{1}{2}}}{\xi^2}d\xi\right], \quad (1)$$

where $\kappa=mc/\hbar$ is the reciprocal Compton wavelength of the electron. At separations between the particles of order 10^{-13} cm, the correction to the ordinary Coulomb law is of the order of one-half percent, and increases logarithmically at smaller separations.

This deviation from Coulomb's law leads, among other effects, to a shift of the energy levels in atoms,[4] and, in particular, it contributes about -27 mc/sec of the 1051 mc/sec difference between the $2p_{1/2}$ and $2s_{1/2}$ levels (the Lamb shift) in ordinary hydrogen. This well-known vacuum polarization contribution seems essential in order to obtain agreement[6] between theory[7] and experiment[8] for this Lamb shift, and hence the experiment may be considered as providing direct evidence for the reality of vacuum polarization phenomena.

While in the ordinary (electronic) hydrogen atom the effect of vacuum polarization in shifting atomic levels is much smaller than other electrodynamic phenomena (radiation reaction), this will no longer be the case when one deals with problems involving the electrical interaction of particles much heavier than electrons, since the effects of radiation reaction are inversely proportional to the square of the mass of the particles involved. In fact, for such heavy particles the polarization of the vacuum leads to the principal deviations from predictions based on Coulomb's law alone for the electrical behavior of heavy, slowly moving, point-charged particles. From this follows the experimental possibility of exploring vacuum polarization phenomena apart from other electrodynamic effects; it also implies that consideration must be given to the effects of vacuum polarization in analyzing experimental results which involve the electrical interaction of heavy particles. We follow with a consideration of some special situations where the effects of vacuum polarization play a significant and perhaps experimentally observable role.

LEVEL SHIFTS IN LIGHT MU-MESONIC ATOMS

Just as in the case of ordinary electronic atoms, vacuum polarization leads to a displacement of the atomic energy levels of atoms composed of a mu meson and a nucleus. Such atoms have recently become accessible to experimental study.[9] For mu-mesonic

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TABLE I. Fractional displacement arising from vacuum polarization, relativity, and finite nuclear size of the $2p_{3/2}$, $2p_{1/2}$, and $2s_{1/2}$ level separations from the $1s_{1/2}$ level from the value predicted by the elementary Bohr theory for light mu-mesonic atoms.

Atom:	H^1	He^4	Li^7	Be^9	B^{11}	C^{12}	N^{14}	O^{16}
$\frac{\Delta E_{2p_{3/2}}-\Delta E_{1s_{1/2}}}{E_2-E_1}\times 10^3$								
Vac. pol.	1.00	2.22	3.17	3.88	4.49	5.01	5.48	5.88
Relativity	0.02	0.07	0.15	0.27	0.42	0.60	0.82	1.07
Nuc. size	**	−0.26	−0.88	−1.87	−3.34	−5.11	−7.72	−11.06
Total	1.02	2.03	2.44	2.28	1.57	0.50	−1.42	−4.11
$\frac{\Delta E_{2p_{1/2}}-\Delta E_{1s_{1/2}}}{E_2-E_1}\times 10^3$								
Vac. pol.	1.00	2.22	3.17	3.88	4.49	5.01	5.48	5.88
Relativity	0.01	0.05	0.11	0.20	0.31	0.44	0.60	0.78
Nuc. size	**	−0.26	−0.88	−1.87	−3.34	−5.11	−7.72	−11.06
Total	1.01	2.01	2.40	2.21	1.46	0.34	−1.64	−4.40
$\frac{\Delta E_{2s_{1/2}}-\Delta E_{1s_{1/2}}}{E_2-E_1}\times 10^3$								
Vac. pol.	0.89	2.02	2.92	3.61	4.20	4.71	5.13	5.54
Relativity	0.01	0.05	0.11	0.20	0.31	0.44	0.60	0.78
Nuc. size	**	−0.23	−0.77	−1.63	−2.92	−4.47	−6.76	−9.67
Total	0.90	1.84	2.26	2.18	1.59	0.68	−1.03	−3.35
E_2-E_1 (kev)	1.898	8.216	18.71	33.37	52.26	75.32	102.66	134.22

atoms the shifts due to vacuum polarization are relatively much larger than for electronic atoms, and in the case of atoms with nuclei having atomic numbers $Z\leq 6$, are larger than the level displacements associated with any other cause, including relativistic shifts (relativistic fine structure), radiation reaction, and the effect of the finite size of the nucleus. The study of such atoms thus provides an ideal case for investigation of vacuum polarization phenomena apart from other higher-order electrodynamic effects and makes possible the establishment of the reality of vacuum polarization phenomena in a situation in which no *real* electrons play a role.

The computation of the shifts of energy levels in hydrogenic atoms is easily carried out by the use of the potential given in Eq. (1) treating the vacuum polarization correction by first-order perturbation theory. We have calculated the displacements of the $1s_{1/2}$, $2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$ levels in mu-mesonic atoms for nuclear atomic numbers ranging from $Z=1$ to $Z=8$. These results are summarized in Table I where there is presented a tabulation of the fractional deviations of the $2s_{1/2}-2p_{1/2}$, $2p_{1/2}-1s_{1/2}$, and $2p_{3/2}-1s_{1/2}$ level separations from the value predicted by the elementary Bohr theory for the separation of $n=2$ and $n=1$ levels: $E_2-E_1=3Z^2\mu e^4/8\hbar^2$, where μ is the reduced mass of the meson-nucleus system. For comparison the relative shifts arising from relativistic effects and from the finite size of the nucleus have also been tabulated together with the total relative shifts arising from these three effects jointly. Further small shifts might be expected to arise from the following effects:

(a) polarization of the nucleus by the mu meson.[10,11] This effect is difficult to estimate accurately since it is sensitive to the nuclear model. Rough estimates indicate it to lead to shifts which are a relatively small fraction of those associated with the finite size of the nucleus.

(b) Shifts associated with radiation reaction. Other higher-order electrodynamic effects ("ordinary Lamb shift") will be of the order of a few percent of those associated with vacuum polarization.

(c) Shifts associated with extranuclear electrons. The presence of electrons in Bohr orbits about the mu-mesonic atom leads to *relative* shifts of the $2p$ and $1s$ levels which are very small compared to any of the shifts tabulated.

Thus the tabulated total shifts will not be significantly affected by these residual effects; the largest uncertainties in the figures for the total shift probably arise from the uncertainties in nuclear radii and nuclear polarizability. The computations have been performed for a mu-meson mass of 207.0 electron masses[12] and for nuclear radii computed from the formula $R=r_0A^{1/3}$ with $r_0=1.3\times 10^{-13}$ cm. Since the shifts associated with finite nuclear size are proportional to the square of the nuclear radius, a ten percent change in the latter will lead to a twenty percent change in the figures quoted for the former.

The present experimental uncertainty in the mu-meson mass (± 0.4 electron mass) will not appreciably affect the figures for the fractional level shifts quoted in Table I, though it will lead to a 0.2 percent uncertainty in the energy separation of the $2p$ and $1s$ levels as computed from the Bohr formula. This uncertainty is of some importance therefore in attempting to verify the existence of the vacuum polarization effect

[10] L. Cooper and E. Henley, Phys. Rev. **92**, 801 (1953).
[11] W. Lakin and W. Kohn, Phys. Rev. **94**, 787 (1954).
[12] Smith, Birnbaum, and Barkas, Phys. Rev. **91**, 765 (1953).

experimentally. It means that measurements of the $2p-1s$ level separations in a single atom will hardly suffice to verify unambiguously this effect and that measurements in two or more light atoms of the transition energy to an accuracy of the order of 0.1 percent will be necessary. The fact that $(\Delta E_{2p}-\Delta E_{1s})/(E_2-E_1)$ first increases and then decreases as a function of Z would appear to be the experimental feature which would provide the most direct verification of the vacuum polarization effect. The energy splitting of the $2p_{1/2}$ and $2s_{1/2}$ levels, if resolvable, would also serve this purpose. While measurements of the x-rays from these light mu-mesonic atoms to the required accuracy does not seem possible at present, it is hoped that experimental techniques may soon be refined to the point where these predictions can be subjected to test.

For heavy mu-mesonic atoms on which experiments have already been performed,[9] the effect of vacuum polarization on level displacements is only a small fraction of the shifts associated with finite nuclear size. For these atoms the level shifts have been employed to determine nuclear radii, and the recognition of the effect of vacuum polarization leads to a correction for the radii so derived of about one percent, as has been pointed out previously.[13] There will also be a small effect of vacuum polarization on the Coulomb scattering of mu mesons, but very much refinement of scattering measurements would be required to make these effects accessible to detection.

PROTON-PROTON SCATTERING

The contribution of vacuum polarization to the electrostatic interaction of two protons requires a small modification of the theoretical analysis of proton-proton scattering data to obtain information about nuclear forces. The long range of the vacuum polarization potential relative to the range of nuclear forces means that in the analysis of proton-proton scattering by means of the f function of Breit, Condon, and Present[14] or the K function employed by Blatt and Jackson[15] there will be a slight curvature in the plot of either of these functions against energy, particularly at the lower energies. This curvature cannot be seen in the presently available data because of the magnitude of the experimental errors. However, the vacuum polarization potential also leads to a slight displacement and change in slope of the plot of either of these functions against energy in the energy region 0.2–4 Mev where the best experimental data exists.

Preliminary calculations of this last effect have been performed. The results may be expressed as follows (we employ the notation of Blatt and Jackson[15]): the contribution to the K function arising from vacuum polarization at several energies are:

Energy (Mev)	ΔK
0.670	0.031
1.200	0.023
1.830	0.020
3.04	0.019
3.53	0.018

A fit of these values by a least-squares straight line yields:

$$\Delta K=0.043-(0.007\times10^{-24})k^2,$$

with k the wave vector magnitude measured in cm^{-1}. If we now take the constants of the K plot as derived by Blatt and Jackson,

$$-R/a=3.755\pm0.024,\quad r_e=(2.65\pm0.07)\times10^{-13}\text{ cm},$$

and correct them to apply to the idealized case where vacuum polarization is absent, one obtains

$$-R/a=3.726\pm0.024,\quad r_e=(2.66\pm0.07)\times10^{-13}\text{ cm}.$$

Thus we see that the effect of vacuum polarization on the zero energy scattering length a is of the order of the present experimental error. If one further traces the consequences of these changes through Schwinger's analysis[16] of the equality of the nuclear ^1S interaction between two protons and between a proton and a neutron, one finds that for the Yukawa potential the discrepancy between the zero-energy scattering lengths for these two interactions is increased from about 7 percent to about 9 percent. (This is perhaps favorable in expanding this narrow margin in order to allow room for the possibility that the magnetic moments of nucleons are not associated with point dipoles but with a spatial distribution of magnetization.)

It is planned to refine the foregoing calculations and also to extend them to energies below 0.6 Mev in the hope that more accurate proton-proton scattering experiments may eventually allow one to recognize directly the effects of vacuum polarization through the curvature of the K- or f-function plots at these lower energies.

COULOMB ENERGIES OF NUCLEI

Since the average separation of protons in a nucleus is much smaller than the electron Compton wavelength, the electrostatic energy of the nucleus is modified to a small extent by vacuum polarization. One may readily estimate that this energy is about 0.5 percent greater than one would calculate with neglect of vacuum polarization. This means that nuclear radii as derived from the mass differences of mirror nuclei should be increased by this same percentage over their previously derived values. Similarly, the coefficient of the Coulomb energy term in the Weizsäcker semi-empirical mass formula should also be slightly increased.

[13] See note added in proof, reference 10; also H. C. Corben, Phys. Rev. **94**, 787 (1954).

[14] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936); Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952).

[15] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

[16] J. Schwinger, Phys. Rev. **78**, 135 (1950).

Unfortunately, so many uncertainties enter into the calculation of nuclear radii from electrostatic energies as to preclude an unambiguous detection of these vacuum polarization effects.

The effects described earlier seem to be those which are least unfavorable for an unambiguous verification of the existence of vacuum polarization phenomena where they are clearly separated from other higher-order electrodynamic effects. It should be kept in mind, however, that such effects are to be expected to be present in all cases where electrostatic interactions at small distances are involved. Further examples of such cases would be: Coulomb penetration factors in nuclear reactions, alpha-particle scattering, alpha-decay theory, fission theory, the astrophysical reaction: $p+p\rightarrow d+e^{+}+\nu$, etc. In all of these cases, however, other uncertainties enter of such magnitude as to make vacuum polarization effects unimportant at the present time.

The present paper represents a preliminary report on these investigations. Details of the calculations and further numerical results will be published elsewhere.

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Gravitational Field Equations

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Arguments are given to show that the laws that determine the motion of the ether should not refer to the motion of anything that is not in the immediate vicinity of the point whose motion is being considered. From this assumption and from Newton's laws of gravitation, the equations of motion of the ether are derived.

I. INTRODUCTION

IN classical mechanics it was assumed that the acceleration of any massive body relative to a primary inertial system was proportional to the force that acted on it. The motion of the primary inertial system was presumed to be known from observation, and it was frequently assumed to be fixed relative to the fixed stars, an assumption which agreed well with Newton's very successful theory of planetary motion. However, this assumption has sometimes been questioned on the intuitive grounds that it is difficult to see how there can be an intimate connection between the motions of two systems that are as widely separated as the solar system and the fixed stars when there is no tangible, physical means of contact between them. It is partially for this reason that many attempts were made during the last century to introduce a fluid medium, called the ether, which could provide the desired connection and which at the same time could provide a medium for the propagation of electromagnetic disturbances.

In a previous paper,[1] it was shown that Einstein's principle of equivalence makes it possible to interpret the gravitational field in terms of an ether flow in a three-dimensional Euclidean space and that all of the verified results of the general theory of relativity can be deduced using this interpretation. It was assumed that coordinates that are fixed in the ether form both a primary inertial system of mechanics and a system in which the velocity of light is the same in all directions, and on the basis of this assumption the equations of motion of a mass point and of a light ray moving through the ether were derived for an arbitrary ether velocity. The velocity of the ether in the spherically symmetric gravitational field was determined from Newton's inverse square law of gravitational attraction, but no attempt was made to derive the laws that determine the motion of the ether in general. It is the object of this paper to discuss one fundamental aspect of this problem and to use the conclusions of this discussion and Newton's laws of gravitation to determine the equations of motion of the ether. As in the previous article, it will be sufficient to assume that a single universal time variable has been introduced. If, as is postulated in the special theory of relativity, there are several completely equivalent ways of introducing such a variable, then any one of them may be used. When the time variable has been defined, the velocity of the ether is uniquely determined everywhere by the fact that the one-way velocity of light relative to the ether is the same in all directions.

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