Some Prospects on the Experiments of Very High Energy Hadronic Reactions*)

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Some interpretations and prospects on the multi-particle production experiments which have been advancing nowadays are given.

The representations of the rapidity space in more than two dimensions are reviewed for the reasonable choice of variables which describe exclusive and inclusive experiments.

On the basis the reduced rapidity variables, the scaling of the multiplicity distribution functions is derived. The multiplicity distribution expressed in the Wigner Surmise form for complex systems in general is interpreted as a possible indirect manifestation of the multiplicity distribution of H-quanta which occupy an available rapidity box.

The significances of the discrete structure in the rapidity space are discussed from the theoretical viewpoints on the question of the indivisibility of the space-time manifold.

Introduction

Since the first conjecture by Heisenberg[1] on the existence of the multiple particle production in very high energy particle collisions, more than thirty years have passed. At that time, this process was considered to be an unusual one associated with the discussion on the applicability condition of quantum field theory. Nowadays, multi-particle production processes are apparently very familiar ones in the high energy particle collisions, and extensive works both from the experimental and theoretical sides have been accumulated day by day. Though recent phenomenological studies based on the S-matrix theory, which was also proposed by Heisenberg for the first time, have revealed various important features, it may be considered that the present understanding of those processes is a preliminary one. Concerning to these situations, we would like to begin with the following considerations. First, it seems for us rather hard to have a sufficient understanding of physics of elementary particles without considering any space-time background, because actually the experimental procedures for the measurements of non-spatio-temporal quantities are deeply related to the measurements of the space-time coordinates themselves, and the usual human abilities for the recognition of nature may have a certain tendency which prefers the space-time descriptions. In this

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sense, it is essentially important to ask how far our space-time concepts can describe the nature, especially in the smallest domains where a straightforward interpolation of the usual space-time concept (Minkowskian space-time) has been questioned and sometimes even a complete abandonement of it has been advised.

Although there may be various ways to achieve the goal, we have pointed out on several occasions the importance of recognizing a particular kind of phenomena which may appear as the discrete activities in the velocity space of the final state particles in the multi-particle processes.[2] We have also pointed out that such a discrete activity in the velocity space can be traced back up to the discrete space-time background,[3] though at the present stage of the development these reciprocal interrelations are rather numerological and we can only give plausibility arguments on this problem. For doing that, first of all, a reasonable kinematical basis which was proposed several years ago by Bubelev,[4] is reviewed. This scheme is very transparent and more stereographic, when we are forced to deal with the exclusive jet data and our interest lies essentially on the massive state. Secondly, we shall consider the reduced rapidity variable[20] which is analogous to the Feynman's x[5] in the rapidity space. On the basis of the reduced rapidity variables, KNO[6] scaling of the multiplicity distribution is derived. The recent experimental data of giant accelerators favour Wigner Surmise[7] type of the multiplicity distribution function. The original Wigner distribution was concerned with the level spacing distributions of complex atoms and atomic nuclei, then we shall try to trace back from the spacing distribution of the prongs in the longitudinal rapidity axis up to the multiplicity distribution of the production of H-quanta[8] (or emission centres), where we assume that a fundamental unit exists along the longitudinal rapidity axis and in average H-quanta decay into equal number of particles. Then, we shall discuss the correspondence between discrete space-time structure and the appearance of discrete velocities.

§1. The rapidity space

Recently, single particle spectra in the multi-particle final states produced by energetic hadronic reactions are greatly clarified experimentally. Presently, experimental data are expressed in the fractional longitudinal momenta x[5] in the center-of-mass system or in the longitudinal rapidity y,[8] where y is defined in terms of longitudinal velocity v:

$$y=\int_0^v dv/(1-v^2).$$

Very recently, Carruthers and Minh Duong-van[45] introduced a transverse rapidity, however as we shall show in the present paper, their variable may be limitted to the region near zero longitudinal rapidity.

Actually, y had been known widely among the cosmic ray physicists as log $\tan\theta/2$, where θ is the emission angle in a particular coordinate system. The essentially important feature lies in the translational invariance along the log $\tan\theta/2$ axis except for very slow particles, though there is a minor difference in the factor of log e. The former defect was improved by taking y*) as the appropriate kinematical variables. By analogy with the hodograph method in the classical mechanics, the velocity space seems to be more fundamental than the momentum space at least for kinematical purposes. In the longitudinal rapidity space, additive nature is recovered with respect to y. In the early period of this century, its mathematical correspondence was found with the Lobachevskian Geometry,[10] and it was found that the hodograph method is possible in this space.[11]

Let us begin with the four velocity $u(u_0, u_1, u_2)$ in the three dimensional space-time. The generalization to the four dimensional case is also possible, but graphical representation is difficult for this case. If the masses of either the observed particles in the final state or the constituents in the intermediate states are fixed, a tip of the four velocity vector always lies on the surface of one of the sheets of a hyperboloid,

$$u^2=u_0^2-u_1^2-u_2^2=1, \qquad u_0>0.$$

The metric in the four velocity space and the velocity components $\bar{u}_1$, and $\bar{u}_2$ are given by Eqs. (1) and (2) respectively;

$$dS_u^2=du_0^2-du_1^2-du_2^2 \tag{1}$$

and

$$u_0\bar{u}_1=u_1,\ u_0\bar{u}_2=u_2. \tag{2}$$

By making use of the relation $u_0du_0-u_1du_1-u_2du_2=0$, we obtain the line element in the velocity space $(\bar{u}_1, \bar{u}_2)$ as follows,

$$ds^2=\frac{d\bar{u}_1^2+d\bar{u}_2^2-(\bar{u}_1d\bar{u}_2-\bar{u}_2d\bar{u}_1)^2}{(1-\bar{u}_1^2-\bar{u}_2^2)^2}, \tag{3}$$

which gives the two dimensional metric of the Lobachevskian Geometry. There is 1 : 1 correspondence between the point on one of the sheets of the hyperboloid and the point inside the circle $D(\bar{u}_1^2+\bar{u}_2^2=1)$ on the plane $\pi(u_0=1)$.

We shall consider, in the present paper, the geodesic parallel coordinate, though there are various coordinate systems.

The line element of the geodesic parallel coordinate[4),12)] is

*) Recently Carruthers et al. introduced a parameter

$$y=\frac{1}{2}\ln\frac{(P+P_{\parallel})}{(P-P_{\parallel})}.^{9)}$$

$$ds^2=\cosh^2\left(\frac{\xi}{k}\right)d\left(\frac{\eta}{k}\right)^2+d\left(\frac{\xi}{k}\right)^2, \tag{4}$$

and the relation of the variables are

$$\bar{u}_1=\tanh\frac{\eta}{k} \text{ and } \bar{u}_2=\frac{\tanh \xi/k}{\cosh \eta/k}. \tag{5}$$

The choice of a specific value for k is equivalent to selecting a unit of length. Thus the absolute unit can be determined by a certain kind of the geometric conditions which corresponds to a particular nature of the physical phenomena. If we take $k=1$, we have the usual definition of y and our variable is additive. The additivity is essentially important for the kinematical analysis. If we wish to treat the transverse motions on the same footing with the longitudinal one, this variables may be one of the convenient ones. The transverse rapidity variable ξ is related to the transverse velocity associated with the longitudinal rapidity (divided by the Lorentz factor). In general the transverse momentum is $m\sinh \xi$, and, at $\eta=0$, $\bar{u}_2=v_T=\tanh \xi$. This coordinate system corresponds to the nets of ellipses whose hyperbolic distances are constant from the $\bar{u}_1=v$ axis and the lines $\bar{u}_1=$const. which are perpendicular to the family of ellipses. The area element for this two dimensional case is

$$d\omega=\cosh\frac{\xi}{k}d\xi d\eta. \tag{6}$$

We may have a room to choose a proper normalization k. For example, if we take $k=Y=\ln s$, then the slow $\ln s$ decrease may be expected due to the purely geometrical reasons with respect to the transverse direction, where $\sqrt{s}$ is the total centre-of-mass energy of the colliding two hadrons and of course there may be a certain applicability region of this type of scaling. The choice of the coordinate systems can be determined in a close correspondence with the experimental data.

We have recapitulated the mathematical representation of the two-dimensional velocity space which was noted by Bubelev for the first time. In order to represent the individual jet data, a further elaboration was made by him, a generalization of von Lindern plot ($\log_{10} \tan\theta_L$-plot) was accomplished by a conformal mapping onto the Euclidean band. Unfortunately the method has not been used so frequently except for by Bubelev et al. and a few authors. This is partly due to lack of the accessible experimental data. However, if in future experiments the complete data including the informations on the particle identifications and momentum vectors become to be available, Bubelev's scheme is one of the most transparent way of the presentation of the experimental data. The three dimensional case was also studied by Kato and Mori.[46)]

§2. "Time" and rapidity

In classical mechanics, by Hamilton's hodograph method, the time t is represented by

$$t=\int\frac{ds}{\sqrt{X^2+Y^2+Z^2}},$$

where X, Y and Z are the coordinate of the points on the hodograph, and ds is the infinitesimal line element of the path. Of course, we cannot know the time dependence of the velocity of a certain part of the excited hadronic matter during the collisions and also our problem is relativistic. However, it may be interesting to consider the cases where all the particles or a certain cluster of particles in the multi-body final states in the collisions are originally exploded from a certain point at a certain common time.

For example, when particles are really the explosion products of a certain event which happened at a particular "time"$=0$, and they are moving with constant velocities, we may make the following transformation of variables for the one dimensional movement,

$$t=\sigma\cosh y,\qquad x=\sigma\sinh y.$$

For this case, we have the flat metric,

$$ds^2=d\sigma^2-\sigma^2dy^2,$$

and for null geodesic we have the relation

$$y=\ln\sigma+\text{const.}$$

It may be noted that Milne's version of the Kinematic Relativity has something to do with our problems.[13)]

§3. Inclusive cross sections in the reduced rapidity and scaling of multiplicity distributions

Let us consider the experiments of the inclusive type. For presenting experimental inclusive particle spectra, sometimes it may be convenient to refer the rapidity variables in the longitudinal axis, because the corresponding phase space is simple. Though any coordinate system (equal velocity system, centre-of-mass system, laboratory system, mirror system and so on) could be used at will, we shall take the equal velocity system which is the centre-of-mass system, if we consider proton-proton collisions. The origin of the rapidity y of a secondary is taken as the rapidity of the coordinate system with respect to any other inertial frames. The range of y is

$$-a\frac{Y}{2}<y<a\frac{Y}{2}$$

(a is constant, and$\simeq 1$), where Y is the relative rapidity of a projectile proton and a target proton. The single particle spectra is

$$\frac{1}{\pi}d\sigma=F(y,Y)dy,\ \ y=\frac{1}{2}\ln(E+P_{\parallel})/(E-P_{\parallel}),$$

where E, and $P_{\parallel}$ are the energy, longitudinal momentum vector in the centre-of-mass system respectively, and we shall consider the energy regions in which the transverse momenta are suppressed.

Presently, available experimental data[14] suggest us the slight increase of the height of the central part, when we plot F versus y. Particularly from the compilation of the data in the wide energy range started from 12 GeV/c up to 1500 GeV/c of incident proton momenta,[15] the increase of the height is not inconsistent with Y, though errors are fairly large in the central region. There has been a proposal to fit all the curves with different incident momenta by a single curve, for example Carruthers and Minh Duong-van[16] have proposed the variable $y/\sqrt{Y}$ as a new scaling variable. This possibility was consistent with the hydrodynamical theory worked out by Landau,[17] and essentially $(Y^2-y^2)^{\frac{1}{2}}$ appeared on the exponent in this case.

In the present paper, we shall propose a reduced rapidity variable z which is defined as $2y/Y$. The range of z is bounded by $\pm a$. In the past we mentioned on the possible existence of a certain unit y_0(correlation length). In our viewpoint, the existence of the correlation length of about $1\simeq 2$ in the empirical spectra means the appearance of the fundamental domains in the rapidity space. However, if we consider the coarse-grained spectra, such a mesh size as y_0 may not be necessary. In fact, y/y_0 and Y/y_0 informations are the same with the y and Y in the reduced rapidity. The inclusive cross section is written in terms of z and Y as follows,

$$\frac{1}{\pi}\frac{d\sigma}{dz}=YF(Y)G(z). \tag{7}$$

When $F(Y)$ is constant, $\langle n\rangle\sigma_{\text{inel}}$ is proportional to $\ln s$. If $F(Y)$ is approximated by (constant)$\cdot Y$, in other words the increase of inclusive cross sections at the central region in the rapidity y is approximately proportional to Y, we have the following type of the multiplicity growth,

$$\langle n\rangle=(\ln^2 s)/\sigma_{\text{inel}}. \tag{8}$$

The inelastic cross section may increase, however in the above formula σ_{inel} contains a large constant part, then the effect of the increase of the inelastic total cross sections may not be significant in the above formula (8) up to, say, several TeV. This consequence is interpreted as follows. At a certain energy, particles are produced by the passage of two throughgoing objects each of which can materiarize $\ln s$ particles rather incoherently. However, when the primary energy increases, the number of constituents which consist of each through-going object increases and density increases. Therefore the collision turns out to be the coherent collision between two groups of $\ln s$ particles. In the relativistic collision, the propagation time of the interaction in the direction to the transverse plane is fairly longer than the collision time, therefore constituents can interact with the counter constituents which lie on the collision axis of constituents with the same impact parameter.

This picture gives us $(\ln s)^2$ growth of the multiplicity, if each elementary collision between two constituents gives us $\ln s$ particles. However, as the primary energy increases, the deviation from $(\ln s)^2$ dependence may occur and the multiplicity growth is going to be more and more strong in $(\ln s)$. Perhaps, finally we may have a power dependence too. Anyway, coherence becomes very important in all respects at very high energy regions. On the other hand, the projectile and the target themselves also expand, because of the growing fluctuations[18] inside through-going objects which may be estimated to be $\sqrt{\ln^2 s}$ times the radius of the constituent. Then, we have

$$\sigma_{\rm inel}(\text{coherent}) \sim (\ln s)^2 \tag{9}$$

for the energy regions where $F(Y) \simeq Y$ is a good approximation. This kind of dependence is also obtained by the counting picture of the constituents under the assumption of a constant cross section between two constituents. Recent ISR experiments[47] suggest us this kind of increase, however a power dependence obtained by Kasahara and Takahashi[48] is also possible. For a theoretical interpretation of the latter case, we refer our previous treatment,[2] where y_0 and the number of fundamental constituents which can occupy each fundamental rapidity domain is limitted. Next, we are going to consider the moments of multiplicity distributions. First, we define the multiplicity distribution function $P(n)$ and semi-inclusive cross sections for the case $F(Y) \simeq Y$ as follows,

$$P(n)\sigma_{\rm inel} = \sigma_n, \tag{10}$$

$$d^q\sigma = \sigma_{\rm inel} Y^{2q} G^{(q)}(z_1, \cdots, z_q) dz_1 dz_2 \cdots dz_q. \tag{11}$$

The multiplicity moment is

$$\langle n(n-1)\cdots(n-q+1)\rangle = Y^{2q}\int\cdots\int dz_1 \cdots dz_q G^{(q)}(z_1, \cdots, z_q). \tag{12}$$

By the same procedure with the Koba-Nielsen-Olesen proof,

$$\langle n^q \rangle \sim \langle n(n-1) \cdots (n-q+1) \rangle, \tag{13}$$

where $(\ln s)^{2q}$ terms are assumed to be dominated. It should be noted here that Eq. (11) represents the hypothesis of our scaling, where the meaning of scaling is different from the one currently used. Here, we put emphasis on the dynamical process of scaling but on the limit. Then we have

$$\langle n^q \rangle \sim Y^{2q} \int \cdots \int dz_1 \cdots dz_q G^{(q)}(z_1, \cdots, z_q). \tag{14}$$

On the other hand, $\langle n^q \rangle$ is given by the definition of the multiplicity distribution function;

$$\langle n^q \rangle = \sum_n p_n n^q \sim \int n^q P(n) dn. \tag{15}$$

Equation (14) implies that

$$\langle n^q \rangle \sim \langle n \rangle^q C_q, \tag{16}$$

and C_q is independent of the incident energy. For the derivation of the form of the multiplicity distribution function, we define the variable Z and $g^{(1)}$ as follows,

$$Z = n/g^{(1)} Y^2 \text{ and } g^{(1)} = \int G^{(1)}(z) dz. \tag{17}$$

Finally, we obtain the scaled multiplicity distribution $\Psi(Z)$,

$$P(n) = (g^{(1)} Y^2)^{-1} \Psi(n/g^{(1)} Y^2), \tag{18}$$

where $g^{(1)} Y^2 = \langle n \rangle$. The different point from the previous proof is the introduction of reduced rapidity variables. The reduced rapidity variables was introduced also by van Hove[20] with respect to the different purposes.

At the present stage, by taking into account the experimental uncertainty, we may assume also the non-increase (constant) of the height of the central region at a higher incident energy. For this case, we have the usual multiplicity growth with the primary energy, that is $\langle n \rangle \simeq \ln s$. And the previous arguments should be corrected in the Y dependences (Y^{2q} must be changed into Y^q), the rest of the proof is similar to the previous case. At the present time, we should keep also the possibility of the case studied by Carruthers and Minh Duong-van[16] in mind. However, for the latter, the range of variables $y/\sqrt{Y}$ is dependent on Y, and our previous arguments on the normalization of the rapidity variable based on the fundamental short range correlation length do not apply to this case.

The experimental results recently obtained by NAL experiments indicate the very rapid scaling of the multiplicity distributions above 50 GeV/c of incident momenta,[21] then if $F(Y)$ is assumed to be constant, the above proof of the scaling does not give us the sufficient explanation for the situation. Because, there is certainly some increase of central regions in the energy range between the PS region and the NAL-ISR regions, therefore with regard to the central region scaling is not reached probably at the energy region of NAL experiments.

§4. The Wigner distribution as a scaled multiplicity distribution in the hadronic reactions

Recently, the charged (as well as the neutral) multiplicity distributions at three different momenta (100, 200, and 300 GeV/c) of the incident protons have been obtained by the experiments of the 20" hydrogen bubble chamber at NAL.[27),29)] At the present time, there is a certain ambiguity on the distinction of various different mechanisms (for example, a diffraction dissociation and pionization processes and so on). Then, we assume that the processes with higher multiplicities may have a certain theoretical significance concerning with the dynamical character of the excited hadronic matter. In other words, our interests lie to the behaviour at a higher multiplicity side rather than in the most accurate curve fitting procedure.

From this viewpoint, a scaled multiplicity distribution $\Psi(Z)$ may be represented by

$$\Psi(Z)=\pi Z\exp(-\pi 4^{-1}Z^2) \tag{19}$$

and

$$Z=n/\langle n\rangle. \tag{20}$$

$\Psi(Z)$ has no free parameters and following constraints are satisfied;

$$\int_0^\infty \Psi(Z)dZ=\int_0^\infty Z\Psi(Z)dZ=2. \tag{21}$$

The above type of the distribution was studied and was compared with the empirical data in great detail by Koba et al.,[22)] and we also discussed the significance of the distribution independently.[23)] We would like to point out that the following analogy exists between our scaled multiplicity and a scaled nearest neighbour level spacing of the complex quantum systems in general. The distribution (19) was found by Wigner for the nearest neighbour level spacing distribution of the compound nucleus, where $Z=D/\langle D\rangle$, and D is the level spacing, and normalizations are different by a factor 2.[21)] The experimental evidence on this law for the case of U^{238} nucleus, obtained by the scattering of slow neutrons, is very famous.[24)] If these levels are associated with symmetry properties of the Hamiltonian of the system, the distribution means the repulsion of the levels with the same symmetry.[25] A same phenomena occurs in complex atomic spectra too.[26] The essential point here is the fluctuation laws and it is perhaps system independent, if the system is sufficiently complex. A characteristic feature of the Wigner distribution is represented by the following parameter d,

$$d=\frac{\langle n\rangle}{\sqrt{\langle n^2\rangle-\langle n\rangle^2}}, \qquad (22)^{6)}$$

and $d=1.9$ for this case. Recent experiments have shown that $d=\text{const}=2$ at the incident energy regions above 50 GeV.[27] There are some discrepancies between the theoretical curve and the empirical data near the peak position of the curve, however it may not to be taken as a serious objection to our present standpoint. Because the characteristic feature of the distribution is the Gaussian nature and the other Gaussian form such as given by Eq. (23);

$$\Psi(Z)=64\pi^{-2}Z^2\exp(-4\pi^{-1}Z^2), \qquad (23)^{25)}$$

should be considered too. The difference comes from the different symmetry law for the system, and the common background for both cases is the non-Gibbsian statistical law, in which the every matrix element of the Hamiltonian is equally probable.[28] Equations (19) and (23) correspond to the ensembles with the orthogonal and unitary symmetry respectively. Equation (23) has a slightly sharper peak and a slightly narrower width, however both forms damp already at $Z=3$ in a normal scale height. The interpretation of this number according to the number of leading constituents is tempting, but we do not discuss it here.

Our main task in this section lies in the translation of the argument from the scaled energy level spacing into the scaled multiplicity. Let us consider hadronic reactions along the longitudinal rapidity axis. The final state particle appears in a ordered sequence along this axis within a certain interval Y. (We neglect the cases of the zero mass particles). Then the average nearest neighbour spacing of the rapidity coordinates of N_c cluster (H-quanta[19]) is Y/N_c. The scaled level spacing is given by $(y_i-y_{i+1})/Y\cdot N_c$. If there exist the fundamental length y_0 which characterizes the dynamical processes along the longitudinal rapidity axis, every variable should be measured by this unit. Therefore the scaled level spacing is $(y_i-y_{i+1})(Y/y_0)^{-1}$. (N_c/y_0) and the most dominant contribution comes from $N_c/\langle N_c\rangle$, because the average number of emission centres may be estimated if we divide Y by y_0. If each cluster (H-quantum) decays into an equal number of final particles in the average and at a certain fixed primary energy, the scaled multiplicity for cluster (H-quantum) is nothing but the one for final state particles.

Therefore, we would like to conjecture that final state hadrons can be traced back to a certain kind of objects, which decay into hadrons (mesons, baryons and resonances) and occupy some discrete domains in the velocity space as emission centres. As will be discussed later, the reason why such a discrete structure in the velocity space does not show up very clearly among the experimental data, is due to the noise caused naturally by the decay processes and the elusive origin of the reference system itself. If a fundamental length y_0 is fairly small, then the discrete structual effect is weak. It must be noticed that an analogy to the nuclear and atomic physics suggests us on the nature of degeneracy, where all the levels correspond to the ones with the same quantum numbers. A possible picture is then as follows. Our objects is related to the excited hadronic matter and they simply correspond to the levels with common quantum numbers (for example, the quantum numbers of vacuum or a support with the spin one, however, there may be alternatives). The deviations from the strict multiplicity scaling may occur from the decay mechanism, and they depend on the decay law, on which we shall discuss in the separate paper. The moment problem suggests us the existence of many emission centres.[43] Anyway, at the present stage of the development on this problem, following standpoints are of great worth. Beyond the usual resonant states with various quantum numbers, one tries to trace back up to the states which are complex in constitutions but simple in their gross features (quantum number and the character of the movement). Finally we wish to point out that large fluctuations in the multiplicity of final state hadrons caused by the decay of H-quanta(emission centres) was shown in the frame work of similar kind of theory as the present one in our previous paper,[2] basing on the Gaussian multiplicity distribution. We also pointed out that the available empirical data which had been obtained at that time from the cosmic ray exclusive events were consistent with those features.[2] Nowadays, at the accelerator experiments, a good statistics is available, then the events with very high multiplicities are interesting and the change of the rapidity distributions with the various multiplicities as well as the increase of the prong densities at a particular rapidity ranges may give us important informations on the production mechanism.

§5. The structure in the rapidity space

Firstly let us recall the past development of the analysis of the jet interactions at extremely high energies. Today, such a type of experiments are called as the experiment of exclusive type. Actually, the direct informations on the nature of hadronic interaction at cosmic ray energies have been deduced by this type of experiments. In the course of the development of the so called cosmic ray jet analysis, people gradually noticed that some clustering of prongs of the final state particles in the $\log_{10}\tan\theta$ axis seems to exist. Though the analysis is considerably difficult and the event statistics is rather low at early days, Niu,[30] Polish group[31] and G. Cocconi[32] independently proposed an important working hypothesis which is usually called the two fire ball model. In those kinds of analysis, the isotropic or quasi-isotropic angular distributions in the rest system of a fire ball have been the basic criteria on the existence of the fire ball. In some events such a structure showed up very clearly and in some events two groups were not resolved well. Of course there was another approach for the analysis of the structure in $\log_{10}\tan\theta$ plot, for example there is a certain approximate relation between the separation of two fire balls which is characterized by the Lorentz factor of the fire balls in the centre-of-mass system and the power m of $\cos^m \theta_{\mathrm{CMS}}$ type angular distribution proposed by Hoang.[44] Anyway during the course of the increase of statistics particularly at high energy regions, some people[33] started to notice that two fire balls are not enough to cover available $\log_{10}\tan\theta$ range.*)

In the mean time, Hasegawa proposed so called H-quantum hypothesis. According to this hypothesis, there exist many fire balls (not necessarily two) along the $\log_{10}\tan\theta$ axis, and each fire ball occupy some universal width on this axis. He also determined this width by inspection of some very high energy clean jets. On the basis of these working hypothesis, he obtained the frequency distribution of the Lorentz factor γ_H^* of each fire ball in the centre-of-mass system. The frequency distribution showed the existence of peaks for $\gamma_H^*=1.5$, $6\simeq7$, $40\ldots$. Furthermore, Hasegawa obtained the multiplicity distribution and the average multiplicity of mesons decayed from these fire balls.[19] As soon as these observations were reported, Yukawa noticed the possibility of the discrete velocity at the meeting which was held at Nagoya.[34]

In 1868, Riemann considered the possibility of a discrete manifold in his famous inaugural lecture,[35] and he called it "Quanta". If Hasegawa's discovery is true, there is some possibility to consider this phenomena as the first manifestation of the discrete structure (Quanta) in the Hyperbolic manifold. Before we make a plausibility argumentent on this problem, it seems to be appropriate to discuss the present experimental situation related to this phenomena.

Perhaps many people are now feeling that a new era began after the first operation of CERN-ISR and the NAL machine. The extensive informations are expected to be obtained by these giant accelerators. Up to now the informations on the inclusive type experiment on the proton-proton collisions have been rich. After studying these experiment, one may notice that the inclusive single particle spectra for mesons are rather smooth in the presently available rapidity ranges. We would like to comment on these features of presently available inclusive spectra. As was emphasized by

*) We assume that fire balls decay isotropically or nearly isotropically.

Kittel, there is a certain tendency at PS energy regions in which the inclusive spectra are generally smooth on the one hand and the exclusive spectra are rich in the structure on the other hand. At the present time, above 100 GeV we still do not know on the exclusive event except for 31 log $\tan\theta/2$ plots from NAL.[36] Therefore, we must wait for deriving any definite conclusions on this questions. Turning on the two particle inclusive spectra, there are some structures which seems to be favourable for our hypothesis, however experiments are still preliminary. The good resolutions on the momenta and angles may be required and the smallness of errors are necessary. The heavier particles such as protons, anti-protons and hyperons have shorter wave lengths, then the inclusive spectra for those heavy particle may be interesting. It may be noted that anti-proton may be a very important probe with respect to the pionization processes in a close connection with the present idea.

Cosmic ray data and lower energy bubble chamber data were examined by Iwai[37),38] and Okamoto[38] recently, they found some structure particularly in the two particle correlations between a pion and a recoiled proton at the rest system of the latter. We may expect that proton inclusive spectra in (y_{max}-y) at low P_T may have some structures (1.3 and 1.7 from the origin), however activity may not be lograrithmic in the vertical scale.

Let us discuss the relations between the discrete structure in the velocity space and the discrete space time background. The discrete space time background has been considered occasionally in the past.[39] Above all, the mathematical papers on the discrete Lorentz transformation written by Schild[40] and Coxeter[41] are relevant to our present ideas. One of the essential problems in the introduction of a discrete space-time background is the compatibility with the invariance under the Lorentz group. Schild found that there is a simple model of discrete space-time (4-dimensional Minkowskian hypercubic lattice) which, although is not invariant under all Lorentz transformations, is invariant under a sufficient number (∞^6set) of Lorentz transformations, and which leads to the discrete velocities. There have been other attempts based on the rational numbers[49] and the Galois Fields,[50] we do not consider those cases in the present paper. Das[42] developed a quantum field theory on the hyper-cubic space-time lattice, however this scheme may not be integral Lorentz covariant.

Those Lorentz transformation is of course not trivial ones, and the above feature is due to the indefiniteness of the Minkowskian metric and is distinct from the usual Euclidean rotations. According to this mathematical scheme, all events in the Minkowskian space-time are represented by four coordinates $x^\mu(t, x, y, z)$ which are real integers in the unit of $c=1$. Those events form hypercubic lattice in space-time. Other regular point lattices in space-time may be considered, however, the essential ingredient which is pertinent to our physical ideas, the existence of discrete velocities, may not be altered. The transformations which leave the hypercubic lattice as a whole invariant are called integral Lorentz transformations. The existence of a fundamental length is assumed, but its magnitude cannot be determined by a kinematical consideration alone. A physical theory based on such a discrete space-time background could be free of divergence, and the divergence difficulties have been an essential disease in the relativistic quantum field theory based on the concept of space-time continuum.

Integral Lorentz transformations are regarded as mapping of the points of the hypercubic lattice into other lattice points. However, an integral Lorentz transformations can also be regarded as a transformation to a new coordinate system. Then the integral Lorentz transformation is given by

$$\begin{aligned} &x^{\mu\prime}=L^{\mu}_{\nu}x^{\nu},\ g_{\mu\nu}L^{\mu}_{\lambda}L^{\nu}_{\sigma}=g_{\lambda\sigma},\ L^{0}_{0}>0,\\ &g_{00}=-g_{11}=-g_{22}=-g_{33}=1,\ g_{\mu\nu}=0\ (\mu\neq\nu), \end{aligned} \tag{24}$$

where L^{μ}_{ν} is the Lorentz matrix whose elements are integers. All integral Lorentz transformations are determined by the solution of 10 quadratic diophantine equations in 16 unknown integers. Schild approached to this problem by using the correspondence between a proper Lorentz transformation and a spinor transformation. He associated 4-vector x^{μ} with a Hermitian spin tensor;

$$\begin{aligned} &a^{\dot{1}1}=t+z,\ a^{\dot{1}2}=x-iy,\\ &a^{\dot{2}1}=x+iy,\ a^{\dot{2}2}=t-z. \end{aligned} \tag{25}$$

Therefore his determination of integral Lorentz transformation is essentially based on the very nature of Gaussian integers. Besides the cases of solid crystals, there may be rather few cases in which the nature of integers plays a fundamental role in physics. The generating relations of the integral Lorentz transformation for the three dimensional case are

$$A^2=B^2=D^2=(AD)^2=(BD)^2=(AB)^4=1, \tag{26}$$

where A, B and D are the typical integral Lorentz transformations which are

$$A=\begin{Vmatrix} 3 & -2 & -2 & 0\\ 2 & -1 & -2 & 0\\ 2 & -2 & -1 & 0\\ 0 & 0 & 0 & 1 \end{Vmatrix},\qquad B=\begin{Vmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 \end{Vmatrix},$$

$$D=\begin{Vmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 \end{Vmatrix}. \tag{27}$$

A, B and D also correspond to the transformation of coordinates

$$x^{\mu} \to x^{\mu} + 2(x^0 - x^1 - x^2),\ x^2 \to -x^2 \text{ and } x^1 \leftrightarrow x^2, \text{ respectively.}^{42)}$$

Schild assumed that the movements of a particle corresponds to the temporally ordered sequence of lattice points. The relations between the two lattice points are induced in two ways; a) two points are connected by the null relations, b) two points are connected by the time-like lines which are obtained by the integral Lorentz transform of the time axis that is $x^{\mu}=(1, 0, 0, 0)$. An instantaneous speed is 1 (light velocity) for the case a) and the average velocity is low, due to the discrete change of the direction of velocity. For the case b), the instantaneous velocities are zero with respect to the instantaneous integral Lorentz frames and movements are determined by the diophantine equation of

$$t^2 - x^2 - y^2 - z^2 = +1. \tag{28}$$

Then, the velocity is associated with a particle whose world line coincides with the transform of the t-axis $(x=y=z=0)$ under integral transformation of the vector (1, 0, 0, 0). Allowable velocities are given by the relation

$$v = t^{-1}(t^2 - 1) \text{ and } \gamma = \frac{1}{\sqrt{1-v^2}} = t. \tag{29}$$

Therefore, only following Lorentz factors are realized in this space-time; that is $\gamma=1$, 2, 3, 5 and so on. The case $\gamma=4$ is not allowed in the hyper-cubic lattice and for the three dimensional case we have $\gamma=3$. The corresponding y values are 1.317 and 1.763 for $\gamma=2$ and 3 respectively. Minimum non zero velocity is $\sqrt{3}c/2=0.866c$*) $(\gamma=2)$. In such a continuum theory as was proposed by Landau based on the compressible relativistic hydrodynamics with the equation of state of the photon gas, a particular significance may be attributed to the sound velocity C_s which is $c/\sqrt{3}$.

On the other hand, the discrete space-time which is represented by the lattice structure simulates the closest packing of equally sized spheres (or ovaloids) which are extended both in space and time directions. In the past, such a closest packing idea has been discussed for the case of atoms and atomic nuclei, and sometime, a highly compressed state of matter, which is expected to occur in the very dense stellar objects whose constituents are mostly neutrons (or hyperons such as Ω^-), is discussed by this idea. Our conjecture is that, when energy is concentrated in the very small space-time region the space-time may be deeply connected by the existence of the compressed state of matter itself. Therefore, if such a state of matter consists of a certain kind of fundamental constituents which have a extended structure, the physics of the world

*) c is the velocity of light.

under very high compression may be well approximated by a lattice structure. The problem of the multiple particle production seems to be the very case where from the very small space-time regions many extended particles are exploded and energy density is extremely high during a certain time interval. Then, a pre-quantized space-time created by the assembly of constituents has a fundamental role in the movement of every constituents.

However, a question remains with respect to the identification of the coordinate system to a particular part of the collision systems. More precisely, the particular part of the colliding systems whose proper system coincides with space-time lattice may be either a fragmented projectile (target) or the centre of velocity system (pionization part) which may not be connected simply to the over all centre-of-mass system of the proton proton collisions. However, as a first guiding principle, we may take following cases, $y_{\mathrm{cms}}=0$, $y_{\mathrm{target(projectile)}}=0$ and $y_{\mathrm{recoil\ proton}}=0$. The discrete activities in the y-space of some exclusive events which are usually considered to be fluctuations, if one is ignorant on the above possibilities, should not be smoothed out carelessly by this reasons. We should be very careful for watching any periodicity in the y space.

The Lorentz group bases for constructing the amplitude of S matrix elements should be reconsidered in the light of the underlying discrete group structures too.

Conclusions

A) We have mentioned expressiveness of the rapidity space from various viewpoints throughout the present paper.

First of all, it is the very space in which the velocity hodograph can be constructed, and this feature is very important for the relativistic kinematics. The aspect has been recognized for the long time by the cosmic ray jet analysts as well as by purely geometrical studies of the relativity theory.

Next, we point out that in this space there is some possibilities to consider the time evolution of the multi-particle production processes.

Thirdly, as we have mentioned in this paper as well as in several occasions in the past, in the rapidity space from the configuration of particles we may observe a certain kind of the discreteness which can be translated back to the discreteness of the space time background or the space time extensions of the basic constituents (or excited consitituents) which are elusive under the excited hadronic substratum.

B) A new kind of scaling law for the inclusive particle spectra at a wide incident energy regions is proposed. Associated with these laws, a normalized rapidity variable for each secondary particle along the collision axis is introduced. The reduced rapidity variables (normalized by the half of the rapidity interval between a projectile and a target proton) are shown to be compatible with the scaling of the multiplicity distributions which was proved by Koba, Nielsen and Olesen and was confirmed by the recent measurements of charged particle multiplicities at several incident momenta of proton proton collisions in the Hydrogen Bubble Chamber experiments. Assuming the existence of the correlation length (or a mesh size in the longitudinal rapidity axis) and basing on the general probability functions for the nearest neighbour spacing distribution of longitudinal rapidity variables, we have found that the well-known Wigner Surmise (linear law for the repulsion of levels), which explains satisfactorily the complex energy spectra in Atomic and Nuclear Physics, can fit the empirical multiplicity distributions and the energy dependence of topological cross sections for proton proton collisions at three incident momenta of NAL experiments.

On the other hand, a possibility in which the multiplicity growth is stronger than $\ln s$ and inelastic cross sections increase is considered for the highest energy data of the ISR experiments and any future higher energy data than the presently available ones. This possibility may be expectable, if coherence sets in for the collisions of two beams (a projectile proton and a target proton) each of which consists of a dense sea of constituents.

We have also suggested that an alternative interpretation of the increase of inelastic cross sections exist based on the idea in our previous papers.[2),3)] There, the basic ingredients were the existence of a lattice structure in the longitudinal rapidity axis and a limitted number of fundamental multiplets which occupy each lattice sites. As to the latter assumptions, if the movement of the multiplets are not approximated by 1-dimensional one for a certain kind of violent collisions, the occupation number may apparently increase along the longitudinal axis and the density of multiplets may increase on the axis.

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