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# Some Reflections on the philosophy of Mathematics Education: A Denunciation of the Time and Content Arguments

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**ABSTRACT**

Of all the arguments directed against presenting mathematical topics within a cultural and historical context, the most serious ones are those that we will refer to as the time and the content arguments.

In this paper, after briefly describing these stances, we will endeavor to evaluate and refute their rationales using a social constructivist line of reasoning.

To this end, we will first discuss the basic tenets of radical and social constructivism and then, through some simple concrete examples, show how social constructivism organically heralds the pedagogical methodology that imparts mathematical concepts in conjunction with their historical and cultural contexts.

**Keywords:** mathematics education, philosophy of mathematics education, time argument, content argument

## INTRODUCTION

The intensification of the efforts to isolate mathematics from its cultural and historical contexts, that is, its core essences, has rendered, once again, the justification for such a conflation a much needed obligation, indeed a prodigiously exigent one. Many contentious objections, though mostly improvident, have been presented by those who dispute the importance of such a comprehensive approach to the teaching of mathematics. The most common ones of these remonstrations, and the ones that are least likely to provoke apoplexy, are what I will refer to as the ***time*** argument and the ***content*** argument.

The proponents of the time argument claim that there is not enough time to present mathematical topics within a historical and cultural context. Evidently, this is a corollary of a philosophical point of view that considers the historical and cultural aspects of mathematics as being disconnected from and inferior to its "actual" content matter, and consequently, undeserving of any class time in a setting functioning under a time constraint. Consequently, implicit in this argument is the worthlessness of historic and cultural aspects of mathematics and the sacrosanctity of achieving mechanical proficiency.

The content argument mainly adheres to the principle that history of mathematics is not mathematics and hence should not be a part of the traditional mathematical classroom content. This, of course, promotes the point of view that mathematics is merely a static collection of facts that are vastly specialized and firmly compartmentalized.

Recognizably, at the crux of both of these arguments that try to put an indelible stain on an all-encompassing methodology of teaching mathematics, lays one's definition of what constitutes mathematics, that is, one's perception of the nature of mathematics, ergo, one's philosophy of mathematics: is mathematics a cultural product or is it culture-free? Is it to be considered a stagnant compendium of facts or as a human activity replete with cultural, historic, and humanistic aspects, that is as "a dynamic, continually expanding field of human creation and invention, a cultural product" (Ernest 1988, p. 2)? Is it a collection of irrefutable statements of logical perfection and utter probity or is it, as any human created structure, to be envisioned as uncertain, falsifiable, and open to refutation and revision? Are concepts of mathematics discovered or invented? Is mathematics to be presented as a finished, well-polished product or as a process? Is it a means of inquiry or a static field of knowledge?

All mathematicians consciously or inadvertently, choose which side they are on with respect to these issues, and use this choice to compile and constitute their course content and the manner in which to present it to the learners. For example, if one views mathematics as an accumulation of irrefutable facts, logical rules, and expedient skills to be used in the service of sciences, then one chooses to become the master dispenser of static verities. In this case, it is not so important to show why and how those rules and facts have ascended to their present eminent positions; all that matters is that the students develop the mastery of mechanical skills needed at that level. The master presents the facts and the compliant student learns them without feeling the need to question their validity: if the book says so, and if the master says so, it must be so.

As another example, if one views mathematics as an anthology of knowledge which is discovered (***and not invented***) through methodical and logical investigation, that is, if mathematical concepts are extant in a Platonic netherworld of shadows independent of human inquiry and activity, then the teacher becomes the envoy who picks up and carries these facts to the student's world. The teacher's role is then to be a paraphraser, tasked with the presentation of these rather mystical mathematical findings as a unified system of knowledge.

Thus, clearly, the answer to any question involving the organization and dispensation of material in a mathematics class, and hence to both the time and the content arguments, rests on one's understanding of the nature of mathematics, that is one's philosophy of mathematics, and, consequently, should involve at least a brief expedition through this discipline. Obviously, this is not at all a novel or inventive assertion. Indeed, many authors have written about the role of personal philosophies in the teaching of mathematics: Thom (1973), Ball (1988), and Ernest (1991), to name just a few.

Since philosophy of mathematics is closely connected with one's perception of knowledge, and in particular, scientific knowledge, a discussion of philosophy of mathematics, in particular on issues involving pedagogy, should start out by clearly defining the concept of scientific knowledge. Accordingly, in the next section we will give, in a rather epigrammatic manner, a characterization of scientific knowledge. Following this, in Section 3, we will apply this delineation to mathematics, and in the fourth and the last section we will present our conclusion.

## CONSTRUCTIVISM AND RADICAL CONSTRUCTIVISM

The classical depiction of science was a means of arriving at the Truth or Reality. This classical approach is now almost completely abandoned in favor of more modern interpretation of the concepts of science as, in essence, being mental constructs proposed to explain our sensory experiences, the main tenet of constructivist epistemology. Yet, unfortunately, as incongruous as the classical view might be with the prevailing exposition of scientific facts, it still, albeit implicitly, enthralls unsuspecting educators with its oppressive hegemony.

The claim that scientific knowledge is constructed by human beings, that is, the fact that meaning and knowledge are always human constructs and not discovered from the world, is not a denunciation of the existence of an external ontological Reality independent of human thought; it is merely the assertion of its inaccessibility. Indeed, a basic postulation of constructivism is that ontological Reality is utterly incoherent as a concept, since there is no way to verify that one has attained a definitive knowledge of it. For, one must already know what this Reality consists of in order to confirm that one has finally arrived at it - a contradiction.

Another important precept of constructivist theory is that there is no single valid methodology in science, but rather a miscellany of effective methods. For more information see Schofield (2008), Crotty (1998), and Vygotsky (1978).

Although the term ***constructivist epistemology*** was first used by Jean Piaget in his famous 1967 article *Logique et Connaissance Scientifique* that appeared in the *Encyclopédie de la Pléiade*, (Piaget 1967), one can trace constructivist ideas to early Greek philosophers such as Heraclitus (c.535 BCE - c. 475 BCE), Protagoras (490 BCE - 420 BCE), and Socrates (c. 469 BCE - 399 BCE). Indeed, Heraclitus' adage *panta rhei* (everything flows), Protagoras' claim that *man is the measure of all things*, and the Socratic maxim "*ἓν οἶδα ὅτι οὐδὲν οἶδα*" (I only know that I know nothing), can clearly be interpreted as harbingers of the constructivist paradigm. This perspective is even more discernible in the works of Pyrrhonian skeptics, who rejected the prospect of attaining truth either by sensory means or by reason, who, in fact, even considered the claim that nothing could be known to be dogmatic. See Bett (2000) or Svavarsson (2010) for more information.

Remaining dormant for a few centuries, constructivist epistemology was revitalized by the Italian philosopher, historian, and rhetorician, Giambattista Vico (1668-1744). Vico, a critic of rationalism and an apologist of classical antiquity, was opposed to all kinds of reductionism, and hence to Cartesian synthesis. In his masterpiece, ***La scienza nuova***, he posited the principle *verum esse ipsum factum* (the truth itself is made), a proposition that can certainly be interpreted as an early instance of constructivist epistemology (Bizzell and Herzberg 2000). Indeed, by the same token, the British idealist philosopher George Berkeley (1685-1753), the Bishop of Cloyne, whose claim *esse est percipi* (to be is to be perceived) challenged metaphysics, can also be construed as one of the forefathers of constructivist epistemology.

The constructivist epistemology was amended in the 1970s by the German-American philosopher Ernst von Glasersfeld (1917-2010) by his ***radical constructivist*** epistemological model. The term *radical* was used to emphasize the fact that from an epistemological perspective, any type of constructivism had to be radical in order not to revert back into some form of realism.

Von Glasersfeld's epistemology was influenced by the works of Vico and Piaget. Indeed, although he was deemed to be "an even more radical constructivist than Piaget" by the likes of Hermine Sinclair, Piaget's successor in Geneva (Sinclair 1987, 29), von Glasersfeld himself referred to Piaget as "the great pioneer of the constructivist theory of knowing" (von Glasersfeld 1990a) and as "the most prolific constructivist in our century" (von Glasersfeld 1996).

At the crux of the radical constructivist theory are the arguments that

i. Knowledge is not passively received but actively built up by the cognizing subject
ii. The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (Glasersfeld 1989, p. 162).

Consequently, the basic tenet of radical constructivism is that *any* kind of knowledge is constructed rather than perceived through senses. As von Glasersfeld put it

> ... knowledge is the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication (von Glasersfeld 1990b, p. 37).

Knowledge is, thus, a self-organized cognitive process of the human brain, the objective of which is not the attainment of a true image of the real world but the formation of a viable organization of the world as it is experienced.

For further information, see for example, von Glasersfeld (1983, 1989, 1990a, 1990b, 1996).

## APPLICATION TO PHILOSOPHY OF MATHEMATICS: SOCIAL CONSTRUCTIVISM

As we mentioned in Section 1, it is now widely accepted that all theories of teaching and learning of mathematics rest on an epistemology, whether openly stated or not:

> In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (Thom 1973, p. 204).

And yet, efforts to explicitly institute epistemological concepts within the realm of pedagogy have usually invoked some antagonistic resistance. In fact,

> To introduce epistemological considerations into a discussion of education has always been dynamite. Socrates did it, and he was promptly given hemlock. Giambattista Vico did it in 18th century, and the establishment could not bury him fast enough (von Glasersfeld 1983, p. 41).

The rather contentious nature of epistemological issues in pedagogy, and in particular in the pedagogy of mathematics, has led to many conflicts and discords within the field. I believe the existence of these divergences which follows from the fact that educators, in general, have different perspectives, beliefs, and epistemologies, is very healthy for the field of pedagogy of mathematics and is a major contributor to its overall development. It is to our credit that, as mathematicians, we question the fundamental philosophical tenets of our field constantly unlike in many others in fields such as engineering or computer science. Let us now proceed to elaborate on one of these perspectives, namely the radical constructivist perspective as it applies to mathematics.

The application of radical constructivism to mathematics is known as ***social constructivism,*** which maintains that human development is socially situated and knowledge is constructed through interaction with others. Following Ernest (1998), with the added assumptions of the existence of social and physical reality, we extend the principles of radical constructivism to elaborate the epistemological basis of social constructivist philosophy of mathematics:

(i) The personal theories which result from the organization of the experiential world must 'fit' the constraints imposed by physical and social reality;
(ii) They achieve this by a cycle of theory-prediction-test-failure-accommodation-new theory;
(iii) This gives rise to socially agreed theories of the world and social patterns and rules of language use;
(iv) Mathematics is the theory of form and structure that arises within language.

Thus, according to the social constructivist perspective, mathematics is a changing and evolving human product. Even the criteria for the ratification of proofs vary and change over time: mathematical proofs follow different standards in different periods of history (Ernest 1998). For more information see also Kilpatrick (1987).

## CONCLUSION

It should, by now, be categorically apparent that the choice to present mathematical concepts within or without their historical and cultural contexts is actually a subtle philosophical argument concerning the structure and constitution of mathematics. Let us contend with this predicament using the social constructivist approach.

Social constructivists advocate that the incorporation of its historical and cultural constituents is essential to the teaching of mathematics, and that without the inclusion of these aspects, the student could acquire, at best, a pedestrian awareness of where a theory comes from, or how it is advanced, or why it is developed. There would be no appreciation or indeed no consciousness of mathematics being driven by both concrete and abstract necessities and constraints. Let us now try to justify this assertion.

First of all, the pedagogical methodology based on presenting mathematical concepts within their cultural and historical milieus is a very good fit for the much heralded problem-solving approach to mathematics as opposed to a one based on perfecting and refining mechanical and computational facility. Using the historical approach to a mathematical topic, open-ended problems can be presented to the students with little more than a rudimentary overview. The role of the teacher would be one of a facilitator tasked with the assertive, proficient, and knowledgeable exposition of problems. The learners are motivated to question, challenge, and critically analyze information rather than blindly accept what is being taught, and are expected to construct their own knowledge through vigorous and active investigation, possibly through group collaboration, and learn mathematics through problem solving, the true measure of proficiency in mathematics.

Secondly, by taking a historical approach to the subject, students get to appreciate that a particular mathematical idea was indeed needed at a specific point in time to explain a certain phenomenon. They also get to experience the "theory-prediction-test-failure-accommodation-new theory" cycle, through carefully assigned projects.

Let us illustrate these points through a very simple example. Suppose we want to talk about the sum of the interior angles of a triangle. We could simply claim that this is equal to $180^0$ and prove this claim by some simple geometric and algebraic manipulations. Now, how would we do that using a historical and cultural perspective?

We would probably start with why the idea of a triangle had to be devised. This would, of course take us to ancient Egypt where some of the earliest known tangible needs for geometry transpired - for building the pyramids and measuring areas of parcels of land for taxation purposes. We would thus argue that it was essential to invent and amend such concepts as ratios, similar triangles, and areas of quadrilaterals, mentioning along the way, that contemporaneously, other civilizations, namely, the Babylonian, Hindu and Chinese, all contributed to the development of geometry as was necessitated by their needs. Of course, we would emphasize that the development of these concepts was entirely empirical and solely pertinent to the particular impending instance.

Now that we have established the *raison d'être* of what we want to teach and its concrete realization, we would try to present it in a more abstract form. This would inevitably lead us to deductive reasoning and hence to ancient Greece, for, deductive reasoning, that is, starting by some known (or assumed) facts, and using laws of logic to arrive at some conclusions, started and later dominated all aspects of thought in Greece. We would hypothesize that the reason for the enactment of this stratagem might be due to the fact that in the quasi-democratic city state system, orators had to convince listeners of the validity of their claims using logical arguments. Or we might posit that the Greeks viewed the universe as a perfect entity and believed that in such an unflawed realm, one should be able to obtain results without the impurity and inaccuracy of empirical techniques. Anyhow, at this stage, it should not at all be astonishing to the learners that the Greek mathematicians, who most certainly borrowed geometric ideas from the abovementioned civilizations, moved them, effectively, into the resplendent domain of theory, culminating, around 300 BCE, in *The Elements* of Euclid of Alexandria, where he axiomatized geometry using five postulates:

i. Given two points, there is a straight line that joins them.
ii. A straight line segment can be prolonged indefinitely.
iii. A circle can be constructed when a point for its center and a distance for its radius are given.
iv. All right angles are equal.
v. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which the angles are less than the two right angles.

We would, then, be in a position to describe an abstract geometric concept called the triangle and prove the theorem regarding the sum of its interior angles.

Now that we achieved an abstract status, further abstractions and generalizations could be introduced. Is the theorem we proved absolutely true in all possible settings? Can there be other geometries where the sum of the angles of a triangle is actually different than $180^0$?

We would then mention that mathematicians have long noticed that the fifth postulate was clearly more complicated than the other four, and, over the years, they tried to derive it from the first four, to no avail. This would take us to the mid-nineteenth century, and to mathematicians like Carl Friedrich Gauss (1777-1855), Nikolai Lobachevsky (1792-1856), János Bolyai (1802-1860), Bernhard Riemann (1826-1866) and Felix Klein (1849-1925), to name just a few, who began to explore alternative geometries where this fifth postulate was not true, in other words, who tried to contrive constant curvature geometries based on the first four of Euclid's postulates, but using alternate versions of the parallel postulate.

Euclidean geometry (also called the parabolic geometry) assumes that given a line and point not on the line there is a unique line passing through that specific point parallel to the given line. However, one could envision a geometry where there are several lines through a given point that are parallel to the given line. This type of geometry is called hyperbolic geometry (or Lobachevsky-Bolyai-Gauss geometry), a mathematical description of a space of negative curvature. In this setting the sum of the interior angles of a triangle is less than $180^0$. Conversely, a geometry could exist where it would be impossible to draw a line through the given point that is parallel to the given line; this type of geometry is called elliptical geometry (or Riemannian geometry), a mathematical description of a space of positive curvature. In this geometry the sum of the interior angles of a triangle is greater than $180^0$.

Because there are only three options (given a line and a specific point not on the line, there can be either multiple parallel lines through a specific point, one parallel line through a specific point, or no parallel lines through a specific point), there are only three basic types of geometry possible. Indeed, in 1868, the Italian mathematician Eugenio Beltrami (1835-1900) proved that non-Euclidean geometries were as logically consistent as Euclidean geometry.

Now from theory we can go back to application. Is curved space just an abstract construct? Patently, it remained so until Albert Einstein (1879-1955) developed his general theory of relativity in 1915, and posited that gravity was the result of the curvature of space and time. Although the differences between the relativistic estimates and the classical ones were quite insignificant for most observable occurrences, the relativistic model, impressively, accounted for some inconsistencies that could not be explained by the classical model, for instance, the small deviation in Mercury's orbit; and thus allowed the idea of curved space, or non-Euclidean geometry, to find a very concrete germaneness.

Here we should emphasize that this concrete-abstract-concrete cycle persists *ad infinitum*: the relativistic approach to cosmology will require new abstract constructs which in turn will find new applications, and so forth.

Of course, each one of this steps could be assigned as a group project, for instance:

Group 1: Egyptian and contemporaneous geometries

Group 2: Geometry in Greece, deductive reasoning

Group 3: Euclidean axioms

Group 4: Geometries where the sum of the interior angles of a triangle is $\neq 180^0$

Group 5: Beltrami and Equivalence of the geometries

Group 6: Curved space and general relativity

As can be seen from the preceding example, historical approach not only gives the educators a chance to elaborate on the concrete-abstract-concrete cycle, but also allows them to provide the learners with crucial sources of inspiration, insight, and motivation.

Thirdly, again as can be seen from the preceding example, presenting mathematics within a historical and cultural context corroborates a notion of this discipline as a dynamic, continuously transmuting field of study that is open to refutation and revision.

Fourthly, the historical approach also shows the internationalist character of mathematics. As teachers, if we are, as we should be, concerned about cultural chauvinism and parochial nationalism, and try to instill an unbiased perspective of mathematics, the historical approach is the needed panacea to convince learners that transnational collaboration, universal solidarity, and fellowship of scholars have always been the epitomes of mathematical development.

Let us justify this assertion by an example; let us, for instance, take probability. Of course, the Italian scholar Gerolamo Cardano's (1501-1576) 1525 book ***Liber de ludo aleae*** (Book on Games of Chance), which was published posthumously in 1663, can be considered a harbinger of mathematical theory of probability. However, the date historians accept as the beginning of modern probability theory is 1654, when two of the most prolific and resourceful mathematicians of the time, the two Frenchmen, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665), began a correspondence addressing the ***problem of points***: suppose a game of chance is to be played several times by two players each of whom have equal chances of winning and each of whom contribute equally to the pot. Each time a player wins the game, he gets one point. The first player to win a certain number of points collects the entire prize. Now suppose that the game is interrupted by external circumstances before either player has achieved the required number of points. How does one then divide the pot fairly?

Soon after, to this mixture of nationalities we add one more: the Dutch mathematician Christiaan Huygens (1629-1695) with his 1657 book titled ***De ratiociniis in aleae ludo*** (Calculations in Games of Chance) further advances the topic.

Now the list goes on with Jacob Bernoulli (1654-1705), a well-known member of the celebrated Swiss family of Basel that dominated mathematics for nearly two centuries, and his famous oeuvre ***Ars Conjectandi*** (The Art of Conjecturing), a seminal work on probability, written between 1684 and 1689, and published in 1713, eight years after his death, by his nephew Nikolaus Bernoulli (1687-1759). Here we see many modern results on permutations, combinations, expected values, and a proof of a special case of the law of large numbers.

Then comes to stage the French-born English mathematician Abraham de Moivre (1667-1754) and his famous book ***The Doctrine of Chances*** of 1718, where we witness the first proof of a special case of the famous Central Limit Theorem by which he was able to approximate the binomial distribution with the normal distribution. Thenceforward, the French mathematician Pierre-Simon Laplace (1749-1827) takes the torch and publishes his ***Théorie analytique des probabilités*** in 1812. Of course, the litany of contributors goes on: the French mathematicians Siméon Denis Poisson (1781-1840) and Émile Borel (1871-1956), the Russian mathematicians Pafnuty Chebyshev (1821-1894), Andrey Markov (1856-1922), Aleksandr Khinchin (1894-1959), and Andrey Kolmogorov (1903-1987), the Italian mathematician Francesco Paolo Cantelli (1875-1966), and many others from many different countries and cultures. Thus, even by this one simple example, the "historically educated" student of mathematics might come to an awareness of the great cultural and national diversity that has contributed to the development of the subject.

If we cannot afford to devote some time for historical and cultural dimensions in mathematics instruction, then we are unabashedly renouncing some extremely vital sources that foster creativeness, inventiveness, intuition, and vision. If mathematics is to be presented detached of its historic and cultural backgrounds, then mathematics is an entity disconnected from human activity; it is an unnatural construct born in perfection and bound to remain in the intellectual domain of a few geniuses.

Worse yet, devoid of any context, mathematics becomes nothing more than a series of computations. However, computational proficiency is not necessarily a measure of mathematical comprehension. It is known that students who display adequate mastery of routine tasks might actually exhibit quite mediocre perceptions of what mathematics truly is (Schoenfeld 1985).

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