Some Remarks on the Non-local Field Theory

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The non-local field theory, proposed by H. Yukawa[1], has two remarkable possibilities which the ordinary theory hasn't. The first of them is a possibility of free from divergence difficulties, and the second is that of a new interpretation of spins of elementary particles. The latter is a formal result of the fundamental equations of non-local fields. As Fierz[2] pointed out, these equations describe not only the behaviours of particle of a given spin, but also that of a group of particles of various spins. But the former has not been studied decisively. Starting from Yukawa's fundamental equations, however, the divergence difficulties would remain obviously, unless particles of various spins satisfy some cancellation relations fortunately, Moreover, the infinite freedoms of spin may raise the difficulties[3].

The methods of avoiding these duplicate difficulties have not been known, but the most natural method[4] seems the reduction of infinite freedoms of spin into irreducible parts in the fundamental equations at the outset. It seems that this reduction can be made for real fields in the present form, but not for virtual fields, and this fact is the very origin of the difficulties. Starting from this assumption, Yennie[5] and Rayski[6] showed the possibilities of avoiding the divergence difficulties. However, they have some mistakes and considered partly, and not all of the self-energies of three particles (considering difference of charge) interacting with each other converge. Moreover, their methods are too artificial. In this letter, we discuss whether the difficulties of ordinary field theories can be removed, assuming that we can restrict spins (l-freedoms).

We define a associated local field quantity a(X) from a non-local $A(x, r)=(x+\frac{r}{2}|A|x-\frac{r}{2})$ by

$$a(x)=\int dr \quad (x+\frac{r}{2}|A|x-\frac{r}{2}) \qquad (1)$$

and consider a simplest possible interaction Lagrangian between a complex spinless field A and a real spinless B, as $L=gA^*BA$. g is a interaction constant. Then
dropped this term.

$$L(x)=\int dr\ (x+\frac{r}{2}\mid L\mid x-\frac{r}{2})$$

$$=O\ a^*(x)b(x)a(x)=OL_{local}\ (x). \quad (2)$$

O is a operator which operates on a^*, a and b. This shows that non-local field introduces a (semi-convergence) factor in local field interaction

From equation (2), we can easily take calculation following to the covariant formalism. In our case, the order of divergence does not differ from ordinary theories except a factor, because vacuum fields contribute in ordinary way, which differs from Yukawa-Yennie's considerations. A analytic form of a operator O depends sensibly on a type of non-local interaction Lagrangians, and then it influences on the divergency of self-energies. The results are listed in Table I. In this Table, *div* means that a divergence of a same order to ordinary theories exist, because in this case the factor O plays no role. And $A^*BA=(x'\mid A^*\mid x''')\ (x'''\mid B\mid x'''')\ (x''''\mid A\mid x'')$, and so on.

Table I. Self-Energy

Type of Lagrangian	A^+	A^-	B
A^*BA	*div*	*div*	*conv*
A^*AB	*conv*	*div*	*div*
BA^*A	*conv*	*conv*	*div*

From these results we know that a factor O plays a convergence factor only for one particle, and can not expect that one gets converging results for each particles simultaneously. To remove these defects we are obliged to give up a matrix character of non-local quantity, and then we can construct a Lagrangian which gives converging results for each particle formally as follows;

$$L(x)=g\int A^*(x+\frac{r_2}{2}+\frac{r_3}{2},\ r_1)\ B(x+\frac{r_1}{2}-\frac{r_3}{2},$$

$$r_2)\ A(x-\frac{r_3}{2}-\frac{r_2}{2},\ r_3)\ dr_1\ dr_2\ dr_3. \quad (3)$$

This is very trivial. Anyhow, we can not expect a powerful result for divergence problems, even if we restrict spins.

1) H. Yukawa, Phys. Rev. **77** (1950), 219; Part II., to appear shortly. We express our thanks to Prof H. Yukawa for having sent us kindly a copy of his work before publication.
2) M. Fierz, Phys. Rev. **78** (1950), 184.
3) O. Hara and H. Shimazu, Prog. Theor. Phys. **5**, (1950).
4) C. Bloch, Det Kgl. Dansk. Vid. Selsk. XXVI, Nr. I (1950); Prog. Theor. Phys. **5** (1950), 606.
5) D. Yennie, Phys. Rev. in press.
6) J. Rayski, Acta Phys. Polonica **10** (1950), 103.