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SOME REMARKS ON THE TREATMENT OF FULLY-SUPERSONIC OBLIQUE FLAMES AS GAS-DYNAMICAL DISCONTINUITIES.

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Some remarks on the treatment of fully-supersonic oblique flames as gas-dynamical discontinuities.

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SUMMARY

The conservation equations are solved for the changes occurring across a steady plane exothermic discontinuity. Using the deflagration branch of these solutions, conditions for the flow to be supersonic on both the "burnt" and "unburnt" sides of the flame are established. Simple flows capable of sustaining such fully-supersonic flames can therefore be constructed quite readily, and two examples are given. The second of these corresponds to an experimentally observed case of shock-induced combustion. The analysis is simplified as much as possible and the work is purely heuristic.

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1. Introduction.

The addition of heat to a flowing gas is usually effected by the release of chemical energy which results from the burning of an appropriate fuel. The details of such a process are numerous and complicated and are certainly rendered no less so by the essential interactions between the flow pattern and the (chemical) heat-liberating mechanisms. However, under favourable conditions the portions of the flow field within which the reactions take place are often very small in comparison with the overall extent of the field and it becomes possible to treat them as surfaces of discontinuity. There is nothing new in such a suggestion, but it is interesting to note that Emmons (1958) felt it necessary to remark that "The diagram of possible steady flows in a combustion wave .... has not been very extensively used to date." and that ".... the more extensive use of the steady flow diagram seems desirable".

We are primarily interested in the problem of heat addition to (or burning in) a supersonic stream and, even though a considerable body of information has lately been gathered together by a number of workers, amongst whom we may mention Ferri and his associates (see, e.g. Ferri, 1964), on the hydrogen-air reaction under these conditions, it still seems profitable to take heed of Emmons' remarks and to use the discontinuous flame-sheet model to construct possible supersonic flows with embedded flames. To start with we shall reiterate some of Emmons' (1958) analysis, making use of a few mathematical rearrangements which help one to comprehend the changes taking place across a discontinuous flame (and, incidentally, bringing to light an error in the flame diagram in his article.) With the aid of this analysis we then go on to construct some simple flow patterns containing fully-supersonic flames (which are defined in Section 3.)

In order to simplify the present purely heuristic analysis as much as possible we shall assume that both burnt and unburnt gases are ideal in the usual sense and have the same specific heats and molecular weights.

2. The flame front as a discontinuity.

Fig. 1 shows a section of a plane flame front, treating the flame as a discontinuity, and also illustrates the notation to be used below. θ_W will be called the flame angle and δ the flow deflection angle. Conditions ahead of the flame are denoted by a subscript $_1$ and those behind the flame by a subscript $_2$. The equations of conservation of mass, momentum and energy are, respectively,

$$\rho_1 u_1 = \rho_2 u_2 , \qquad (1)$$

$$v_1 = v_2 , \qquad (2)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 , \qquad (3)$$

$$h_1 + Q + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2 . \qquad (4)$$

p and ρ are pressure and density, and h is the temperature - dependent part of the specific enthalpy. Q is the heat released by unit mass of gas as a result of combustion; it is treated here as a constant. In order to simplify the work which follows we shall assume that the specific heat ratio γ and the gas constant R are the same on both sides of the flame, so that

$$h_n = \frac{\gamma R T_n}{\gamma - 1} \quad ; \quad n = 1, 2, \tag{5}$$

where T is the absolute temperature. The system is completed by the thermal equation of state

$$p_n = \rho_n R T_n \; ; \quad n = 1, 2. \tag{6}$$

Equation 2 simply shows that there is no change of momentum parallel to the flame front. Since, as can be seen from Fig. 1,

$$V_1^2 = u_1^2 + v_1^2 \; ; \; V_2^2 = u_2^2 + v_2^2 , \tag{7}$$

the energy equation 4 reduces to

$$h_1 + Q + \tfrac{1}{2} u_1^2 = h_2 + \tfrac{1}{2} u_2^2 , \tag{8}$$

and equations 1, 3 and 8, together with 5 and 6, are the equations satisfied by a stream flowing normal to the discontinuity, with a speed in the unburnt gas equal to u_1. The latter is frequently referred to as the flame speed.

Assuming that conditions ahead of the flame, together with the value of Q, are known, equations 1, 3, 5, 6 and 8 can be solved to find conditions in the burnt-gas flow. In particular, a relationship of some importance exists between the velocity ratio u_2/u_1 (or equivalently, the density ratio ρ_1/ρ_2) and what we may call the flame Mach number, namely u_1/a_1. a_1 is the speed of sound in the unburnt gas;

$$a_1^2 = \gamma R T_1 \; . \tag{9}$$

Defining the quantities λ and m such that

$$\lambda \equiv u_2/u_1 \; ; \quad m \equiv u_1/a_1 \tag{10}$$

we can show (Emmons, 1958) that

$$\lambda = \frac{1}{(\gamma + 1)} \left\{ \gamma + \frac{1}{m^2} \right\} \pm \left[\left(\frac{m^{-2} - 1}{\gamma + 1} \right)^2 - 2m^{-2} \left(\frac{\gamma - 1}{\gamma + 1} \right) Q a_1^{-2} \right]^{\frac{1}{2}} \quad . \tag{11}$$

If we adopt the point of view that m and Qa_1^{-2} are known for a particular combustible mixture, equation 11 enables us to calculate the density ratio λ across the flame.

The nature of the λ-versus-m curves for a given Qa_1^{-2} can be inferred as follows. First we note that λ is a two-valued function of m. Second, it is clear that λ is only real provided that

$$m^2 \left(\frac{1}{m^2} - 1 \right)^2 \geq 2(\gamma^2 - 1) \, Q/a_1^2 \; . \tag{12}$$

The value m_c of m which results from the use of the equality symbol in equation 12 gives the location of a branch point λ_c, m_c in the solution curves; i.e.

$$m_c^2\left(\frac{1}{m_c^2}-1\right)^2 = 2(\gamma^2-1)\,Q/a_1^{\,2}\ , \tag{13}$$

$$\lambda_c = \frac{1}{\gamma+1}\left\{\gamma+\frac{1}{m_c^2}\right\} \ . \tag{14}$$

Equation 13 can only be meaningful if $Q \geq 0$, but this condition is quite in accordance with the notion that burning "adds heat" to the flow.

With the relation 13, equation 11 can conveniently be re-written in the form

$$(\gamma+1)(\lambda-1) = \left(\frac{1}{m^2}-1\right) \pm\left[\left(\frac{1}{m^2}-1\right)^2-\left(\frac{m_c}{m}\right)^2\left(\frac{1}{m_c^2}-1\right)^2\right]^{\frac{1}{2}} \ . \tag{15}$$

We can now show that

$$(\gamma+1)\frac{d\lambda}{dm} = \left(\frac{-2}{m^3}\right)\left\{1 \pm \frac{\left[(m^{-2}-1)-\frac{1}{2}m_c^2(m_c^{-2}-1)^2\right]}{\left[(m^{-2}-1)^2-(m_c/m)^2(m_c^{-2}-1)^2\right]^{\frac{1}{2}}}\right\}, \tag{16}$$

from which it follows that

$$\frac{d\lambda}{dm} = \pm\infty \text{ when } m = m_c \neq 1 . \tag{17}$$

When $m_c = 1$ we observe from equation 13 that $Q = 0$ and no burning takes place. Then equation 15 gives

$$(\gamma+1)(\lambda-1) = \begin{cases} 2(m^{-2}-1)\ , \\ 0 \end{cases} \tag{18}$$

and 16 gives

$$(\gamma+1)\frac{d\lambda}{dm} = \begin{cases} -4/m^3\ , \\ 0\ . \end{cases} \tag{19}$$

The first of equations 18 and 19 give the familiar adiabatic shock wave solutions, whilst the second represent the trivial, "no change", solutions of the conservation equations. In this special case ($m_c = 1$) both solution curves pass through the point $\lambda_c = 1$, $m_c = 1$ with finite slopes.

Turning to the remaining cases for which $Q > 0$, let us write equation 13 as

$$m_c^2\left(\frac{1}{m_c^2}-1\right)^2 = q \equiv 2(\gamma^2-1)Q/a_1^{\,2} > 0\ . \tag{20}$$

Then it follows that

$$m_c^2 = (1+\tfrac{1}{2}q)\left\{1 \pm \left[1-\frac{1}{(1+\frac{1}{2}q)^2}\right]^{\frac{1}{2}}\right\} \ . \tag{21}$$

The positive sign gives $m_c > 1$ whilst the negative sign gives $m_c < 1$.

If we plot λ as ordinate versus m as abscissa (as in Fig. 2) the ordinates of curve λ_c lie exactly half way between those of the solution curves in equation 18.

When $m \to \infty$, equation 15 shows that

$$(\gamma + 1)(\lambda - 1) \to \begin{cases} 0 \\ -2 \end{cases} \tag{22}$$

and equation 16 shows that the slopes $d\lambda/dm \to 0$ as follows:

$$(\gamma + 1)\frac{d\lambda}{dm} \to \left.\begin{cases} q/m^3 \\ -(4 + q)/m^3 \end{cases}\right\} \to 0 \quad . \tag{23}$$

When $m \to 0$, equation 15 shows that

$$(\gamma + 1)(\lambda - 1) \to \begin{cases} 2/m^2 \to \infty , \\ q/2 \quad , \end{cases} \tag{24}$$

and equation 16 shows that the slopes $d\lambda/dm$ behave as follows:

$$(\gamma + 1)\frac{d\lambda}{dm} \to \begin{cases} -4/m^3 \to -\infty , \\ 0 \end{cases} \tag{25}$$

It can also readily be shown that $d\lambda/dm \neq 0$ for any m in the interval $0<m<\infty$.

The information that we have elicited so far enables us to sketch the λ, m curves which are presented in Fig. 2. These are not to scale and are not drawn for any particular γ or q values, but it is evident that Fig. 6, 4b in Emmons' (1958) article is incorrect in the rather important regions near $m = 0$ (using our notation: Emmons writes m as M_1.)

Referring to Fig. 2 the curves labelled ABC and DEF are solution curves for a given q, B and E being the branch points referred to earlier. The sections AB and EF are those corresponding to the minus sign in equation 11, and it is with the section AB that we shall be concerned here since, as Emmons demonstrates, it is this section which gives the locus of possible flame solutions for a given q. He points out that EF gives strong detonation wave solutions, whilst BC and DE (representing strong deflagrations and weak detonations, respectively) are generally inadmissable as solutions to physically plausible situations. The line labelled $q = 0$ is the locus of possible adiabatic shock solutions, with only the part for which $\lambda < 1$ being physically admissable.

As the foregoing analysis demonstrates, the λ, m relationship is unaffected by the obliquity of the flame front. However, the flow deflection angle δ is related to θ_W and λ as follows. Fig. 1 shows that

$$\frac{u_1}{v_1} = \tan \theta_W \quad ; \quad \frac{u_2}{v_2} = \tan(\theta_W + \delta) = \frac{u_2}{v_1} \quad , \tag{26}$$

the last result following from equation 2. Consequently

$$\lambda = \frac{\tan(\theta_W + \delta)}{\tan \theta_W} \quad , \tag{27}$$

or

$$\tan \delta = \frac{(\lambda - 1)\tan \theta_W}{1 + \lambda \tan^2 \theta_W} \quad . \tag{28}$$

Thus δ is a single-valued function of θ_W for any given λ. In practice we are more likely to know the unburnt flow conditions, the flame speed and the heat of combustion;

i.e. we shall know M_1, m and q where the Mach number M_1 of the unburnt stream is related to m by

$$m = M_1 \sin\theta_W \quad . \tag{29}$$

For the flame solutions (curve AB in Fig. 2) λ is uniquely related to m and q, so that the unburnt stream conditions etc. will lead to a unique value of δ.

We note that, since there is a maximum value m_C of m for a given q, there will be a corresponding maximum for θ_W (written as θ_{WC}), given by

$$m_C = M_1 \sin\theta_{WC} \quad . \tag{30}$$

(N.B. θ_W is essentially positive and $M_1 > m$ under all circumstances.) Equation 28 also shows that there is an absolute maximum of δ for a given λ, namely

$$\tan\delta_{max} = \frac{(\lambda - 1)}{2\sqrt{\lambda}} \tag{31}$$

arising when θ_W has a value $\theta_{W\ max}$ such that

$$\tan\theta_{W\ max} = \frac{1}{\sqrt{\lambda}} \quad . \tag{32}$$

For a given value of q, λ increases with m up to the maximum value λ_C. Hence the least value of $\theta_{W\ max}$ for a given q is given by

$$(\tan\theta_{W\ max})_{min} = \frac{1}{\sqrt{\lambda_C}} \quad . \tag{33}$$

As m increases to m_C, θ_W increases to θ_{WC} as given by equation 30, or alternatively, by

$$\tan\theta_{WC} = \frac{1}{\sqrt{M_1^2/m_C^2 - 1}} \quad .$$

$$= \left\{ [(\gamma + 1)\lambda_C - \gamma]\ M_1^2 - 1 \right\}^{-\frac{1}{2}} , \tag{34}$$

(using equation 14). Then the maximum permissible value of θ_W, namely θ_{WC}, will always be less than $\theta_{W\ max}$ if the condition

$$\lambda_C < [(\gamma + 1)\lambda_C - \gamma] M_1^2 - 1 \quad ,$$

or

$$M_1^2 > \frac{\lambda_C + 1}{(\gamma + 1)\lambda_C - \gamma} \quad , \tag{35}$$

is satisfied. With $\lambda_C \geq 1$ the greatest value of the right-hand side of equation 35 is 2 (when $\lambda_C = 1$) and the least value is $(\gamma + 1)^{-1}$ (when $\lambda_C = \infty$.) One can see that in a supersonic ($M_1 > 1$) unburnt stream the flow deflection angle δ will, in general, be small and, more importantly, the flame front will lie comparatively close to the on-coming streamlines.

A further point of some importance is the value of the Mach number M_2 in the burnt-gas flow behind the flame, where M_2 is defined as

$$M_2 \equiv V_2/a_2 \quad ; \quad a_2^{\,2} = \gamma R T_2 \,. \tag{36}$$

Using equations 1 and 6 in equation 3 gives

$$u_2\left\{ RT_1 + u_1^{\,2} \right\} = u_1\left\{ RT_2 + u_2^{\,2} \right\}$$

which, with equations 9 and 36, can be re-written as

$$\frac{a_2^{\,2}}{u_2^{\,2}} = \frac{a_1^{\,2}}{\lambda u_1^{\,2}} + \gamma\left(\frac{1}{\lambda} - 1\right) .$$

Using the definitions of M_1 and M_2 we readily show from this result that

$$\frac{1}{M_2^{\,2}} = \left\{ \frac{1}{\gamma\lambda M_1^{\,2}\sin^2\theta_W} - \left(1 - \frac{1}{\lambda}\right)\right\}\gamma \sin^2(\delta + \theta_W) \,. \tag{37}$$

From equation 27 we can show that

$$\sin^2(\delta + \theta_W) = \frac{\lambda^2\tan^2\theta_W}{1 + \lambda^2\tan^2\theta_W} ,$$

$$= \frac{\lambda^2\sin^2\theta_W}{1 + (\lambda^2 - 1)\sin^2\theta_W} ,$$

so that equation 37 can be re-written in the form

$$\frac{1}{M_2^{\,2}} = \left\{ \frac{1}{M_1^2} - \gamma(\lambda - 1)\sin^2\theta_W \right\} \frac{\lambda}{1 + (\lambda^2 - 1)\sin^2\theta_W} \,. \tag{38}$$

Thus M_2 is greater or less than unity according as to whether

$$\sin^2\theta_W \gtrless \left(\frac{\lambda}{M_1^{\,2}} - 1\right)\left\{ (\lambda - 1)(1 + (\gamma + 1)\lambda) \right\}^{-1} \,. \tag{39}$$

In particular, if

$$M_1^{\,2} > \lambda > 1 \,, \tag{40}$$

then the upper inequality in 39 is satisfied for all $\theta_W > 0$ and M_2 is always greater than unity. Indeed it is only possible to find $M_2 < 1$ if $M_1^2 < \lambda$ since $\sin^2\theta_W$ is essentially positive.

An alternative form of equation 37 is

$$\frac{1}{M_2^2} = \left\{ \frac{1}{\gamma\lambda m^2} - \left(1 - \frac{1}{\lambda}\right)\right\}\gamma \sin^2(\delta + \theta_W)$$

(using equation 29), so that when $m = m_C$ and $\lambda = \lambda_C$ M_2 has the value M_{2C} given by

$$\frac{1}{M_{2C}^2} = \left\{ \frac{1}{\lambda_C}\left[(\gamma + 1)\lambda_C - \gamma\right] - \gamma\left(1 - \frac{1}{\lambda_C}\right)\right\}\sin^2(\delta_C + \theta_{WC})$$

$$= \sin^2(\delta_C + \theta_{WC}). \tag{41}$$

Under these conditions M_2 is always greater than unity unless $\theta_{WC} = 0 = \delta$, when $M_2 = 1$. It can be seen that the limiting flame Mach number m_C for a given q is such as to make the burnt gas travel at a locally sonic speed normal to the flame front.

3. Fully supersonic flames.

One interesting fact which emerges from the considerations of the previous Section is that it is, at least in principle, possible to find oblique flames embedded in a wholly supersonic flow (i.e. both M_1 and M_2 are supersonic.) Indeed a sufficient condition is given in equation 40, although it is of course a rather more restrictive condition than is absolutely necessary. We may christen flames for which both M_1 and M_2 are greater than one, "fully supersonic".

The advantage of such fully-supersonic flames from a purely analytical point of view is that the limitations of upstream influence in a supersonic flow can be invoked in order to construct flame-flow patterns in a comparatively simple way. Naturally one must be careful to ensure that any elementary patterns made up in this way do not violate the bounds of physical plausibility, but with this in mind we shall go on here to discuss one configuration which immediately suggests itself. This is the two-dimensional V-flame which is sketched in Fig. 3.

The flame front is assumed to be planar and to be symmetrically disposed about the axis of a parallel uniform cold (or unburnt) flow of Mach number $M_\infty > 1$. The expansion of the stream tubes resulting from heat addition at the flame front will give rise to compression waves propagating out from the front and which, in this idealised situation, will coalesce to form a plane oblique shock front attached (as shown in Fig. 3) to the flame tip. If the quantities m and q are taken as known, together with M_1, then both θ_W and λ, and hence δ, are uniquely determined as we have seen. The condition of a parallel uniform hot flow fixes the deflection angle required across the shock at the value δ, and hence results in a unique connection between the hot and cold (M_∞) streams. (The shock must necessarily be of the 'weak' variety if M_1 is to be greater than unity.)

Some numbers for a typical case are shown in the lower half of Fig. 3. They were obtained as follows. First the values $Q/C_pT_1 = 1$ and $\gamma = 1.4$ were selected, so that $Q/a_1^2 = 2.5$ and (from equation 20) $q = 4.8$. Then equation 21 (negative sign) gave $m_C = 0.388$ and equation 14 gave $\lambda_C = 3.36$. Next the values $M_1 = 2$ and $m = 0.2$ were chosen giving (from equation 15) $\lambda = 2.11$ and (from equation 29) $\theta_W = 5° 44'$. Then, from equations 28 and 38 respectively, it was found that $\delta = 6° 13'$ and $M_2 = 1.44$. The usual oblique shock relations were used to find that $M_\infty = 2.22$, the shock angle θ_S being $31° 48'$. With this information it was possible to show that

$$p_1 = 1.44\, p_\infty \;;\quad \rho_1 = 1.3\, \rho_\infty \;;\quad T_1 = 1.11\, T_\infty \;.$$

The pressure change across the flame was found from equation 3, which can easily be manipulated to show that

$$p_2/p_1 = 1 - \gamma m^2(\lambda - 1) \quad . \qquad (42)$$

This pressure ratio is near to unity (0.98 in the present case) owing to the smallness of m. The values of p_2 etc. are quickly found to be

$$p_2 = 1.42\, p_\infty \;;\quad \rho_2 = 0.62\, \rho_\infty \;;\quad T_2 = 2.3\, T_\infty \;.$$

Another quantity of interest is the actual speed of the hot gas flow, V_2, which can conveniently be found from

$$V_2 = \frac{M_2}{M_\infty}\frac{a_2}{a_\infty}\ V_\infty = \frac{M_2}{M_\infty}\ \sqrt{\frac{T_2}{T_\infty}}\ V_\infty = 0.98\ V_\infty$$

in the present case. The very small velocity change is noteworthy.

Another situation which can be easily constructed by the present methods is depicted in Fig. 4. This basic flow configuration has been used by Rubins et al (1963) to study the kinetics of shock-induced combustion in an initially cold but pre-mixed hydrogen air system. The introduction of a solid surface into the gas flow means that boundary-layer effects will be present, but we propose to ignore these here and treat the whole field as inviscid outside the discontinuities. In doing so we imply that the flame angle is sufficient to carry it well away from the influence of the surface.

The paper by Rubins et al which has been referred to contains a photograph of the flow pattern which occurs in their experiments. Unfortunately this picture is not of the best quality in the (photo-) copy of the paper which is available to the present writer, but Fig. 4 constitutes a reasonable distillation of the main facts of the situation which can be discerned from it. The main-stream Mach number, M_∞, is equal to 3 and δ_W, the wedge angle, is 28°. The experimental gas mixture apparently contains a comparatively small amount of hydrogen and it does not seem unreasonable to treat the gas mixture as having a constant value of γ equal to 1.4, at least for present purposes. The shock wave angle θ_S (Fig. 4) can be measured to within a degree or so from the experimental photograph and is equal to about 57°. This is certainly a significant amount greater than the value of $48\frac{1}{2}°$ which would occur with $M_\infty = 3$, $\delta_W = 28°$ and $\gamma = 1.4$ in the absence of combustion. Assuming that the flame front lies along the upstream edge of the luminous emission zone which can be seen in the experimental picture, a reasonable value for θ_f (Fig. 4) is 50° 12'. Now with $M_\infty = 3$, $\gamma = 1.4$ and $\theta_S = 57°$ we can calculate $\delta_W + \delta = 32° \ 20'$. From the geometry of Fig. 4 it follows that $\delta = 4° \ 20'$ and $\theta_W = 10° \ 52'$, and from equation 15 we find that $\lambda = 1.42$. The value of M_1 is equal to 1.23 (using the oblique shock relations once more) whence equation 29 gives $m = 0.23$. The relevant value of M_2 (from equation 38) is 1.06, so that the flame is just fully-supersonic in the sense defined above, a fact which lends a little weight to the application of our analysis to the experimental situation. Knowing m, λ and γ we can now use equations 15 and 20 to find q, and thence Q/a_1^2. It transpires that $q = 1.82$ so that $Q/a_1^2 = 0.948$.

So far we have seen how, with the aid of a flow photograph and the information that $M_\infty = 3$, $\delta_W = 28°$, it is possible with the present simple theory to calculate the heat released in the burning process. It would serve as some confirmation of the whole procedure if we could link this calculated information with some additional experimental knowledge about Q/a_1^2 for the particular flow picture which has been used. Unfortunately this is not given directly in the paper, but we may estimate a value from some of the facts which are presented there. First, it is stated that the air was pre-heated to a maximum temperature of 3,800°R or about 2,100°K. Assuming this to be the stagnation temperature of the stream of Mach number 3, this gives a value for T_∞ of 750°K, and a value for T_1 of about 1,600°K. Now although not explicitly stated, the final results in Rubins' paper are quoted for an equivalence ratio of 0.4, so we may assume that this figure is somewhere near the experimental value. Since some of the oxygen had already been burnt in pre-heating the experimental air, we shall assume that there are 0.2 grams of O_2 in each gram of gas mixture, which means that there will be about 0.01 grams of H_2 in each gram of mixture. The complete oxidation of one gram of H_2, using 8 grams of O_2, liberates about 28,000 calories, so that Q will be roughly 280 calories in the present case. Taking 29 for the molecular weight of the gas mixture, a_1^2 is about 155 calories and $Q/a_1^2 \simeq 1.8$. Bearing in mind the necessarily rather rough calculations and the assumptions involved here, this figure and the previous one are really quite surprisingly close. If, as is suggested by Rubins' results, only as little as 60% of the hydrogen is consumed initially the agreement becomes even better.

4. Conclusions.

Following a re-examination of the changes occurring across a flame-like discontinuity it has been shown that the flow on either side of the discontinuity may be supersonic. This fact enables one to construct simple flow patterns capable of supporting such fully-supersonic flames and some evidence for their experimental existence has been evaluated. The technique of assuming that the complex flow and chemical changes (which occur in a real flame) can be compressed into a sheet of discontinuity in the overall flow field may well prove useful in evaluating the gross fluid dynamical effects in more awkward situations than those examined here.

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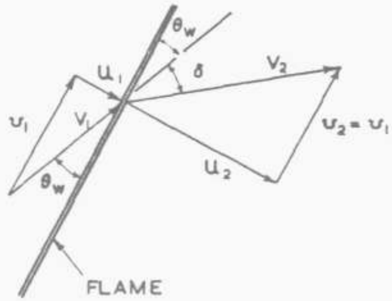

FIG.1. THE FLAME AS A DISCONTINUITY.

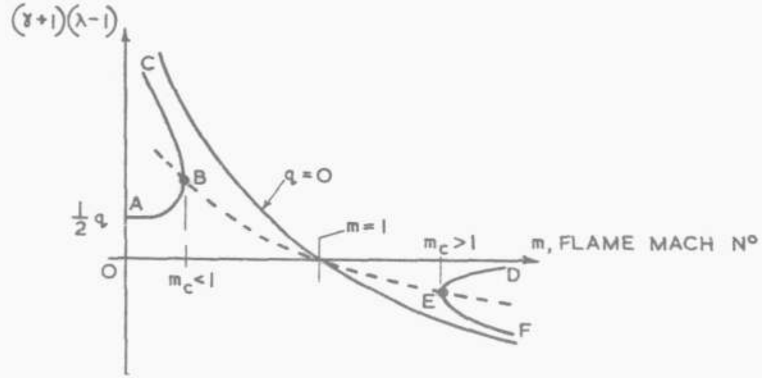

FIG. 2. DENSITY RATIO VERSUS FLAME MACH NUMBER.
(λ IS THE RATIO OF UNBURNT GAS DENSITY TO BURNT GAS DENSITY.)

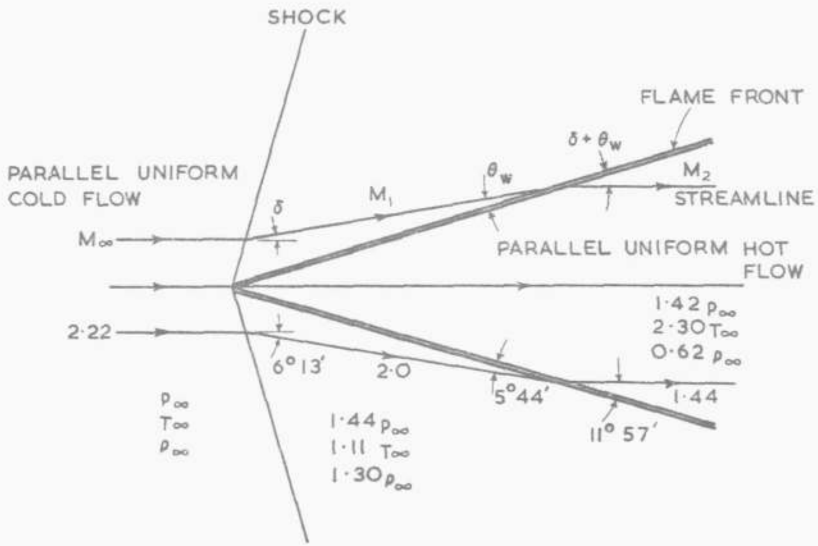

FIG. 3. TWO-DIMENSIONAL FULLY-SUPERSONIC V-FLAME.

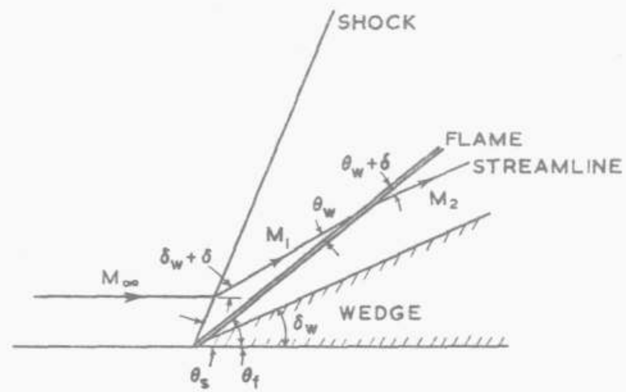

FIG.4. SHOCK-INDUCED COMBUSTION.