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SOME SCALES OF EQUIVALENT WEIGHT CHARACTERIZATIONS OF HARDY'S INEQUALITY: THE CASE $q < p$

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Abstract. We consider the weighted Hardy inequality

$$\left(\int_0^\infty \left(\int_0^x f(t)dt\right)^q u(x)dx\right)^{1/q} \leqslant C\left(\int_0^\infty f^p(x)v(x)dx\right)^{1/p}$$

for the case $0 < q < p < \infty,\ p > 1$. The weights $u(x)$ and $v(x)$ for which this inequality holds for all $f(x) \geqslant 0$ may be characterized by the Mazya-Rosin or by the Persson-Stepanov conditions. In this paper, we show that these conditions are not unique and can be supplemented by some continuous scales of conditions and we prove their equivalence. The results for the dual operator which do not follow by duality when $0 < q < 1$ are also given.

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