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Author's Reply and Corrigendum

Some shortcomings in Michell's truss theory*

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Abstract This reply outlines further classes of problems for which Michell's original theory is valid.

1 Introduction

In a private communication, György Károlyi (Technical University of Budapest, currently of Essen University) presented examples, for which Maxwell's (1872) theorem and hence Michell's (1904) original optimality criteria are valid, although the support conditions are statically indeterminate. The author fully agrees with Károlyi's remarks. In fact, in addition to statically determinate support conditions – with layout-independent reactions – Michell's original optimality criteria give the correct optimal layout for the following, rather narrow, classes of problems.

2 Optimal members in one direction only (R-type regions)

One possible optimal region for plane Michell trusses, termed by Prager (1974) "R-type", has the characteristics that at any point of the plane there is a nonvanishing member in at most one direction (principal direction I). Considering problems with unequal permissible stresses ($\sigma_0^- < \sigma_0^+$) in compression and tension, both Michell's (1904) original optimality criteria and the corrected optimality conditions by Hemp (1973) are valid if we have everywhere the adjoint principal strains $|\bar{\varepsilon}^I| \geq |\bar{\varepsilon}^{II}|$, $\bar{\varepsilon}^I = -1/\sigma_0^-$ and $-1/\sigma_0^- \leq \bar{\varepsilon}^{II} \leq 1/\sigma_0^+$. The above statement is not restricted to statically determinate support conditions. In general, the considered classes of layouts consist of R-type regions.

Proof. Hemp's (1973) relevant criteria

$$(\text{for}\, f < 0)\ \bar{\varepsilon} = -1/\sigma_0^-\,, \quad (\text{for}\, f = 0)\, -1/\sigma_0^- \leq \bar{\varepsilon} \leq 1/\sigma_0^+ \quad (1)$$

are clearly fulfilled by the above adjoint strains. However, Michell's (1904) less restrictive criteria

$$(\text{for}\, f < 0)\quad \bar{\varepsilon} = -k\,, \quad (\text{for}\, f = 0) \quad -k \leq \bar{\varepsilon} \leq k \qquad (2)$$

are also clearly satisfied with $k = 1/\sigma_0^- > 1/\sigma_0^+$.

Example. Consider a vertical line support and a point load at $0 \leq \beta \leq \pi/6$ to the horizontal (Fig. 1a). Minimize the truss weight for the stress constraint $-1/3 \leq \sigma \leq 1$.

Assuming that the optimal layout consists of a single member in the direction of the point load (Fig. 1b), Hemp's (1973) optimality criteria give

$$\bar{\varepsilon}^I = -1/\sigma_0^- = -3\,, \qquad (3)$$

and kinematic admissibility requires $\bar{\varepsilon}_y = 0$ in the vertical direction if we assume a location-independent adjoint strain tensor. The corresponding Mohr circle (Fig. 1c) implies

$$\bar{\varepsilon}^{II} = 6/[1 - \cos(\pi - 2\beta)] - 3\,. \qquad (4)$$

For the range $\beta = [0,\, \pi/6]$, $\bar{\varepsilon}^{II}$ takes on his highest value for $\beta = \pi/6$, which is $\bar{\varepsilon}^{II} = 1$. The smallest value of $\bar{\varepsilon}^{II}$ is zero for $\beta = 0$. This means that for all considered values of β we have $0 < \bar{\varepsilon}^{II} \leq 1/\sigma_0^+$ and hence Hemp's inequality condition in (1) is satisfied. However, the same adjoint strain field also fulfills Michell's less strict condition (2). Therefore, for this problem the latter would also yield the correct solution.

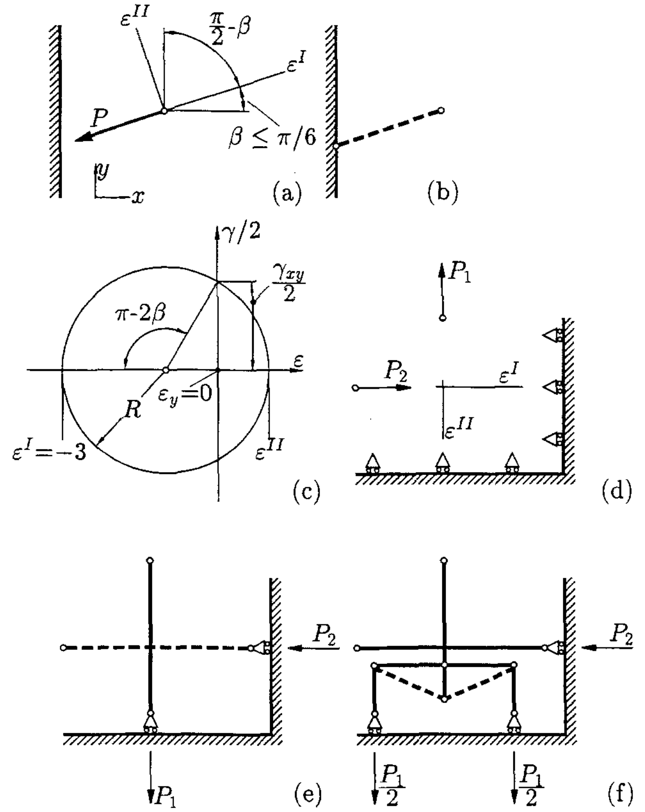

Fig. 1. Examples of layout problems for which Michell's original criteria are valid

3 Support conditions for which a uniform dilatation is kinematically admissible

Since Maxwell's theorem is based on a virtual displacement field which is a uniform dilatation, Michell's original criteria can be used for any support conditions that admit such displacements. As an example, a system of supports is shown

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in Fig. 1d, which also indicates two point loads in the horizontal and vertical directions. Hemp's criteria would give for this problem $\varepsilon^I = -1/\sigma_0^-$, $\varepsilon^{II} = 1/\sigma_0^+$ and Michell's criteria $\varepsilon^I = -k$, $\varepsilon^{II} = k$. Both are kinematically admissible and result in the optimal layout in Fig. 1e. Note that in this case the reactions are *not* layout-independent, because the (nonoptimal) layout in Fig. 1f, for example, gives different vertical reactions from the one in Fig. 1e.

4 Conclusions

Whilst Michell's (1904) original optimality criteria give the correct optimal solutions for some narrow classes of problems with $\sigma_0^+ \neq \sigma_0^-$, in the great majority of classes it is necessary to employ Hemp's (1973) criteria, which have also been confirmed and used in applications by the author.

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Corrigendum

On p. 245, in line 1, 'a distance "a" ' should read 'a distance "1" '.

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