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Research Article

Some Subordination Results on q-Analogue of Ruscheweyh Differential Operator

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We derive the q-analogue of the well-known Ruscheweyh differential operator using the concept of q-derivative. Here, we investigate several interesting properties of this q-operator by making use of the method of differential subordination.

1. Introduction

Recently, the area of q-analysis has attracted the serious attention of researchers. This great interest is due to its application in various branches of mathematics and physics. The application of q-calculus was initiated by Jackson [1, 2]. He was the first to develop q-integral and q-derivative in a systematic way. Later, from the 80s, geometrical interpretation of q-analysis has been recognized through studies on quantum groups. It also suggests a relation between integrable systems and q-analysis. In ([3–5]) the q-analogue of Baskakov Durrmeyer operator has been proposed, which is based on q-analogue of beta function. Another important q-generalization of complex operators is q-Picard and q-Gauss-Weierstrass singular integral operators discussed in [6–8]. The authors studied approximation and geometric properties of these q-operators in some subclasses of analytic functions in compact disk. Very recently, other q-analogues of differential operators have been introduced in [9]; see also ([10, 11]). These q-operators are defined by using convolution of normalized analytic functions and q-hypergeometric functions, where several interesting results are obtained. From this point, it is expected that deriving q-analogues of operators defined on the space of analytic functions would be important in future. A comprehensive study on applications of q-analysis in operator theory may be found in [12].

We provide some notations and concepts of q-calculus used in this paper. All the results can be found in [12–14]. For $n \in \mathbb{N}, 0 < q < 1$, we define

$$[n]_q = \frac{1-q^n}{1-q},$$
$$[n]_q! = \begin{cases} [n]_q [n-1]_q \cdots [1]_q, & n = 1, 2, \ldots; \\ 1, & n = 0. \end{cases} \tag{1}$$

As $q \to 1, [n]_q \to n$, and this is the bookmark of a q-analogue: the limit as $q \to 1$ recovers the classical object.

For complex parameters a, b, c, q $(c \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}, |q| < 1)$, the q-analogue of Gauss's hypergeometric function ${}_2\Phi_1(a, b; c, q, z)$ is defined by

$${}_2\Phi_1(a, b; c, q, z) = \sum_{k=0}^{\infty} \frac{(a,q)_k (b,q)_k}{(q,q)_k (c,q)_k} z^k, \quad (z \in \mathbb{U}), \tag{2}$$

where $(n, q)_k$ is the q-analogue of Pochhammer symbol defined by

$$(n,q)_k = \begin{cases} 1, & k = 0; \\ (1-n)(1-nq)\left(1-nq^2\right)\cdots\left(1-nq^{k-1}\right), & k \in \mathbb{N}. \end{cases} \tag{3}$$

The q-derivative of a function $h(x)$ is defined by

$$D_q(h(x)) = \frac{h(qx) - h(x)}{(q-1)x}, \quad q \neq 1, \ x \neq 0, \tag{4}$$

and $D_q(h(0)) = f'(0)$. For a function $h(z) = z^k$ observe that

$$D_q(h(z)) = D_q(z^k) = \frac{1-q^k}{1-q} z^{k-1} = [k]_q z^{k-1}; \tag{5}$$

then $\lim_{q\to 1} D_q(h(z)) = \lim_{q\to 1}[k]_q z^{k-1} = kz^{k-1} = h'(z)$, where $h'(z)$ is the ordinary derivative.

Next, we state the class $\mathcal{A}$ of all functions of the following form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{6}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. If f and g are analytic functions in $\mathbb{U}$, we say that f is subordinate to g; written $f \prec g$, if there is a function w analytic in $\mathbb{U}$, with $w(0) = 0, |w(z)| < 1$, for all $z \in \mathbb{U}$, such that $f(z) = g(w(z))$ for all $z \in \mathbb{U}$. If g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathbb{U}) \subseteq g(\mathbb{U})$.

For each A and B such that $-1 \le B < A \le 1$, we define the function

$$h(A, B; z) = \frac{1 + Az}{1 + Bz}, \quad (z \in \mathbb{U}). \tag{7}$$

It is well known that $h(A, B; z)$ for $-1 \le B \le 1$ is the conformal map of the unit disk onto the disk symmetrical with respect to the real axis having the center $(1-AB)/(1-B^2)$ for $B \neq \pm 1$ and radius $(A-B)/(1-B^2)$. The boundary circle cuts the real axis at the points $(1-A)/(1-B)$ and $(1+A)/(1+B)$.

Definition 1. Let $f \in \mathcal{A}$. Denote by $\mathcal{R}_q^\lambda$ the q-analogue of Ruscheweyh operator defined by

$$\mathcal{R}_q^\lambda f(z) = z + \sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!} a_k z^k, \tag{8}$$

where $[a]_q$ and $[a]_q!$ are defined in (1).

From the definition we observe that, if $q \to 1$, we have

$$\begin{aligned} \lim_{q\to 1} \mathcal{R}_q^\lambda f(z) &= z + \lim_{q\to 1}\left[\sum_{k=2}^{\infty} \frac{[k+\lambda-1]_q!}{[\lambda]_q![k-1]_q!} a_k z^k\right] \\ &= z + \sum_{k=2}^{\infty} \frac{(k+\lambda-1)!}{(\lambda)!\,(k-1)!} a_k z^k = \mathcal{R}^\lambda f(z), \end{aligned} \tag{9}$$

where $\mathcal{R}^\lambda$ is Ruscheweyh differential operator which was defined in [15] and has been studied by many authors, for example [16–18].

It can also be shown that this q-operator is hypergeometric in nature as

$$\mathcal{R}_q^\lambda f(z) = z\,{}_2\Phi_1\left(q^{\lambda+1}, q, q, q; z\right) * f(z), \tag{10}$$

where ${}_2\Phi_1$ is the q-analogue of Gauss hypergeometric function defined in (2), and the symbol $(*)$ stands for the Hadamard product (or convolution).

The following identity is easily verified for the operator $\mathcal{R}_q^\lambda$:

$$q^\lambda z\left(D_q\left(\mathcal{R}_q^\lambda f(z)\right)\right) = [\lambda+1]_q \mathcal{R}_q^{\lambda+1} f(z) - [\lambda]_q \mathcal{R}_q^\lambda f(z). \tag{11}$$

2. Main Results

Before we obtain our results, we state some known lemmas.

Let $P(\beta)$ be the class of functions of the form

$$\phi(z) = 1 + c_1 z + c_2 z^2 + \cdots, \tag{12}$$

which are analytic in $\mathbb{U}$ and satisfy the following inequality:

$$\operatorname{Re}(\phi(z)) > \beta, \quad (0 \le \beta < 1; z \in \mathbb{U}). \tag{13}$$

Lemma 2 (see [19]). *Let $\phi_j \in P(\beta_j)$ be given by* (12), *where* $(0 \le \beta_j < 1; j = 1, 2)$; *then*

$$(\phi_1 * \phi_2) \in P(\beta_3), \quad (\beta_3 = 1 - 2(1-\beta_1)(1-\beta_2)), \tag{14}$$

and the bound β_3 is the best possible.

Lemma 3 (see [20]). *Let the function ϕ, given by* (12), *be in the class $P(\beta)$. Then*

$$\operatorname{Re}\phi(z) > 2\beta - 1 + \frac{2(1-\beta)}{1+|z|}, \quad (0 \le \beta < 1). \tag{15}$$

Lemma 4 (see [21]). *The function $(1-z)^\gamma \equiv e^{\gamma\log(1-z)}$, $\gamma \neq 0$, is univalent in $\mathbb{U}$ if and only if γ is either in the closed disk $|\gamma - 1| \le 1$ or in the closed disk $|\gamma + 1| \le 1$.*

We now generalize the lemmas introduced in [22] and [23], respectively, using q-derivative.

Lemma 5. *Let $h(z)$ be analytic and convex univalent in $\mathbb{U}$ and $h(0) = 1$ and let $g(z) = 1 + b_1 z + b_2 z^2 + \cdots$ be analytic in $\mathbb{U}$. If*

$$g(z) + \frac{zD_q(g(z))}{c} \prec h(z), \quad (z \in \mathbb{U}; c \neq 0), \tag{16}$$

Then, for $\operatorname{Re}(c) \ge 0$,

$$g(z) \prec \frac{c}{z^c}\int_0^z t^{c-1} h(t)\,dt. \tag{17}$$

Proof. Suppose that h is analytic and convex univalent in $\mathbb{U}$ and g is analytic in $\mathbb{U}$. Letting $q \to 1$ in (16), we have

$$g(z) + \frac{zg'(z)}{c} \prec h(z), \quad (z \in \mathbb{U}; c \neq 0). \tag{18}$$

Then, from Lemma in [22], we obtain

$$g(z) \prec \frac{c}{z^c}\int_0^z t^{c-1} h(t)\,dt. \tag{19}$$

□

Lemma 6. *Let $q(z)$ be univalent in $\mathbb{U}$ and let $\theta(w)$ and $\phi(w)$ be analytic in a domain D containing $q(\mathbb{U})$ with $\phi(w) \neq 0$ when $w \in q(\mathbb{U})$. Set $Q(z) = zD_q(q(z))\phi(q(z))$, $h(z) = \theta(q(z)+Q(z))$ and suppose that*

(1) *$Q(z)$ is starlike univalent in $\mathbb{U}$;*

(2) $\mathrm{Re}(zD_q(h(z))/Q(z)) = \mathrm{Re}((D_q(\theta q(z)))/\phi(q(z))) + (zD_q(Q(z))/Q(z))) > 0$ $(z \in \mathbb{U})$.

If $p(z)$ is analytic in $\mathbb{U}$, with $p(0) = q(0)$, $p(\mathbb{U}) \subset D$, and

$$\begin{aligned}&\theta(p(z)) + zD_q(p(z))\phi(p(z))\\&\quad\prec \theta(q(z)) + zD_q(q(z))\phi(q(z)) = h(z),\end{aligned} \tag{20}$$

then $p(z) \prec q(z)$ and $q(z)$ is the best dominant.

The proof is similar to the proof of Lemma 5.

Theorem 7. *Let > 0, $\alpha > 0$, and $-1 \leq B < A \leq 1$. If $f \in \mathcal{A}$ satisfies*

$$(1-\alpha)\frac{\mathcal{R}_q^{\lambda} f(z)}{z} + \alpha\frac{\mathcal{R}_q^{\lambda+1} f(z)}{z} \prec h(A,B;z), \tag{21}$$

then

$$\begin{aligned}&\mathrm{Re}\left(\left(\frac{\mathcal{R}_q^{\lambda} f(z)}{z}\right)^{1/n}\right)\\&\quad> \left(\frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{1/n}\\&\qquad(n \geq 1).\end{aligned} \tag{22}$$

The result is sharp.

Proof. Let

$$g(z) = \frac{\mathcal{R}_q^{\lambda} f(z)}{z}, \tag{23}$$

for $f \in \mathcal{A}$. Then the function $g(z) = 1 + b_1 z + \cdots$ is analytic in $\mathbb{U}$. By using logarithmic q-differentiation on both sides of (23) and multiplying by z, we have

$$\frac{zD_q(g(z))}{g(z)} = \frac{zR_q^{\lambda} f(z)}{R_q^{\lambda} f(z)} - 1; \tag{24}$$

by making use of identity (11), we obtain

$$\frac{zD_q(g(z))}{g(z)} = \frac{[\lambda+1]_q}{q^{\lambda}}\frac{R_q^{\lambda+1} f(z)}{R_q^{\lambda} f(z)} - \frac{[\lambda]_q}{q^{\lambda}} - 1. \tag{25}$$

Taking into account that $[\lambda+1]_q = [\lambda]_q + q^{\lambda}$, we obtain

$$\frac{q^{\lambda}}{[\lambda+1]_q} zD_q(g(z)) + g(z) = \frac{\mathcal{R}^{\lambda+1} f(z)}{z}. \tag{26}$$

From (11), (23), and (26), we get

$$g(z) + \frac{q^{\lambda}\alpha}{[\lambda+1]_q} zD_q(g(z)) \prec h(A,B;z). \tag{27}$$

Now, applying Lemma 5, we have

$$g(z) \prec \frac{[\lambda+1]_q}{q^{\lambda}\alpha} z^{-[\lambda+1]_q/q^{\lambda}\alpha}\int_0^1 t^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1+At}{1+Bt}\right)dt, \tag{28}$$

or by the concept of subordination

$$\frac{\mathcal{R}_q^{\lambda} f(z)}{z} = \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1+Auw(z)}{1+Buw(z)}\right)du. \tag{29}$$

In view of $-1 \leq B < A \leq 1$ and $\lambda > 0$, it follows from (29) that

$$\mathrm{Re}\left(\frac{\mathcal{R}_q^{\lambda} f(z)}{z}\right) > \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1-Au}{1-Bu}\right)du, \tag{30}$$

with the aid of the elementary inequality $\mathrm{Re}(w^{1/n}) \geq (\mathrm{Re}\, w)^{1/n}$ for $\mathrm{Re}\, w > 0$ and $n \geq 1$. Hence, inequality (22) follows directly from (30). To show the sharpness of (22), we define $f \in \mathcal{A}$ by

$$\frac{\mathcal{R}_q^{\lambda} f(z)}{z} = \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1+Auz}{1+Buz}\right)du. \tag{31}$$

For this function, we find that

$$\begin{aligned}&(1-\alpha)\frac{\mathcal{R}_q^{\lambda} f(z)}{z} + \alpha\frac{\mathcal{R}_q^{\lambda+1} f(z)}{z} = \frac{1+Az}{1-Bz},\\&\frac{\mathcal{R}_q^{\lambda} f(z)}{z} \longrightarrow \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(\frac{1-Au}{1-Bu}\right)du\\&\qquad \text{as } z \longrightarrow -1.\end{aligned} \tag{32}$$

This completes the proof. □

Corollary 8. *Let $A = 2\beta - 1$ and $B = -1$, where $0 \leq \beta < 1$ and $\alpha, \lambda > 1$. If f satisfies*

$$(1-\alpha)\frac{\mathcal{R}_q^{\lambda} f(z)}{z} + \alpha\frac{\mathcal{R}_q^{\lambda+1} f(z)}{z} \prec h(2\beta-1,-1;z), \tag{33}$$

then

$$\begin{aligned}&\mathrm{Re}\left(\left(\frac{\mathcal{R}_q^{\lambda} f(z)}{z}\right)^{1/n}\right)\\&\quad> \left((2\beta-1)u^{[\lambda+1]_q/q^{\lambda}\alpha}\right.\\&\qquad\left.+\frac{2(1-\beta)[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 \frac{u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}}{1+u}du\right)^{1/n}\\&\qquad(n \geq 1).\end{aligned} \tag{34}$$

Proof. Following the same steps as in the proof of Theorem 7 and considering $g(z) = \mathscr{R}_q^\lambda f(z)/z$, the differential subordination (27) becomes

$$g(z) + \frac{q^\lambda \alpha}{[\lambda+1]_q} z D_q(g(z)) \prec \frac{1+(2\beta-1)z}{1+z}. \quad (35)$$

Therefore,

$$\begin{aligned}
\operatorname{Re}&\left(\left(\frac{\mathscr{R}_q^\lambda f(z)}{z}\right)^{1/n}\right) \\
&> \left(\frac{[\lambda+1]_q}{q^\lambda\alpha}\int_0^1 u^{([\lambda+1]_q/q^\lambda\alpha)-1}\left(\frac{1+(1-2\beta)u}{1+u}\right)du\right)^{1/n} \\
&= \left(\frac{[\lambda+1]_q}{q^\lambda\alpha}\int_0^1 u^{([\lambda+1]_q/q^\lambda\alpha)-1}\right. \\
&\qquad\left.\times\left((2\beta-1)+\frac{2(1-\beta)}{1+u}\right)du\right)^{1/n} \\
&= \left((2\beta-1)u^{[\lambda+1]_q/q^\lambda\alpha}\right. \\
&\qquad\left.+\frac{2(1-\beta)[\lambda+1]_q}{q^\lambda}\int_0^1 \frac{u^{([\lambda+1]_q/q^\lambda\alpha)-1}}{1+u}du\right)^{1/n}.
\end{aligned} \quad (36)$$

□

Theorem 9. *Let $\lambda > 0$ and $0 \le \rho < 1$. Let γ be a complex number with $\gamma \neq 0$ and satisfy either $|2\gamma(1-\rho)([\lambda+1]_q/q^\lambda) - 1| \le 1$ or $|2\gamma(1-\rho)([\lambda+1]_q/q^\lambda) + 1| \le 1$. If $f \in \mathscr{A}$ satisfies the condition*

$$\operatorname{Re}\left(\frac{\mathscr{R}_q^{\lambda+1} f(z)}{\mathscr{R}_q^\lambda f(z)}\right) > \rho, \quad (z \in \mathbb{U}), \quad (37)$$

then

$$\left(\frac{\mathscr{R}_q^\lambda f(z)}{z}\right)^\gamma \prec \frac{1}{(1-z)^{2\gamma(1-\rho)([\lambda+1]_q/q^\lambda)}}, \quad (z \in \mathbb{U}), \quad (38)$$

where $q(z)$ is the best dominant.

Proof. Let

$$p(z) = \left(\frac{\mathscr{R}_q^\lambda f(z)}{z}\right)^\gamma, \quad (z \in \mathbb{U}). \quad (39)$$

Then, by making use of (11), (37), and (39), we obtain

$$1 + \frac{q^\lambda z D_q(p(z))}{\gamma[\lambda+1]_q p(z)} \prec \frac{1+(1-2\rho)z}{1-z}, \quad (z \in \mathbb{U}). \quad (40)$$

We now assume that

$$\begin{aligned}
q(z) &= \frac{1}{(1-z)^{2\gamma(1-\rho)[\lambda+1]_q/q^\lambda}}, \quad \theta(w) = 1, \\
\phi(w) &= \frac{q^\lambda}{\gamma[\lambda+1]_q w};
\end{aligned} \quad (41)$$

then $q(z)$ is univalent by condition of the theorem and Lemma 4. Further, it is easy to show that $q(z)$, $\theta(w)$, and $\phi(w)$ satisfy the conditions of Lemma 6. Note that the function

$$Q(z) = z D_q(q(z))\phi(q(z)) = \frac{2(1-\rho)z}{1-z} \quad (42)$$

is univalent starlike in $\mathbb{U}$ and

$$h(z) = \theta(q(z)) + Q(z) = \frac{1+(1-2\rho)z}{1-z}. \quad (43)$$

Combining (40) and Lemma 6 we get the assertion of Theorem 9. □

Theorem 10. *Let $\alpha < 1$, $\lambda > 0$ and $-1 \le B_i < A_i \le 1$. If each of the functions $f_i \in \mathscr{A}$ satisfies the following subordination condition,*

$$(1-\alpha)\frac{\mathscr{R}_q^\lambda f_i(z)}{z} + \alpha\frac{\mathscr{R}_q^{\lambda+1} f_i(z)}{z} \prec h(A_i, B_i; z), \quad (44)$$

then,

$$(1-\alpha)\frac{\mathscr{R}_q^\lambda \Theta(z)}{z} + \alpha\frac{\mathscr{R}_q^{\lambda+1}\Theta(z)}{z} \prec h(1-2\gamma, -1; z), \quad (45)$$

where

$$\Theta(z) = \mathscr{R}_q^\lambda (f_1 * f_2)(z), \quad (46)$$

$$\begin{aligned}
\gamma = 1 &- \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)} \\
&\times\left(1 - \frac{[\lambda+1]_q}{q^\lambda\alpha}\int_0^1 \frac{u^{([\lambda+1]_q/q^\lambda\alpha)-1}}{1+u}du\right).
\end{aligned} \quad (47)$$

Proof. we define the function h_i by

$$\begin{aligned}
h_i(z) = (1-\alpha)\frac{\mathscr{R}_q^\lambda f_i(z)}{z} &+ \alpha\frac{\mathscr{R}_q^{\lambda+1} f_i(z)}{z} \\
&(f_i \in \mathscr{A}, i = 1,2);
\end{aligned} \quad (48)$$

we have $h_i(z) \in P(\beta_i)$, where $\beta_i = (1-A_i)/(1-B_i)$ $(i = 1,2)$. By making use of (11) and (48), we obtain

$$\mathscr{R}_q^\lambda f_i(z) = \frac{[\lambda+1]_q}{q^\lambda\alpha}\int_0^1 t^{([\lambda+1]_q/q^\lambda\alpha)-1} h_i(t)\,dt, \quad (i = 1,2), \quad (49)$$

which, in the light of (46), can show that

$$\mathscr{R}_q^\lambda \Theta(z) = \frac{[\lambda+1]_q}{q^\lambda\alpha} z^{1-([\lambda+1]_q/q^\lambda\alpha)}\int_0^1 t^{([\lambda+1]_q/q^\lambda\alpha)-1} h_0(t)\,dt, \quad (50)$$

where, for convenience,

$$h_0(z) = (1-\alpha)\frac{\mathscr{R}_q^{\lambda}\Theta(z)}{z} + \alpha\frac{\mathscr{R}_q^{\lambda+1}\Theta(z)}{z}$$
$$= \frac{[\lambda+1]_q}{q^{\lambda}\alpha} z^{1-([\lambda+1]_q/q^{\lambda}\alpha)} \times \int_0^1 t^{([\lambda+1]_q/q^{\lambda}\alpha)-1}(h_1 * h_2)(t)\,dt. \quad (51)$$

Note that, by using Lemma 2, we have $(h_1 * h_2) \in P(\beta_3)$, where $\beta_3 = 1 - 2(1-\beta_1)(1-\beta_2)$.

Now, with an application of Lemma 3, we have

$$\operatorname{Re}(h_0(z))$$
$$= \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\operatorname{Re}((h_1 * h_2)(uz))\,du$$
$$\geq \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(2\beta_3 - 1 + \frac{2(1-\beta_3)}{1+u|z|}\right)du$$
$$> \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}\left(2\beta_3 - 1 + \frac{2(1-\beta_3)}{1+u}\right)du$$
$$= 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)} \times \left(1 - \frac{[\lambda+1]_q}{q^{\lambda}\alpha}\int_0^1 \frac{u^{([\lambda+1]_q/q^{\lambda}\alpha)-1}}{1+u}du\right) = \gamma, \quad (52)$$

which shows that the desired assertion of Theorem 10 holds. □

Conflict of Interests

The authors declare that they have no competing interests regarding the publication of this paper.

Authors' Contribution

Huda Aldweby and Maslina Darus read and approved the final manuscript.

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