NBER WORKING PAPER SERIES

# SOME THEORETICAL ASPECTS OF BASE CONTROL

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Working Paper No. 650

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

March 1981

This paper was originally prepared for the first John Deutsch Round-Table on Economic Policy held at Queen's University, October 31 - November 2, 1980. The author would like to thank Gordon Thiessen whose probing comments aided the development of many of the ideas discussed in this paper. The opinions expressed in this paper are those of the author and no responsibility for them should be attributed to the Bank of Canada or the National Bureau of Economic Research. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics.

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## ABSTRACT

This paper focuses on the implications of using the monetary base or bank reserves as an instrument to control a monetary aggregate. Following analysis of a series of theoretical models of increasing complexity, it is concluded that in a system with either institutional or structural lags base control may entail very sharp and possibly undamped oscillations of short-term interest rates. The shorter the time period over which the authorities choose to bring the monetary aggregate back to its target, the more volatile will be the movements of interest rates. Furthermore, there is an asymmetry in the U.S. institutional structure such that rigid implementation of the base control system will under certain circumstances lead to a decline in short-term interest rates to very low levels. The final section of the paper is devoted to the examination of the Canadian institutional structure emphasizing the differences between the U.S. and Canadian systems.

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# SOME THEORETICAL ASPECTS OF BASE CONTROL

## 1. Introduction

One can divide into two groups the decisions surrounding the implementation of a monetary policy based on the use of monetary aggregates as intermediate targets -- one group is composed of strategic decisions, the other of tactical decisions. The former include such questions as which monetary aggregate provides the most appropriate target (narrow versus broad aggregates) and how rapidly should the rate of growth of that target be brought down (gradualism versus cold-shower policies). In addition there is the even more basic question as to how one interprets a strategy of targeting on a monetary aggregate. Of the three principal contending approaches one can be derived from the work of Poole (1970), one is related to the reduced-form types of equations, and the third can be thought of as feedback mechanism in which interest rates respond in the appropriate direction to nominal income growth that is too rapid or too slow.[1]

The tactical questions presuppose that a choice has been made regarding the target rate of growth of a specific monetary aggregate over some horizon period. In this category of questions one can place the choice of fan versus band and the width of the fan or band (i.e., the difference between upper and lower targets), whether the authorities are prepared to allow the monetary aggregate to remain outside the limits for a period of time or will respond to movements of the monetary aggregate within the band, and the horizon over which they attempt to bring the monetary aggregate back within the band should it move outside the limits. Perhaps the most important of these tactical questions is whether the authorities should try to achieve their monetary target by operating on interest rates, i.e., by sliding up and down the demand curve for money, or by operating on the base or bank reserves.[2] A recent interchange on this question of the relative merits of the use of base control versus interest rate control as the mechanism for influencing the movements of the monetary aggregates can be found in White (1979) and Courchene (1979).

In Canada the authorities have used interest rates as the proximate instrument in the achievement of the intermediate-run narrow money target. Until October 1979 this was also the mechanism used in the United States where the authorities placed most emphasis on the federal funds rate in their operating procedures. However, on October 6, 1979, the Federal Reserve announced a change in these procedures that moved the focus away from the federal funds rate and towards reserve measures as a means of controlling the monetary aggregates.[3] The subject of short run procedures is also a current issue in the United Kingdom where debate is now going on over the optimal methods of implementing a monetary aggregate policy.[4] Switzerland, which appears to have opted for a system of base as both a target and instrument, is used as an example by those who wish the authorities to drop monetary aggregates altogether and focus only on base.[5] In Germany, central bank money (which is very similar to base) has been used as a target but a combination of interest rates and other control techniques have been used to achieve this target. Thus there is a variety of international experience on which one could draw in assessing the relative merits of different techniques although one would have to take into account the different institutional environments in different countries.

In this paper I carry out a theoretical analysis of the implications of using base or bank reserves to control a monetary aggregate. That is, I examine the role of base as an instrument but not as a target. Most studies of such a form of control have focussed on the stability and predictability of the money multiplier.[6] This approach has serious drawbacks for those who do not believe in the usefulness of the money multiplier notion, particularly in a world with both institutional and economic lags. It is also the case that the adherents of base control and the money multiplier approach have tended to emphasize the "supply of money"[7] and have tended to downplay or ignore the demand for money. But it is the interaction of demand and "supply" functions that determines both the resulting level of money and the level of interest rates. And at least in some cases it is possible that rigid adherence to some forms of base control will lead to explosive (i.e., undamped) oscillations in interest rates and perhaps even in the monetary aggregate itself. Hence it is worth analyzing the movements of interest rates in cases where the authorities are using base control to see whether this instability problem can arise. Furthermore, by investigating in depth the implications of base control in a variety of models one can gain insights into the strengths and weaknesses of base control and the institutional settings in which it is more or less likely to lead to desirable outcomes. Furthermore, the limitations of base control in achieving the monetary targets over a short-run horizon will also become clear.

The paper is structured as follows. A series of models of increasing complexity are introduced and the implications of base control are examined in each case. I begin with a simple model without institutional or economic lags and without uncertainty, with non-interest bearing demand deposits as the only liability of the banking system, and without currency. In this model banks adjust their portfolio by buying and selling liquid assets since no liability side management is possible. I then introduce the following modifications to the basic model but for the sake of simplicity each modification is added to the simple basic model and therefore in general I do not allow for the complexities that might arise in making two or more modifications at the same time. In the first change to the model I introduce currency holding by the public as a part of money demand. In the next model I assume that the banks issue time deposits but still use changes in liquid assets as the mechanism for adjusting their portfolios. The following version of the model, probably the most important one in terms of substantive conclusions, allows for the existence of lags in the system, both economic as in the demand for money, and institutional, as in lagged reserve accounting. The role of excess reserves and borrowed reserves is explored in detail in the context of this model. Using these theoretical models as a basis I carry out an analysis of the U.S. system of base control and an assessment of what can be learned from its operation thus far. This is followed by the construction of a theoretical model of the Canadian institutional structure that emphasizes the differences between the U.S. and the Canadian system. Finally, I offer some conclusions that can be drawn from the analysis.

There are a number of limitations to the analysis to which attention should be drawn. First, it accepts as given the two-stage procedure in which base is used to target on an intermediate monetary aggregate and the latter in turn is related to the final target variable. However, Friedman (1975) has argued that in general there is no need for an intermediate monetary target and that the monetary instrument (base or interest rate) should be directed towards achieving the final target. Second, it is assumed that the authorities are trying to hit actual money and hence are not adjusting their targets to take into account the stochastic nature of money demand. This is in line with the prescription of most advocates of base control although it may be non-optimal in cases where there are stochastic movements in the demand for money equation.[8] Third, I offer no empirical work in this paper. The significance of some of the theoretical results discussed below will depend on whether the relevant conditions are fulfilled in practice or not. Fourth, the analysis is limited to the financial sub-sector. I am thus implicitly assuming that over the period of time analyzed in the paper (i.e., the short run) there is little or no effect on the real sector or price sector of movements in financial variables.[9] This limitation of the analysis can be removed by adding the other sectors to the model but, at least in the more complex analysis with lags, the resulting model may not be analytically tractable and hence one may have to have recourse to computer simulations. Fifth, I use only one interest rate in the model, thereby obviating the need to deal with the relationships between the one-day rate that is determined principally by the banking system in their attempt to adjust their reserve position, the 30- to 90-day rates that are more important in the demand for money equation, and the long-term rates that enter into investment functions. The oscillations of the one-day rate that are the focus of this paper would not be very important if they did not result in similar movements in the longer-term money market and bond market rates. However, recent theoretical work by Shiller (1979) and Pesando (1980) on the volatility of longer-term rates and the U.S. experience of the past year suggest that there can be a great deal of volatility even in the longer-term rates engendered by sharp movements in the very short-term rates. The effect of the movements of one-day rates on longer-term rates makes the argument of this paper more general than might otherwise be thought. Sixth, the models are all deterministic in nature. Introducing additive uncertainty to the initial model gives results similar to those found by Pierce and Thomson (1972).

## 2. The Models

### 2.1 Model 1: No lags, one bank liability, no currency

There is the very simplest textbook case. Money is defined as non-interest bearing demand deposits held with the banks and the banks are assumed to have no liabilities other than demand deposits. Reserve requirements are contemporaneous and all relationships are deterministic with no stochastic component. There are also assumed to be no lags in the demand for money equation.

(1) $RR(t) = dDD(t)$

(2) $RT(t) = RR(t)$

(3) $DD(t) = a - b\ i(t) + cY(t)$

(4) $MA(t) = DD(t).$

In equation (1), required reserves (RR) are a constant fraction of contemporaneous demand deposits (DD). Equation (2) sets out the equality of required reserves and total reserves (RT), i.e., there are no excess reserves held by the banks. One can think of required reserves as the demand for reserves and total reserves as the supply of reserves. Note also that there are no borrowed reserves in this system and therefore that the central bank is directly controlling total reserves. In equation (3) the public's demand for demand deposits is expressed as a function of the one interest rate in the system (i) and of real income (Y).[10] Finally, the monetary aggregate on which the authorities are targeting (MA) is equal to demand deposits in this system. Throughout the paper, all coefficients are positive.

Solving (1), (2), and (4) one gets

$$(5) \quad MA(t) = \frac{1}{d}(RT(t)).$$

This is a standard money multiplier result in which the target money aggregate is tightly linked to the reserve variable under the control of the central bank. An even more basic formulation would have an equation linking the supply of reserves to the asset side of the central bank's balance sheet and would thus relate changes in the money supply to open market purchases and sales.

There is a second equation implicit in the system developed above, that linking the interest rate to total reserves.

$$(6) \quad i(t) = \frac{1}{b}(a + cY(t) - \frac{1}{d}RT(t)).$$

An increase in total reserves leads to an immediate decline in the interest rate, the magnitude of the decline being a function of both the reserve requirement d, and the interest rate coefficient in the demand for money, b. The smaller is either of these parameters, the larger the effect on the interest rate of a given change in reserves.

Even in the context of this very simple model attention should be drawn to several important points. First, although reserves can be used to hit the target monetary aggregate, interest rates can be used equally well as shown by replacing DD by MA in the demand for money equation and then treating i as the instrument. Second, although the focus of the base or reserve control approach is on the supply of reserves, it is the case that the demand for money equation must always be satisfied.[11] The way the demand for money equation is satisfied in the short run is via the interest rate movements shown in equation (6). It must not be assumed that the authorities somehow force an increased amount of money into a non-bank public that is unwilling to hold it or force an unwilling public to give up money that it would like to retain. All exchanges are voluntary in our system and there is no "money rationing". The public is induced to increase or decrease its holdings of money by interest rate changes and, indeed, it is these interest rate changes that are the fulcrum of the effect of changes in the monetary aggregate on output, employment, the exchange rate and prices in most macro-economic models.

A third element in this analysis is the implicit structure of the banking system assumed in this model. This structure may play an important role in determining whether or not a change in reserves leads towards or away from the new equilibrium, i.e., whether or not the model is stable. In many base or reserve control models there is implicitly or explicitly a banking system balance sheet in which the banks hold liquid assets and reserves as their assets and demand deposits as their liabilities. Furthermore in response to an increase (decrease) in reserves created by the central bank through open market operations the banks buy (sell) liquid assets thereby pushing down (up) interest rates and increasing (reducing) demand deposits. This dynamic story of the reactions of the chartered banks to a reserve change is the usual one discussed in the textbooks as part of the explanation of the money multiplier. What is sometimes omitted in the textbooks is the fact that it is the interest rate changes that induce the non-bank public to exchange liquid assets for deposits and vice versa. Furthermore, models in which the banks rely on liability management to adjust their balance sheets can give different results. This subject will be addressed in a future paper.

One final element worth noting in this model is the absence of a money supply equation. In this model and all the succeeding models, the crucial equation is that in which the supply of reserves created by the central bank is equated to the demand for reserves which is a function of the magnitude of reservable deposits, the reserve requirement, and in some cases of the desired holding of excess reserves. These supply and demand functions are the basic building blocks of the analysis and they can be related to the behaviour of the central bank and the banking system, respectively. An alternative approach to analyzing the model is to solve out for the monetary aggregate as a function of reserves as in equation (5). Instead of treating this result as one of the reduced-form equations of the model (along with equation (6)) it is often treated as a money supply equation. Since the money supply combines the behaviour of the banks and the central bank (and indeed in some models the behaviour of the non-bank public and the government as well) it is not a supply equation in the usual sense of the word. This causes no problems as long as one realizes that the money supply defined in this way is an artificial construct and does not simply represent the behaviour of any single group in the model or the economy. Nonetheless it seems to me to be much simpler to discuss the financial side of the economy in terms of the basic transactors whose behaviour is being modelled and I shall continue to do so throughout the analysis.[12]

## 2.2 Model 2: No lags, one bank liability plus currency

Thus far it has been assumed that the target monetary aggregate consisted only of demand deposits and that the liabilities of the central bank consisted only of the reserves of the chartered banks. In fact, currency is part of the narrow monetary aggregate and it is also a liability of the central bank.[13]

(7) $RR(t) = dDD(t)$

(8) $B(t) = RR(t) + C(t)$

(9) $DD(t) + C(t) = a - b\ i(t) + cY(t)$

(10) $MA(t) = DD(t) + C(t).$

In equation (8) the supply of base (B) is equated to the demand for base, which is the sum of required reserves and currency (C). The target monetary aggregate is the sum of demand deposits and currency and the demand curve for these money balances is assumed to be the usual function of income and the interest rate.

Carrying out the usual algebraic manipulations one gets the following two equations:

$$(11) \quad MA(t) = \frac{1}{d} B(t) - (\frac{1}{d} -1) \; C(t)$$

$$(12) \quad i(t) = \frac{1}{b} (a + cY(t) - \frac{1}{d} B(t) + (\frac{1}{d} -1) \; C(t)).$$

To achieve the monetary aggregate target the central bank must be able to respond to shifts between currency and demand deposits. For example, if d were equal to 0.1, a one dollar random shift from demand deposits to currency, with base unchanged, would lead to a $9 decline in total money, and a corresponding increase in the interest rate. The appropriate response of the authorities to a one dollar shift from demand deposits to currency would be to increase base by $0.90. In such a case, currency rises by $1, reserves fall by $0.10, demand deposits fall by $1, and therefore the target monetary aggregate is unchanged.[14] In terms of the multiplier analysis, large random shifts between currency and demand deposits imply a very volatile multiplier. In contrast, if the interest rate is used as the instrument via the demand for money equation, then shifts between currency and demand deposits cause no operational difficulties.

A common way of treating currency in this type of model is to assume a stable relationship between currency and demand deposits held by the non-bank public.

(13) $C(t)/DD(t) = e$.

Then the money supply equation becomes

(14) $MA(t) = \frac{1+e}{d+e} B(t)$.

Thus the simplicity of the base control relationship is re-established if the ratio of currency to demand deposits is constant, i.e., if there are no random shifts between the two components of money.

2.3 Model 3: No lags, two bank liabilities, no currency

In this model I assume that banks issue interest-bearing time deposits but that they do not use these time deposits to conduct liability management. One simple way of introducing these "passive" time deposits into the analysis is to postulate that the banks move the time deposit rate in line with movements of market rates of interest and accept the resulting volume of time deposits.

(15) $RR(t) = dDD(t) + tTD(t) \qquad 0 \leqq t \leqq d$

(16) $RT(t) = RR(t)$

(17) $DD(t) = a - b\, i(t) + cY(t)$

(18a) $MA(t) = DD(t)$

(18b) $MA(t) = DD(t) + TD(t)$

(19) $TD(t) = f + g\, i(t) + hY(t)$.

Required reserves are now a function of both demand deposits and time deposits (TD) with the reserve ratio on time deposits smaller than or equal to that on demand deposits. The target monetary aggregate can be specified either in terms of narrow money i.e., demand deposits in this model where currency does not exist (equation (18a)), or broad money i.e., demand deposits plus time deposits (equation (18b)). The level of time deposits demanded is directly related to the market rate of interest since a rise in the latter brings about a corresponding rise in the time deposit rate and, hence, a shift from demand deposits to both market instruments and time deposits. Note that this argument implies that $b > g$ since some of the reduction of demand deposits in response to a rise in interest rates corresponds to a movement into market instruments.[15] The demand for time deposits is also directly related to the level of nominal income.[16]

The reduced-form equations in this model are more complicated than those in the earlier models because of the movements between time deposits and demand deposits as interest rates increase. The equation corresponding to the use of narrow monetary aggregate as target is labelled (a) and that corresponding to the broad aggregate is labelled (b).

$$(20)\quad i(t) = \frac{d}{bd-tg}\left(a + \frac{t}{d} f + (c + \frac{th}{d})\ Y\ (t) - \frac{1}{d}\ RT(t)\right)$$

$$(21a)\quad MA(t) = \frac{1}{d}\ RT(t) - \frac{t}{d}\ TD(t)$$

$$= \frac{1}{d}\ (1 + \frac{tg}{bd-tg})\ RT(t) - \frac{t}{bd-tg}(bf + ag) - \frac{t}{bd-tg}\ (cg + bh)\ Y(t)$$

$$(21b)\quad MA(t) = \frac{1}{d}\ RT(t) + \frac{d-t}{d}\ TD(t)$$

$$= \frac{1}{d}\ (1 - \frac{(d-t)g}{bd-tg})\ RT(t) + \frac{d-t}{bd-tg}\ (ag+bf) + \frac{d-t}{bd-tg}\ (bh + cg)\ Y(t)$$

Since $b > g$, as discussed above, and $d \geqq t$ by our assumptions regarding the institutional framework, $bd > tg$. Hence the denominators of all the fractions are positive.

If the authorities' preferred aggregate is the narrow monetary aggregate, an increase in income with reserves constant will result in a decline in the monetary aggregate. This result occurs because the increase in income leads to an increase in the demand for time deposits both directly and through the induced increase in interest rates. The increase in time deposits means that less demand deposits can be supported by the given reserves, to use the money multiplier terminology. On the other hand, if the broader aggregate is the focus of policy an increase in income with reserves constant leads to an increase in the monetary aggregate since the rise in time deposits more than offsets the decline in demand deposits when the latter bear a higher reserve requirement than the former.

It is clear that if the authorities wish to simplify their task of linking reserves with the preferred monetary aggregate, they should set t equal to zero if they pursue a narrow aggregate policy and t equal to d if they pursue a broad aggregate policy. In both cases this would have the effect of simplifying the reduced-form equation for money equation to its most basic form, i.e., $MA(t) = \frac{1}{d} RT(t)$. Thus the intuitively obvious conclusion is reached, at least for this simple model -- impose a uniform reserve requirement on all the components of the monetary aggregate but do not impose a reserve requirement on deposits that are not included in the aggregate.[17] With a split reserve requirement or with reserves on deposits not included in the aggregate the authorities must act to offset shifts among components by adjusting the volume of reserves.[18] In the language of multiplier analysis, the authorities must adjust reserves to offset any change in the multiplier brought about by shifts between time deposits and demand deposits.

As pointed out above, with a split reserve requirement an increase in income leads to a fall in the narrow aggregate and an increase in the broader aggregate, if reserves are held constant. To keep the monetary aggregate constant in the face of an increase in income reserves would have to be increased in the case in which the authorities are targeting on the narrow aggregate and reduced in the case in which the authorities are targeting on the broader aggregate. Using (19) one can see that this implies that with a rise in income interest rates would increase more in the case of a broad aggregate target than in the case of a narrow aggregate target.[19] This point has been an important element of the discussion about the choice of a narrow aggregate by the Bank of Canada. See White (1979).

2.4 Model 4: Lags, one bank liability, no currency

The discussion thus far has been limited to fairly simple textbook types of models and has analyzed the implications of base or reserve control in these settings. I now turn to more realistic models of the economic environment in which lags in behaviour play a crucial role. I begin with a version of the demand for money equation in which the response of money demand to interest rate changes takes place via a distributed lag. I then introduce the existence of borrowed reserves and excess reserves into the model. This modification enables us to deal with the possibility of control of non-borrowed reserves rather than total reserves by the central bank. The institutional lag in reserve requirements is then brought into the discussion. In the light of these models, I will discuss the new U.S. approach to reserve control in the next section of the paper.

Model 4a: Economic lags, no borrowed or excess reserves

In this model the demand for money equation includes lagged interest rates as an explanatory variable in addition to current interest rates. This is in line with estimated equations of money demand in virtually all of which interest rates take some time to affect the quantity of money demanded.

(22) $RR(t) = dDD(t)$

(23) $RT(t) = RR(t)$

(24) $DD(t) = a - b_0 i(t) - b_1 i(t-1) + cY(t)$

(25) $MA(t) = DD(t)$.

There would be no difficulty in generalizing the money demand equation to incorporate a lagged income term or an error term.[20] Since there are no borrowed or excess reserves in this version of the model, the reserves equation retains its simple form of an equality between the supply of total reserves and the demand for required reserves. The reduced-form equations for the monetary aggregate and the interest rate are as follows:

(26) $MA(t) = \frac{1}{d} RT(t)$

(27) $i(t) = \frac{1}{b_0} (a + cY(t) - \frac{1}{d} RT(t)) - \frac{b_1}{b_0} i(t-1)$.

The monetary aggregate retains its usual simple form in this model. The interest rate equation, however, now contains a lagged dependent variable with a coefficient that is the ratio of the effect on the demand for money of the lagged interest rate to the effect of the current interest rate. Regardless of the relative magnitudes of these coefficients this formulation implies a sawtooth movement of interest rates in response to a change in income with reserves constant or to a change in reserves with income constant. Furthermore, if, as is not unlikely in practice, $b_1$ is greater than $b_0$, i.e., the lagged effects of interest rate changes dominate the current effects,[21] then the coefficient on the lagged interest rate in equation (27) will be greater than one in absolute value and the movement of interest rates will follow a path of explosive oscillations. Thus, a change in income or reserves (or in the error term for the demand for money if it were added to equation (24)) would lead to ever increasing upward and downward movements of the interest rates.[22]

The economics behind this result is fairly straightforward. Suppose nominal income began to increase and the authorities held total reserves constant. The increase in the demand for money and hence in the demand for reserves in the face of an unchanged quantity of reserves supplied by the central bank, would lead to a rise in interest rates. This increase would have to be sufficient to offset the desired increase in money, i.e., it would be $c\Delta Y/b_0$, where the numerator represents the increase in the quantity of money demanded as a result of the income increase and the denominator represents the effect of a unit increase in interest rates on money demanded in the current period. If this new interest rate prevailed in the subsequent period, it would entail a further downward movement in the demand for money of the order $(b_1 \frac{c\Delta Y}{b_0})$ because of the lagged effect of interest rates on money demanded. But such a downward movement in money demand and hence in reserves demanded would be inconsistent with the unchanged supply of reserves. Hence interest rates have to fall in the second period. The lower interest rates of the second period would lead to an increase in the quantity of money demanded in the third period if left unchanged. Hence interest rates would have to rise in the third period. Whether these oscillations are damped eventually leading to a stable equilibrium, or undamped, leading to explosive oscillations, depends on the ratio $b_1/b_0$, the relative size of lagged effects and current effects of interest rates on the demand for money, as shown above.

One further point should be noted in the context of this model. If $b_0$ equalled zero, i.e., over the current period the demand for money did not respond to movements of interest rates, the interest rate is not determinate. A change in the quantity of money demanded or quantity of reserves supplied cannot be equilibrated in the current period by an offsetting interest rate change. For example, suppose income were to increase and the demand for money increased as a result. The demand for reserves would rise as transactors tried to hold more money balances. The banks would try to sell liquid assets to re-establish their equilibrium and this would drive up interest rates. But the movement in interest rates cannot affect the quantity of money demanded until the next period. Hence there is no interest rate at which the system would be in equilibrium in the current period.

<u>Model 4b:</u> Economic lags, borrowed reserves and excess reserves

In the U.S. institutional structure borrowed reserves play an important role in the transmission of the policy impulse from the supply of reserves to interest rates and monetary aggregates. With the introduction of borrowed reserves one must divide total reserves into non-borrowed reserves (RNB) which are under the control of the central bank, and borrowed reserves (RB) which are created at the initiative of the banks.[23] Total reserves can also be defined as the sum of required reserves and excess reserves (RE).

(28) $RT(t) = RNB(t) + RB(t)$

(29) $RT(t) = RR(t) + RE(t).$

The rest of the model remains as before with contemporaneous reserve requirements and lagged responses of the demand for money to interest rate changes; i.e., equations (22), (24), and (25) continue to hold. To close the model, it is necessary to add a pair of equations describing the demand for excess reserves and borrowed reserves by the banking system.

(30) $RB(t) = q(i(t) - rdis(t)) \qquad i(t) \geqq rdis(t)$

$\qquad = 0 \qquad i(t) < rdis(t)$

(31) $RE(t) = p \qquad 0 < i(t)$

As long as the market rate[24] exceeds the discount rate (rdis) charged by the central bank on borrowings by the banks, there is a relationship between borrowed reserves and the difference between the two rates.[25] The greater is this differential, the larger is the amount of borrowing by the banks. If the market interest rate falls below the discount rate I assume that all borrowing is repaid. Regardless of the level of interest rates the banks are assumed to hold a relatively small and fixed amount of excess reserves. I also assume that the banks would not be willing to hold more excess reserves unless the market interest rate fell very close to zero.[26] At such very low interest rates, however, the demand for excess reserves becomes almost infinitely elastic, as was the case in the 1930s.[27]

Solving this model for the case in which the market rate is above the discount rate and the banks are borrowing reserves from the central bank gives the following pair of equations.

$$(32)\quad i(t) = \frac{q}{q+db_0}\, rdis(t) + \frac{1}{q+db_0}\,(p + ad + cdY(t) - RNB(t)) - \frac{db_1}{q+db_0}\, i(t-1)$$

$$(33)\quad MA(t) = \frac{1}{q+db_0}\,(b_0 RNB(t) + b_1\, RNB(t-1) - qb_0\, rdis(t) - qb_1\, rdis(t-1) + cqY(t) + qa - (b_0 + b_1)\, p) - \frac{db_1}{q+db_0}\, MA(t-1).$$

In comparison with equation (27) in which there was no discount window one can see that the coefficient on the lagged dependent variable is now smaller than before and therefore the system is less likely to be explosive. The coefficient on the lagged dependent variable can be written as $b_1/(b_0 + \frac{q}{d})$ as compared to $b_1/b_0$ in the earlier case. The damping in the oscillation arises from the fact that any increase in money demanded can now be offset not only by rising interest rates but also by increased borrowing at the discount window which permits the monetary aggregate to increase in the short run. Nevertheless the sawtooth movement in interest rates continues to hold and the monetary aggregate now also follows the same sawtooth movement.

It is worth emphasizing that holding non-borrowed reserves constant in this case does not imply that the monetary aggregate will return to its target value after a shock to the system such as an increase in income. This can be seen by solving for the equilibrium equations corresponding to equations (32) and (33).[28]

$$(34)\quad i = \frac{1}{q+d(b_0+b_1)}(q\ rdis + p + ad + cdY - RNB)$$

$$(35)\quad MA = \frac{1}{q+d(b_0+b_1)}((b_0+b_1)\ RNB - q(b_0+b_1)\ rdis + cqY + qa - (b_0+b_1)p)$$

An increase in income leads to an increase in interest rates and an increase in the monetary aggregate even after the system settles down. The reason is that the higher interest rate (relative to the discount rate) results in an increase in borrowing from the central bank that allows the monetary aggregate to expand. In this version of the model, therefore, an increase in income leads to an increase in the interest rate which is not sufficient to bring money back to its target.[29] If the authorities wish to keep money on target in the relatively short run they will have to push up the interest rate from the level that is consistent with a constant amount of non-borrowed reserves either by reducing the level of non-borrowed reserves or by raising the discount rate. Thus an element of discretion is re-introduced to the system in that the central bank has to decide on the horizon over which the monetary target should return to its target and the combination of discount rate increase and decrease in non-borrowed reserves needed to achieve this result.[30]

Rather than pursuing this general model further I wish to examine more closely the economics of the slightly simpler model in which $b_0$ equals zero, i.e., interest rates affect money only with a lag. In the case of the model with no discount window this assumption led to the result that the interest rate was indeterminate. In the case of a model with a discount rate the solution to the system is as follows:

$$(36)\quad i(t) = rdis(t) + \frac{ad+p}{q} + \frac{cd}{q} Y(t) - \frac{1}{q} RNB(t) - \frac{db_1}{q} i(t-1)$$

$$(37)\quad MA(t) = \frac{b_1}{q} RNB(t-1) - b_1\, rdis(t-1) + (a - \frac{b_1 p}{q}) + cY(t) - \frac{db_1}{q} MA(t-1).$$

Suppose income were to increase and hence cause an increase in money demanded. The increase in the quantity of money demanded results in an increase in required reserves and hence an increased demand for reserves by the banks. The banks would drive up the interest rate either by trying to sell liquid assets or by trying to increase their borrowing in the federal funds market. Interest rates would continue to increase until the banks were prepared to increase their borrowing from the central bank by the amount of the increase in required reserves. In the next period this increase in interest rates would lead to a reduction in the demand for money and hence a fall in required reserves and thus a fall in interest rates. Unlike the earlier models, the equilibration of the amount of reserves supplied by the central bank and that demanded by the banks is brought about in the first instance not by an adjustment of money demanded (and hence required reserves) as a result of interest rate movements but by an increase in borrowing at the discount window. Indeed no adjustment of the current level of demand deposits demanded is possible in this model given that $b_0$ is equal to zero, i.e., that there is no response of money demanded to interest rates in the current period. The increase in the interest rate is initially therefore determined by the magnitude of q since that represents the response of borrowed reserves to changes in interest rates.[31]

It is important to realize that in these models the apparent direct linkage between non-borrowed reserves and the monetary aggregate no longer exists. For example, as can be seen from equation (37), a change in non-borrowed reserves in the current period has no effect on the aggregate until the following period and the monetary aggregate will then oscillate until a new short-run equilibrium is reached. And in this particular case it is very clear that the linkage from non-borrowed reserves to the monetary aggregate operates via the rise in interest rates and borrowed reserves in the current period to the demand for money in the following period. Whether it is useful to call such a relationship a money supply curve is a moot point. What is clear is that the relationship is a good deal more complex than in the earlier models. It is also clear that it is not possible to hit the monetary target period-by-period in this model even if one were willing to accept the implied movements of interest rates because of the lag between non-borrowed reserves and the monetary aggregate. It follows, therefore, that total reserves are also not controllable in this model.[32] As discussed above, in the more general model with $b_0$ not equal to zero, the authorities might be able to achieve their target each period but only by manipulating non-borrowed reserves and the discount rate appropriately and accepting possibly explosive oscillations in interest rates.

In circumstances where the excess reserves provided by the central bank increase to amounts above the frictional level (p in equation (31)) desired by the banks it may easily be the case that the interest rates will settle at just above zero. This result will occur, for example, when there is a lag in the demand for money. Assume a decline in income which leads to a decline in money demanded. With the fall in required reserves, and non-borrowed reserves constant, banks will try to lend federal funds or buy liquid assets, thereby driving down interest rates. With lower interest rates and a constant discount rate the banks will repay their borrowings to the central bank. If the fall in required reserves is sufficient to force them into a position where they are holding excess reserves larger than their desired holdings then the market interest rates will continue to fall until they are willing to hold the excess reserves. As indicated above, the banks will probably not be willing to hold these excess reserves at interest rates much above zero.[33] I am thus arguing that there is a serious asymmetry in the base control mechanism that arises from the difference between the demand for borrowed reserves and the demand for excess reserves with the former very sensitive to market rates of interest and the latter very insensitive to market interest rates. This asymmetry becomes significant when current rates of interest have little effect on the current quantity of money demanded.

Formally, the model of positive excess reserves can be treated by setting q equal to zero in equations (32) and (33). The equations collapse (except for terms in p) to equations (26) and (27), i.e., the discount window ceases to matter. If, in addition, $b_0$ is very small or indeed zero then interest rates will fall towards zero very rapidly and the authorities will still not necessarily be able to achieve the aggregate target over any short horizon.

<u>Model 4c:</u> Institutional lag, borrowed reserves and excess reserves

In this version of the model I introduce lagged reserve accounting of the sort currently being used in the United States and Canada. I continue with the U.S. institutional environment of borrowing at the discount window but drop the economic lags in the demand for money. Since the results correspond fairly closely to those in the case of an economic lag we can treat them in fairly cursory fashion.

(38) $RR(t) = dDD(t-1)$

(39) $DD(t) = a - b\ i(t) + cY(t)$

(40) $MA(t) = DD(t).$

Required reserves are now a function of demand deposits one period earlier where the period is defined in terms of the length of the accounting lag. In the case of the United States, for example, this period is two weeks. The equations describing reserve behaviour, i.e., equations (28), (29), (30), and (31), are the same as in the previous model.

Solving in the usual fashion for the situation in which equation (30) holds one gets:

$$(41)\quad i(t) = rdis(t) + \frac{1}{q}\,(p + ad + cdY(t-1) - RNB(t)) - \frac{db}{q}\; i(t-1)$$

$$(42)\quad MA(t) = a - \frac{bp}{q} - b\ rdis(t) + \frac{b}{q}\,RNB(t) + cY(t) - \frac{db}{q}\,MA(t-1).$$

These results are formally similar to those in the case of a one period economic lag in the demand for money and contemporaneous reserve requirements (equations (36) and (37) above) although with slightly different lag structure. The economic interpretation is somewhat different, however. Suppose there is an increase in income which leads to an increase in money demanded. There is no effect on required reserves in the current period and hence no change in the interest rate in the current period. In the next period, however, required reserves increase. If the central bank holds non-borrowed reserves constant, the banks will find themselves short of reserves. The attempt by the banks to increase their reserves will lead to an increase in the interest rate as banks attempt to purchase federal funds, and to sell liquid assets to the public. As the short-term interest rate rises relative to the central bank discount rate banks will turn to the central bank and borrow reserves and the shortage of reserves will therefore disappear. As a result of the increase in interest rates there will be a decline in demand deposits held at the banks. This in turn will lead to a decline in required reserves in the following period which will lead to a fall in interest rates and a rise in demand deposits held at the banks. The by-now familiar sawtooth will be the outcome of this system.[34]

Turning to the situation in which there are positive excess reserves in the system, there is no longer any reduced-form relationship between interest rates or the monetary aggregate and non-borrowed reserves. If the central bank increases non-borrowed reserves in such a situation the extra reserves entail an increase in the excess reserves of the banking system since required reserves are predetermined and are not affected by movements in current demand deposits. With the banks unwilling to add excess reserves to their portfolio at interest rates much above zero, interest rates will clearly fall to very low levels in such circumstances.[35] This scenario becomes of practical significance in a situation where nominal income falls or grows more slowly than the targeted rate of growth of money. The fall in income leads to a fall in current demand deposits which, in turn, leads to a decline in the following period's required reserves. With non-borrowed reserves constant, this implies an increase in excess reserves with the consequent fall in interest rates to very low levels as described.[36]

## 3. The New Federal Reserve Procedures

As part of its October 6, 1979 program the Federal Reserve announced that it was "placing greater emphasis on day-to-day operations on the supply of bank reserves and less emphasis on confining short-term fluctuations in the federal funds rate".[37] Although the details of the new operating procedures were not spelled out by the Federal Reserve in its October 6, 1979 press release, they have since been set out in a staff paper entitled "The New Federal Reserve Technical Procedures for Controlling Money".[38] In this section of the paper I set out my interpretation of these procedures in the light of the analysis of the earlier sections. Although I rely heavily on the staff paper for the Federal Reserve's own views on how the system is being implemented, the evaluation I am offering is based in large part on the experience of the first year of the operation of the reserves control system.

The principal steps in the new methods of controlling the monetary aggregates can be characterized as follows:

(1) Set growth rates for those aggregates for which the Federal Reserve establishes targets, i.e., M1A, M1B and M2. In compliance with the Humphrey-Hawkins Act, target growth rates are set over a one-year horizon.

(2) Given these targeted growth paths and expectations as to desired excess reserves, the growth of certificates of deposit and that of other reservable liabilities not in the targets, the growth of currency, and the split of deposits between member banks and non-member banks, the Federal Reserve calculates the implied path for the family of reserve measures such as total reserves, monetary base, non-borrowed reserves. The Federal Reserve is prepared to supply whatever reserves the banks are required to hold against certificates of deposit and other deposits that are not components of the narrower monetary aggregates, i.e., M1A, M1B and M2.

Two comments are in order here. First, the Federal Reserve is engaged in a reserves control process rather than a base control process for the reason outlined in Section 2.2. A random shift from demand deposits to currency would lead to much larger movements of the monetary aggregates under a system of base control than under a system of reserves control. Second, in Section 2.3, I argued that in an optimal system reserves would be imposed only against deposits included in the monetary aggregates and that reserve requirements against these deposits would be uniform. Given that there are reserves in the U.S. system against deposits that are not in the monetary aggregates and that reserve requirements against the deposits within the aggregate are far from uniform the Federal Reserve must allow for growth in the former types of deposits and for shifts within the latter types of deposits. It is easiest to think of the Federal Reserve adjusting non-borrowed reserves to deal with these "nuisance" elements.[39]

(3) The planned path of non-borrowed reserves is calculated from the path of total reserves by initially assuming a level of borrowing near that prevailing in a recent period.

(4) Although total reserves are the principal overall objective of reserve setting, only non-borrowed reserves are directly under the control of the Federal Reserve through open-market operations.

(5) Suppose, for example, M1A began to grow faster than targeted. Given the path for non-borrowed reserves, the increase in required reserves would lead the banks to attempt to raise funds in the federal funds market and hence bid up the federal funds rate and to borrow more at the discount window. The higher interest rates would eventually feed back on M1A and slow down

its growth rate. In the interim period while the higher interest rates are having their effect on M1A, required reserves and hence total reserves will remain above their initial targeted path. Thus during that period the Federal Reserve can control only the supply of non-borrowed reserves and not that of total reserves.

(6) If the Federal Reserve wished to speed up the effect of its policy and bring total reserves down more quickly, it could lower the non-borrowed reserve path and/or raise the discount rate. Both these actions would have the effect of raising interest rates more than otherwise and hence would reduce the growth of M1A (and thus of required reserves and total reserves) more quickly than using the strategy of holding non-borrowed reserves to their initial path and leaving the discount rate unchanged.

The analysis of model 4b throws some light on this aspect of the analysis. As shown there, holding non-borrowed reserves to their target path is likely not to have a sufficiently large effect on interest rates to bring the monetary aggregate back to its target in a relatively short period of time. Hence the Federal Reserve can speed the impact of its actions on the monetary aggregate (and therefore on total reserves) by increasing the discount rate or reducing the level of non-borrowed reserves. The Federal Reserve must thus make a choice between a fairly automatic policy in which it adheres to a target path for non-borrowed reserves while allowing the monetary aggregates and total reserves to deviate from their target paths for lengthy periods of time (a long horizon policy) or a policy with substantial discretion over non-borrowed reserves and discount rates which aims at bringing the monetary aggregates and total reserves back to their target path in a short period of time (a short horizon policy). One element entering the decision between long horizon and short horizon policy is the substantially greater credibility the Federal Reserve would achieve by controlling the aggregates and total reserves over a shorter time period. Offsetting this, however, is the increase in interest rate volatility from choosing a shorter horizon. Indeed, as indicated in the various models in Section 2.4, if the Federal Reserve tries to hit its aggregate targets over too short an horizon period, it might create an explosive oscillation of interest rates. Furthermore, with the lag structures at work in the economy it is possible that both interest rates and the monetary aggregates will display ever-increasing cycles.[40]

The principal conclusion of this part of the analysis is that the choice of horizon is crucial. A relatively short horizon for bringing monetary aggregates and total reserves back to their target path would imply very sharp movements in interest rates and non-borrowed reserves and possibly even explosive oscillations in these variables. With a longer horizon, interest rates and non-borrowed reserves would be less volatile but total reserves and monetary aggregates would remain above target for a longer period of time following the increase in income.

Another aspect of the control system that has become apparent over the year is the asymmetry on the upside and on the downside. As noted above (in Section 2.4) with non-borrowed reserves remaining on their target path, an increase in income would lead to a rise in interest rates and, in the short run, the size of this increase would be dependent on the response of borrowed reserves to the differential between the interest rate and the discount rate. Conversely, if income fell substantially and non-borrowed reserves remained on their growth path, then interest rates would fall, borrowed reserves would be repaid and the system would likely move into a position in which it was holding excess reserves. With banks generally unwilling to hold more than frictional amounts of excess reserves even when interest rates are relatively low, the interest rates have to fall to very low levels to induce the banks to hold the excess reserves. Furthermore the fall in interest rates would probably be very rapid. The asymmetry between the two cases arises because the demand for borrowed reserves depends on the differential between the interest rate and the discount rate whereas the demand for excess reserves above a minimum frictional level is zero for all interest rates above a very low level. This asymmetry implies that the Federal Reserve would have to live with total reserves (and non-borrowed reserves) below target paths for a longer period of time than it would like unless it is prepared to see interest rates fall to very low levels. Since such an outcome is unlikely to be palatable, the Federal Reserve will not be able to follow an automatic non-borrowed reserves path when the monetary aggregate falls substantially below its target.

## 4. The Canadian Institutional Structure

Detailed descriptions of the way in which cash setting by the Bank of Canada affects the excess reserves of the chartered banks and thus influences short-term interest rates in Canada can be found in

Dingle, Sparks and Walker (1972) and White and Poloz (1980). The model underlying these descriptions can be characterized as a disequilibrium model and hence does not fall into the same category as the equilibrium models discussed in this paper. Nonetheless it is possible to capture important elements of the Canadian structure by adding one equation to the set of equations used in earlier models.

The flavour of the type of analysis of the Canadian system coming out of the Bank of Canada is suggested by the following quotation from Clinton and Lynch (1979):

> "Suppose for example the Bank were to embark upon a more expansionary policy. Initially, the chartered banks would be confronted with an excess supply of cash reserves. In their efforts to eliminate the excess they would buy assets, causing interest rates to decline and the money supply to increase, just as in the familiar textbook credit multiplier. However, because of the lagged reserve requirement, expansion of the banking system does not bring about a reduction in excess reserves. Thus, there is no definite limit on the expansion of the system that will follow from a given increase in excess reserves. As long as an excess supply remains in the system a disequilibrium persists and the banks continue to expand. Analytically the problem is that if the demand for excess reserves is not a function of the level of this month's deposits or interest rates then the demand for total reserves is a predetermined function of lagged deposits, and the supply of reserves is given by monetary policy. Equilibrium thus requires the mutual coincidence of two predetermined variables and the system is overdetermined. In practice the process is typically brought to a halt not by a self-equilibrating market mechanism but by the central bank itself withdrawing the excess, having achieved its desired effect on short-term interest rates or some other proximate target. The point to note is that at the end of the month the level of bank reserves will not necessarily indicate an expansionary policy."

It is of interest to try to describe this type of behaviour in the context of the types of financial models developed earlier in this paper.

(43) $RR(t) = dDD(t-1)$

(44) $RT(t) = RNB(t)$

(45) $RT(t) = RR(t) + RE(t)$

(46) $DD(t) = a - b_0\, i(t) - b_1\, i(t-1) - b_2\, i(t-2) + cY(t)$

(47) $MA(t) = DD(t)$

(48) $RE(t) = p \qquad\qquad 0 < i(t).$

Required reserves are a function of lagged demand deposits under the system of lagged reserve requirements in use in Canada. I ignore for the sake of simplicity the fact that there are reserves against time deposits.[41] Total reserves are equal to non-borrowed reserves and also to the sum of excess reserves and required reserves. Although there is occasional borrowing by the chartered banks from the Bank of Canada, this usually occurs at the end of an averaging period and is often the result of an unexpected clearing swing against a bank on the last day of the averaging period. Without going into detail on this point, I believe that it is fair to characterize the Canadian system as an excess reserves system despite occasional borrowings.[42] Demand deposits are assumed to respond to interest rate movements with a distributed lag. The monetary aggregate is defined in terms of demand deposits. Of course, in Canada, the narrow aggregate which the authorities use as an intermediate target is M1, the sum of currency and demand deposits, but once again for the sake of simplicity, I ignore currency in the discussion that follows. Finally, the excess reserves desired on average by the banking system are equal to a constant, p.[43] In practice, in Canada, the average amount of excess reserves is normally on the order of less than 0.05 of 1 per cent of statutory deposits or about \$60 million.

As can easily be seen by attempting to solve the model, there is no determinate equilibrium for interest rates or the monetary aggregate if one were to treat RNB as the driving variable of the model. To characterize what actually happens in the Canadian system, albeit crudely, one has to add a disequilibrium equation which links the change in interest rates to the difference between the actual amount of excess reserves in the system and the desired holdings of the chartered banks. Thus equation (48) is replaced by the following equation.

$$(49)\quad i(t) - i(t-1) = -s(RE(t) - p)$$

If excess reserves exceed (fall short of) those desired by the banks (i.e, p) interest rates fall (rise). The coefficient s reflects the magnitude of the change in interest rates in each period to the excess or shortfall in excess reserves. Note that if excess reserves are held above the desired level indefinitely interest rates would eventually fall to levels close to zero as in the earlier equilibrium models. It is the fact that it takes time for such a fall to occur that is the crux of the Canadian system since the surplus of excess reserves will be removed by the Bank of Canada when interest rates reach the desired levels, i.e., well before they reach zero.

One can now sketch out the mechanism used by the Canadian authorities to achieve monetary aggregate targets.[44] Suppose the aggregate is initially on target[45] but that nominal income subsequently begins to grow at a faster rate than that consistent with the targets. The level of money demanded will rise above the target as a result of the income increase. The authorities then decide to raise interest rates to the level required to bring money back to target.[46] Because of the lags in the money demand equation and the danger of instrument instability or excessive volatility of interest rates, the increase in interest rate is designed to bring money back to target not immediately but in a reasonable period of time where the latter is determined from the properties of the money demand equation. To achieve this increase in interest rates, the authorities temporarily reduce the amount of reserves to the system, reducing the amount of excess reserves below the desired level, and thus forcing the banks to sell liquid assets and bid aggressively for large blocks of time deposits. Both these actions result in an increase in short-term interest rates. When the desired level of interest rates is reached, the reserves previously removed are returned to the system, the banks no longer are short of funds, and interest rates remain at their new level. This in turn eventually brings money back to its target value.

The movement of total reserves over the period under discussion is largely the result of the movements of other variables and is therefore of little interest to the policymakers. There are four elements affecting the level of reserves in the example. First, as a result of the increase in money demand required reserves rise in the following period and hence total reserves increase. Second, the reduction in excess reserves is mirrored by a decline in total reserves but the amounts involved are very small (i.e., s is large). Third, until the higher interest rates influence the quantity of money demanded total reserves will remain high. Fourth, when money demand falls back to the target required reserves will fall and so will total reserves.

To make these points in a slightly more concrete fashion we can use the system of equations (43) to (47) and (49). Suppose nominal income rises in period t by one unit. The monetary aggregate rises in period t by c units. Suppose the policymakers decide in period (t+1) that the cause of the increase in money demand was a rise in income and it was not simply random noise.[47] If $b_2$ were larger than the sum of $b_0$ and $b_1$ it would not be possible to re-achieve the target money balances in less than three periods without introducing explosive oscillations in interest rate. Hence the authorities would act to increase the interest rate in period (t+1) by $c/(b_0 + b_1 + b_2)$ and hold it there, thereby aiming at bringing money back to target in period (t+4). To achieve the higher interest rate in period (t+1) non-borrowed reserves and hence excess reserves would be reduced by $\frac{1}{s}\left(\frac{c}{b_0+b_1+b_2}\right)$ during period (t+1). Once the interest rate rose to its new level these reserves would be put back into the system.

Using these elements one can calculate the movement in total reserves brought about by the change in income as follows. In period t, there is no change in total reserves since the increase in demand deposits affects required reserves only with a lag. The change in total reserves in period (t+1) is composed of an increase of dc units in required reserves (one unit increase in income results in c units increase in demand deposits which in turn causes an increase of dc units in required reserves). Offsetting this in part is the decline in excess reserves set into motion by the authorities of $\frac{1}{s}\frac{c}{b_0+b_1+b_2}$ units which is needed to raise interest rates. Thus the net increase in total reserves in period (t+1) is $cd - \frac{1}{s}\frac{c}{b_0+b_1+b_2}$. In period (t+2) these excess reserves are put back into the system but required reserves fall because of the effect of the increase in interest rates on money demanded in period (t+1). Hence, compared to the initial position in period t, total reserves are higher by $cd - cd\frac{b_0}{b_0+b_1+b_2}$. In period (t+3) required reserves are pushed down further by the effect of the continuing high interest rates on demand deposits in period (t+2). Thus compared to the initial position total reserves are higher by $cd - cd\frac{b_0+b_1}{b_0+b_1+b_2}$.

Finally, in period (t+4) total reserves are back to their initial equilibrium since money returned to its initial value in period (t+3). As can be seen by this outline the movement of total reserves is very much a resultant of other behavioural actions and hence is not a useful guide to central bank actions.

Clearly the above characterization is very crude. The relationship between excess reserves and interest rates is much more complex than indicated, as can be seen in the two articles cited at the beginning of this section. Nonetheless two principal conclusions can be drawn from the analysis. First, there is a recursive element in the Canadian policy structure which runs from (1) movements of the monetary aggregate, to (2) the desired setting of interest rates intended to re-establish the target levels of the monetary aggregate at some future date, to (3) the temporary setting of excess cash in order to achieve the desired level of interest rate. Second, the movement of total reserves is a combination of the movements in required reserves resulting from the movement in deposits the previous period and of the temporary movements of excess reserves needed to achieve changes in interest rates. In sheer size the former overwhelm the latter[48] and hence total reserves are not a variable which can be used to interpret central bank actions.

## 5. Conclusions

In this paper I have examined some of the implications of the rigid version of base control. It seems clear in the case of those models that incorporate either institutional or economic lags that base control may entail very sharp and possibly explosive oscillations of very short-term interest rates. These in turn are likely to lead to sharp oscillations in longer-term money market and bond market rates. There may be forms of base control which are sufficiently flexible to avoid the problems discussed in this paper. I believe that it is incumbent on the proponents of base control to specify with some precision the kind of less rigid rules for base control that would give sensible results. Furthermore they must consider whether changes in institutional structure would be needed to make their system workable and whether the new rules would be likely to lead to the requisite changes in behaviour by market participants. Finally, they must show that this type of system would perform better than the system currently in place. It is only in the context of a well-defined proposal for base control embedded in a moderately realistic model of the financial system (i.e., one with lags and stochastic error variables) that the debate can proceed.

## NOTES

1. A recent theoretical analysis of these questions of strategy can be found in Freedman (1980).
2. In one sense this formulation of the problem is inaccurate since the authorities operate on reserves (or excess reserves in Canada) to influence interest rates even when the latter are used to affect the monetary aggregate. Nonetheless, in a more basic sense the distinction remains since in a base control system the authorities aim at money directly via the movement of base without using interest rates as a proximate instrument. This does not mean, however, that interest rates play no role in bringing about the movement of money in response to a change in base, as will be seen below.
3. A detailed discussion of the new procedures can be found in Board of Governors of the Federal Reserve System (1980) and Axilrod and Lindsey (1980). The former has been excerpted and discussed in Lang (1980).
4. United Kingdom (1980).
5. Comments by Karl Brunner at Federal Reserve Bank of Boston Conference, October 1980. But see Büttler et al (1979) in which base is treated as the instrument for achieving the desired money target.
6. See, for example, Clinton and Lynch (1979) and the references cited therein. More recent studies include Johannes and Rasche (1979) and Büttler et al (1979).
7. The question of the usefulness of the concept of the supply of money will be addressed below.
8. The magnitude of these random movements in the very short run is a significant element in the controversy whether to use base or interest rate control to achieve monetary aggregate targets. If the demand for money were volatile in the very short run but stable over an intermediate run, the Poole analysis would imply use of interest rates in the very short run to control the monetary aggregate in the intermediate run.
9. In the Canadian case this assumption is particularly suspect since there is a very quick linkage from interest rates to exchange rates and then to prices.
10. The price level is thus implicitly assumed to be constant throughout the analysis.
11. This statement is true only for the class of models in which money demand is assumed always to be in (short-run) equilibrium. The class of disequilibrium models permit a distinction between the supply of money and demand for money. However, as suggested below, the concept of the supply of money is not necessarily a useful one.

12. This point becomes more substantive when one reaches the case of lagged reserve accounting where the notion of a money supply based on the reserves of the banking system becomes much more difficult to conceptualize whereas the notions of the demand for and supply of reserves continue to hold without modification. This point will be developed in section 2.4(b) below when lagged reserve accounting is introduced.
13. I ignore the fact that in some jurisdictions the government is the issuer of at least part of the currency (e.g., coin).
14. If the central bank finds it difficult in the short run to track shifts between currency and deposits, a case can be made for focussing on a reserve path rather than a base path. A shift of $1 from demand deposits to currency in the former case leads to a $1 increase in money whereas in the latter it leads to a $9 decline in money. See Board of Governors of the Federal Reserve System (1980) for such a justification of a reserve target rather than a base target.
15. This formulation presupposes zero or at least fixed interest rates on demand deposits and the assumption of gross substitutability.
16. In this formulation income is proxying for both income and wealth, which would be introduced separately in a correctly specified equation. One could also model the ratio of time deposits to demand deposits but the results of this formulation of the problem are less revealing for our purposes.
17. This simple conclusion might have to be modified for a model in which the banks conduct liability side management rather than liquid asset management. Note also that in certain cases reserve requirements are imposed for equity or efficiency reasons that are unrelated to the question of monetary control.
18. The problems created by imposing reserves on deposits that are not part of the target monetary aggregate or by non-uniform reserve requirements are much easier to handle in a system with lagged reserve requirements than in a system with contemporaneous reserve requirements.
19. This result is simply a reflection of the lower elasticity of the broader aggregate with respect to market interest rates when the rate on time deposits moves in line with market rates.
20. The careful reader will note that if the lag structure of the demand for money with respect to income were identical to that with respect to interest rates the oscillation of interest rates in response to an increase in income would disappear in some of the models of this section. If the mean lag on income were shorter than that on interest rates, a result found in some demand for money equations, the oscillation results would continue to hold however. Perhaps more important, an increase in the price level in those models of money demand in which prices affect money with no lag, or a movement in the error term in the demand for money equation will give rise to the types of oscillations described. Hence the result is more general than indicated in the text.

21. In making this assertion I have implicitly defined the time period in which the analysis is cast as the period over which proponents of base control wish to achieve control over the money supply. This period is sufficiently short that the statement in the text holds.
22. This argument and the corresponding ones in the rest of this section of the paper are closely related to the notion of instrument instability in Holbrook (1972). The fact that the authorities are using reserves rather than interest rates to control money does not cause the problem of potentially explosive oscillations of the interest rate to disappear. It simply reappears in a slightly different guise. Discussion of the possibility that interest rate movements could be explosive can also be found in Pierce and Thomson (1972), Ciccolo (1974) and Lombra and Struble (1979).
23. The central bank has some influence over the amount of borrowing through its control over the discount rate and through its administration of the discount window.
24. In a more complex model one would use the federal funds rate in the borrowed reserves equation and, say, a 90-day rate in the demand for money equation. This would require the addition of another equation linking the 90-day rate to the federal funds rate.
25. The coefficient q is at least partly related to the way in which the discount window is administered. Thus the "reluctance" of banks to borrow from the central bank can be influenced by the behaviour of the central bank. One could also add a constant to equation (30) to represent a certain small amount of borrowing that takes place even when the interest rate falls below the discount rate but this would not change the analysis.
26. A slightly less stringent set of assumptions that would lead to virtually the same results would involve replacing the right hand side of equation (31) by $p - ni(t)$ where n is a small number.
27. Although the behavioural equation from the point of view of the banks treats borrowed reserves as a function of the level of the interest rate (relative to the discount rate), from the point of view of the system as a whole it is preferable to think of borrowed reserves as the given factor and the interest rates as endogenously determined. With the central bank setting non-borrowed reserves, required reserves a function of income and mainly lagged interest rates, and a constant level of excess reserves, borrowed reserves are almost a residual of the system. Thus in the analysis that follows I will consider separately the case in which banks have positive borrowed reserves and that in which the movements of non-borrowed reserves and required reserves result in the banks having no borrowed reserves on their balance sheet, i.e., an excess reserves case.

28. Recall that I am assuming that there is no effect on income of changes in interest rates in the period under study. Hence the notion of equilibrium must be interpreted as a temporary position before the influence of financial variables on the real economy begins to take effect.
29. Eventually, of course, the higher interest rates will lead to a slowdown in income growth which will bring the monetary aggregate back to its target.
30. In the simple model which we are using, if the authorities were to float the discount rate on market rates, then (32) and (33) basically collapse to equations (26) and (27) and the monetary aggregate is always on target. That is, the discount window becomes irrelevant if the margin between the market rate and discount rate never moves. Recall, however, that the oscillation of interest rates may well be explosive in equation (27).
31. In this model, floating the interest rate on the discount rate leads to an indeterminacy of the interest rate, as can be seen from equation (36). The reason is that with a floating discount rate there is no way to equilibrate a shortage of reserves caused by an increase in money demanded since neither the public nor the banks respond to a movement in current rates.
32. If one thinks of excess reserves as constant at some frictional level, total reserves will follow required reserves which, in turn, are a function of the monetary aggregate. If the monetary aggregate is not controllable, neither are total reserves.
33. Floating the discount rate will make no difference to the outcome since interest rates must fall sufficiently to induce the banks to hold the excess reserves and the discount rate is not an argument in the demand function for excess reserves.
34. In principle the central bank could prevent the sawtooth in money and interest rates from arising by offsetting the increase in income by the appropriate decline in non-borrowed reserves. This, however, requires perfect information regarding the movement of income.
35. Even if excess reserves moved slightly in response to changes in interest rates the result would be only slightly modified.
36. One can easily combine the one-period lag in the demand for money and lagged reserve accounting. The resulting interest rate pattern is one in which interest rates oscillate every second period. More complicated models with n period lags can be constructed by the interested reader. The resulting nth order difference equations may be difficult to solve but are very likely to give cyclical movements which may well be undamped.
37. Federal Reserve Bulletin, October 1979, page 830.
38. Board of Governors of the Federal Reserve System (1980). This paper has been discussed in Lang (1980).

39. Note that it is much easier to deal with these problems in a system with lagged reserve accounting than in one with contemporaneous reserve accounting.

40. Adding an expenditure sector and a price sector to the model may, under certain circumstances, strengthen the conclusion in the text regarding an unstable cyclical response. Suppose, for example, that the authorities are following a short horizon policy which has caused interest rates to increase sharply in response to an increase in the monetary aggregate which resulted from an increase in the rate of growth of nominal income. It may be the case that at the same time as the lagged effects of the high interest rates drive the monetary aggregate below its target they also slow down the growth of real income and perhaps prices and, hence, intensify the downward movements of the monetary aggregate. Thus the possibility of explosive oscillations in the monetary aggregate and interest rates may be increased by introducing the interrelationship of the financial sector and the real and price sectors into the model.

41. In any event the Bank of Canada is willing to supply whatever reserves are required to support these time deposits since they are not part of the target monetary aggregate.

42. The purchase and resale arrangements (PRA) between the money market dealers and the Bank of Canada also do not vitiate this judgement.

43. It would make little difference to the analysis if desired excess reserves were weakly related to the level of interest rates.

44. See White (1976) for an early account of this mechanism.

45. For purposes of simplicity I ignore the existence of the band in the analysis that follows.

46. There may be a lag at this point because of the need to decide whether the increase in money was random in which case there should be no interest rate change or was a result of an increase in income.

47. This distinction assumes a random error v also is part of the demand for money equation.

48. This is particularly the case since required reserves also move in response to movements in time deposits and Government of Canada deposits.

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