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Published on: 25 Sep 2015 - Social Science Research Network

Topics: Zero lower bound, Fiscal policy, New Keynesian economics, Nominal interest rate and Monetary policy

Author's Accepted Manuscript

www.elsevier.com/locate/jme

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PII: S0304-3932(16)30114-3
DOI: http://dx.doi.org/10.1016/j.jmoneco.2016.10.012
Reference: MONEC2891

To appear in: *Journal of Monetary Economics*

Received date: 11 November 2013
Revised date: 28 October 2016
Accepted date: 28 October 2016

Cite this article as: Lena Mareen Boneva, R. Anton Braun and Yuichiro Waki, Some unpleasant properties of loglinearized solutions when the nominal rate i z e r o , *Journal of Monetary Economics* http://dx.doi.org/10.1016/j.jmoneco.2016.10.012

Some unpleasant properties of loglinearized solutions when the nominal rate is zero.[☆,☆☆]

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Abstract

Does fiscal policy have large and qualitatively different effects on the economy when the nominal interest rate is zero? An emerging consensus in the New Keynesian (NK) literature is that the answer to this question is yes. Evidence presented here suggests that the NK model's implications for fiscal policy at the zero bound may not be all that different from its implications for policy away from it. For a range of empirically relevant parameterizations, employment increases when the labor tax rate is cut and the government purchase multiplier is less than 1.05.

Keywords: Zero lower bound; Fiscal policy; New Keynesian model
JEL codes: E52; E62

☆This paper previously circulated under the title of "Small and Orthodox Fiscal Multipliers at the Zero Bound."

☆☆We thank Ricardo Reis, the editor, and the referees for their very helpful comments. This paper has also benefited from comments from seminar participants at Australian National University, the Bank of Japan, the Bank of Portugal, the 7th Dynare Conference, the European Central Bank, the Federal Reserve Bank of Atlanta, the University of Tokyo, the University of Groningen, London School of Economics, Vanderbilt University and the Verein for Socialpolitik. We would also like to thank Larry Christiano, Isabel Correira, Luca Fornaro, Pedro Teles and Tao Zha for their helpful comments. Boneva gratefully acknowledges financial support from Cusanuswerk and the Economic and Social Research Council. Part of this research was conducted while Braun and Waki were visitors at the Canon Institute for Global Studies. These views are our own and not those of the Bank of England or the Federal Reserve System.

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1. Introduction

The recent experiences of Japan, the United States and Europe with zero/near-zero nominal interest rates have raised new questions about the conduct of monetary and fiscal policy in a liquidity trap. A large and growing body of new research has emerged that provides answers using New Keynesian (NK) frameworks that explicitly model the zero lower bound (ZLB) on the nominal interest rate. Modeling the ZLB on the nominal interest rate is particularly important in the NK model because the interest rate policy of the monetary authority plays a central role in stabilizing the economy. Very low nominal interest rates constrain the ability of monetary policy to respond to shocks and this may result in macroeconomic instability.

Recent research has found that fiscal policy has very different effects on the economy when the nominal interest rate is zero. Eggertsson (2011) finds that employment falls in response to a cut in the labor tax rate, a property that he refers to as the "paradox of toil." Christiano et al. (2011) and Woodford (2011) conclude that the size of the government purchase multiplier is close to two or even larger. These results have sharp implications for the conduct of fiscal policy in low interest rate environments. If supply-side stimulus is contractionary and demand-side fiscal policies are particularly potent then governments should rely exclusively on demand-side fiscal stimulus when the central bank's actions are constrained by the ZLB.

This paper proposes and solves a tractable stochastic nonlinear NK model that honors the ZLB on the nominal interest rate and that also reproduces the large output and small inflation declines observed during the U.S. Great Recession (GR). We encounter some parameterizations of the model that are consistent with previous results. However, the novel contribution of our paper is that we find other empirically relevant parameterizations of the model where the government purchase multiplier is about one or less and the response of employment to a cut in the labor tax rate is positive.

These new findings are important because they raise the possibility that there might also be a role for using supply-side policies to stabilize the economy in low interest rate environments. On the one hand, the case for demand-side measures is weaker since their efficacy is small. On the other hand, the case for supply-side measures is stronger because they are expansionary.

Why are the results presented here different from previous findings? One reason is the solution method. Previous results are based on a solution method that models the nonlinearity induced by the ZLB on the nominal interest rate but loglinearizes the other equilibrium conditions about a zero inflation steady state. This solution method zeroes out the resource costs of price adjustment which affects the local dynamics of the model at the ZLB. A comparison of loglinear (LL) and nonlinear (NL) solutions reveals that the LL solution sometimes incorrectly predicts that supply-side stimulus is contractionary when in fact it is expansionary.

A second and distinct reason for our findings is the parameterization of the model. The GR was associated with a 7% decline in output but only a 1% decline in the annualized inflation rate (see Christiano et al. 2011). We calibrate the model to these targets and this has implications for the size of the government purchase multiplier. Intuitively, government purchases are primarily a demand shifter and reproducing the GR targets results in a relatively flat aggregate supply schedule. At the ZLB, the government purchase multiplier can still be large in this situation. Indeed, Woodford (2011) has found that the government purchase multiplier can be arbitrarily large in the neighborhood of a point that can be indexed by the expected duration of zero interest rates. This region of the parameter space is small under our calibration scheme. If the expected duration of zero interest rates exceeds seven quarters or is less than five quarters, the government purchase multiplier is small using either the LL or the NL equilibrium conditions.

These points are made in a nonlinear stochastic NK model with Rotemberg (1982) quadratic price adjustment costs. Rotemberg adjustment costs are widely used when studying the ZLB (Benhabib et al. 2001, Evans et al. 2008, Aruoba et al. 2016, Eggertsson et al.

2014) because the dimension of the state space is small. In our setup, output and inflation in the ZLB state solve a system of two nonlinear equations, which are the nonlinear analogues of what Eggertsson and Krugman (2012) refer to as "aggregate demand" and "aggregate supply" schedules. Some merits of our approach are that it provides: a graphical representation of the NL equilibrium conditions; an analytical characterization of the model's key properties; and an easy and accurate strategy for computing all equilibria. This final merit is important because we encounter multiple ZLB equilibria. The LL solution method, in contrast, has the property that aggregate supply and aggregate demand have a single crossing point at the ZLB.

Many NK models use Calvo price adjustment instead. The standard LL solution method also zeroes out the resource costs of price dispersion in Calvo models of price adjustment. It follows that approximation biases can also occur under Calvo price adjustment. This possibility is illustrated with a tractable (but stylized) model of Calvo price adjustment. Thus, our findings may be relevant in a large class of NK models.

Our research is closest to research by Christiano and Eichenbaum (2012) who consider related questions in a similar model. They show that imposing a particular form of E-learnability rules out one of the two equilibria that occur in their model, and find that the qualitative properties of the remaining equilibrium are close to the LL solution. Our main conclusions about the size and sign of fiscal multipliers do not rely on multiplicity of equilibrium at the ZLB. Fiscal multipliers are small and orthodox in regions of the parameter space where equilibrium is unique.

Our research is also related to recent work by Mertens and Ravn (2014) who consider ZLB sunspot equilibria. A major advantage of our setup is that it is straightforward to find all equilibria by finding the zeros of an equation. We encounter new cases of multiplicity, most significantly the possibility of multiple ZLB equilibria. Ascertaining the presence of multiple ZLB equilibria is a daunting task in richer NK models such as those considered by Gust et al. (2016), Aruoba et al. (2016) or Fernandez-Villaverde et al. (2015).

Results presented here offer guidance about the regions of the parameter/shock space where multiplicity is most likely to arise in medium-scale NK models.

The remainder of our analysis proceeds in the following way. Section 2 describes the model and equilibrium concept. Section 3 explains how the model is parameterized. Section 4 characterizes equilibrium using the NL conditions. Section 5 documents that fiscal multipliers may be small and orthodox at the ZLB. Section 6 discusses potential objections to our findings. Finally, Section 7 concludes.

2. Model and equilibrium

We consider a stochastic NK model with Rotemberg (1982) quadratic costs of price adjustment faced by intermediate goods producers. Monetary policy follows a Taylor rule when the nominal interest rate is positive but is restricted from falling below zero. The equilibrium analyzed here is the Markov equilibrium proposed by Eggertsson and Woodford (2003).

2.1. The model

Representative Household. The representative household chooses consumption c_t, labor supply h_t, and bond holdings b_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(\prod_{j=0}^{t} d_j \Big) \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\nu}}{1+\nu} \right\} \tag{1}$$

subject to the budget constraint

$$b_t + c_t = \frac{b_{t-1}(1+R_{t-1})}{1+\pi_t} + (1-\tau_{w,t})w_t h_t + T_t. \tag{2}$$

where ν and σ are the curvature parameters for respectively labor supply and consumption. R_t and π_t are the net nominal interest rate and the net inflation rate, respectively, and the after-tax real wage is $(1-\tau_{w,t})w_t$. The preference discount factor from period t to $t+1$

is βd_{t+1}, and d_t is a preference shock. We assume that the value of d_{t+1} is revealed at the beginning of period t. The variable T_t includes transfers from the government and profit distributions from the intermediate producers. The optimality conditions for consumption and labor supply choices are

$$c_t^{\sigma} h_t^{\nu} = w_t(1 - \tau_{w,t}), \tag{3}$$

and

$$1 = \beta d_{t+1} E_t \left\{ \frac{1 + R_t}{1 + \pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \right\}. \tag{4}$$

Final good producers. Perfectly competitive final good firms use a continuum of intermediate goods $i \in [0, 1]$ to produce a single final good with the technology: $y_t = [\int_0^1 y_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$. The profit maximizing input demands for final goods firms are

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} y_t, \tag{5}$$

where $P_t(i)$ denotes the price of the good produced by firm i and P_t the price of the final good. Thus $\pi_t = P_t/P_{t-1} - 1$. The price of the final good satisfies $P_t = [\int_0^1 P_t(i)^{1-\theta} di]^{1/(1-\theta)}$.

Intermediate goods producers. Intermediate good i is produced according to $y_t(i) = z_t h_t(i)$, where z_t, the state of technology, is common to all producers. Labor is homogeneous and thus real marginal cost for all firms is w_t/z_t. Producer i sets prices to maximize

$$E_0 \sum_{t=0}^{\infty} \lambda_{c,t} \left[(1 + \tau_s) \frac{P_t(i)}{P_t} y_t(i) - \frac{w_t}{z_t} y_t(i) - \frac{\gamma}{2} \Big(\frac{P_t(i)}{P_{t-1}(i)} - 1 \Big)^2 y_t \right] \tag{6}$$

subject to the demand function (5). Producers take the stochastic discount factor, $\lambda_{c,t} \equiv \beta^t (\prod_{j=0}^{t} d_j) c_t^{-\sigma}$, as given. The sales subsidy τ_s satisfies $(1 + \tau_s)(\theta - 1) = \theta$, or that profits are zero in a steady state with zero inflation. The final term in brackets is the cost of price adjustment. We assume it is proportional to aggregate production y_t, so that the share of price adjustment costs in the aggregate production depends only on inflation. The optimality condition for intermediate producers in a symmetric equilibrium with $(P_t(i), y_t(i), h_t(i)) = (P_t, y_t, h_t)$ for all i is

$$\pi_t(1+\pi_t) = \frac{\theta}{\gamma}\left(\frac{w_t}{z_t} - 1\right) + \beta d_{t+1} E_t \left\{ \left(\frac{c_t}{c_{t+1}}\right)^{\sigma} \frac{y_{t+1}}{y_t} \pi_{t+1}(1+\pi_{t+1}) \right\}. \tag{7}$$

Monetary policy. Monetary policy follows a Taylor rule that respects the ZLB on the nominal interest rate:

$$R_t = \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \widehat{gdp}_t), \tag{8}$$

where $r_t^e \equiv 1/(\beta d_{t+1}) - 1$ and $\widehat{gdp}_t$ is the log deviation of GDP from its steady-state value.[1]

The aggregate resource constraint is given by

$$c_t = (1 - \kappa_t - \eta_t) y_t, \tag{9}$$

where $\kappa_t \equiv (\gamma/2)\pi_t^2$ is the resource cost of price adjustment and where government purchases are $g_t = \eta_t y_t$. GDP in our economy, gdp_t, is

$$gdp_t \equiv (1 - \kappa_t) y_t = c_t + g_t. \tag{10}$$

This definition of GDP assumes that the resource costs of price adjustment are intermediate inputs and are consequently subtracted from gross output when calculating GDP. In what follows we will use output to refer to gross output and GDP to refer to consumption plus government purchases.

[1] Our assumption that monetary policy responds directly to variations in d_t and our choice of the functional form of the Taylor rule are made to facilitate comparison with other papers in the literature.

The term κ_t plays a central role in the analysis that follows. Section 4 shows that loglinearizing equation (10) around a zero inflation steady state can result in incorrect inferences about the local dynamics of this economy at the ZLB and relates this result to κ_t. Whenever the inflation rate changes, κ_t also changes and (10) implies that GDP and labor input h_t move differently, possibly even in opposite directions. However, if equation (10) is loglinearized about a zero inflation steady state κ_t disappears and GDP and labor input are identical. A term like κ_t occurs in many NK models. For instance, the resource cost of price dispersion is an analogous term that appears in the resource constraint under Calvo pricing (see Yun 2005). Thus, loglinearizing the resource constraint about a zero inflation rate under Calvo pricing creates the same potential approximation biases. We present results for a model with Calvo price setting in Section 6 that illustrate this point.

2.2. Markov equilibrium with zero interest rates

Following Eggertsson and Woodford (2003), we analyze the ZLB using a two state Markov equilibrium concept. Suppose that s_t denotes the state of the economy which is either low or high, $s_t \in \{L, H\}$, then a *ZLB equilibrium* is an equilibrium with a zero interest rate in state L. The initial state, s_0, is L and s_t evolves under the assumptions that the transition probability from state L to L is $p < 1$ and that H is an absorbing state. All exogenous variables including the preference shock d_{t+1} and the technology shock z_t change if and only if s_t changes: $\{d_{t+1}, z_t\}$ equals $\{d^L, z^L\}$ when $s_t = L$, and $\{1, z\}$ when $s_t = H$.[2]

Under these assumptions, the equilibrium is characterized by two distinct values for prices and quantities. The value in state L is denoted with the superscript L and the value in state H has no superscript. In state H the economy rests in a steady state with a zero inflation rate and a positive nominal interest rate. More formally, $h = \{(1-\tau_w)/(z^{\sigma-1}(1-$

[2]We will treat the fiscal policy variables as parameters in what follows. We fix $\tau_{w,t}$ and η_t at their steady-state values and then consider small perturbations in these parameters in state L when computing the fiscal multipliers. Other restrictions are that $p\beta d^L < 1$, to guarantee that utility is finite, and that prices and quantities are non-negative.

$\eta)^\sigma)\}^{1/(\sigma+\nu)}$ and $\pi = 0$ if $s_t = H$.

2.3. Output and inflation in a ZLB equilibrium

An attractive feature of the model is that the equilibrium conditions for output and inflation (or alternatively employment and inflation) in state L can be summarized by two equations in these two variables. These equations are nonlinear versions of what Eggertsson and Krugman (2012) refer to as "aggregate supply" (AS) and "aggregate demand" (AD) schedules. In what follows, we adopt the same shorthand when referring to these equations.

The AS schedule summarizes intermediate goods firms' price setting decisions, the household's intratemporal first order condition, and the aggregate resource constraint. To obtain the AS schedule, start with (7) and substitute out the real wage using (3). Then use (9) to replace consumption with labor input. In a ZLB Markov equilibrium, the AS schedule in state L is

$$\pi^L(1+\pi^L) = \frac{\theta}{\gamma(1-p\beta d^L)} \left[\frac{\left(1-\kappa^L-\eta\right)^\sigma (h^L)^{\sigma+\nu}}{(1-\tau_w)(z^L)^{1-\sigma}} - 1 \right] \tag{11}$$

where $\kappa^L = (\gamma/2)(\pi^L)^2$.

The AD schedule summarizes the household's Euler equation and the resource constraint. It is obtained by substituting consumption out of the household's intertemporal Euler equation (4) using the resource constraint (9). The resulting AD schedule in a ZLB Markov equilibrium in state L is

$$\frac{h^L}{h} = \frac{z}{z^L} \frac{1-\eta}{1-\kappa^L-\eta} \left(\frac{1-p\beta d^L/(1+\pi^L)}{(1-p)\beta d^L} \right)^{\frac{1}{\sigma}}. \tag{12}$$

3. Parameterization of the model

A principal claim of this paper is that LL solutions of the NK model can break down in empirically relevant situations. This section describes our strategy for producing empirically relevant parameterizations of the model.

Some of our parameters are fixed at common values used elsewhere in the NK literature and others are estimated. Preferences over consumption are assumed to be logarithmic ($\sigma = 1$) because this is a common reference point in the DSGE literature. It is also well known that β is not well identified in DSGE models. Consequently, β is fixed at 0.997 which implies an annual rate of time preference of 1.2% (see also Denes et al. 2013). The parameter θ is set to 7.67, which implies a markup of 15%.[3] This choice of θ lies midway between previous estimates based on industry level and macro data. Broda and Weinstein (2006) find that the median value of θ ranges from 3 to 4.3 using 4-digit industry level data for alternative country pairs. Denes et al. (2013) estimate θ to be about 13 in a NK model that is similar to ours. The government purchase share of output, η, is fixed at 0.2 and the labor tax rate, τ_w, is set to 0.2.

3.1. Parameters that are estimated

Bayesian methods are used to estimate the curvature parameter for labor supply, ν, the price adjustment cost parameter, γ, and the coefficients of the Taylor rule. The data consists of the U.S. inflation rate, the output gap and the Federal Funds rate over a sample period that extends from 1985:I through 2007:IV.

Our estimate of $\nu = 0.37$ with a 90% credible interval of $[0.087, 0.79]$ is typical of values used in the NK literature. Smets and Wouters (2007) estimate a value 0.55, Denes et al. (2013) estimate a value of 0.52, and Gust et al. (2016) assume linear disutility of labor. The posterior means of the Taylor rule parameters are $\phi_\pi = 1.67$ and $\phi_y = 0.31$. These

[3] We found that θ and γ are not individually identified by our estimation procedure. Given the central role played by γ in the NK model, we decided to fix θ and estimate γ.

magnitudes are also representative of other estimates. For instance, Smets and Wouters (2007) estimates are $\phi_\pi = 1.65$ and $\phi_y = 0.31$ respectively. The estimated posterior mean of γ is 495.8 and 90% of its posterior mass lies between 276 and 700. Our posterior mean is larger than Ireland (2003) who estimates a value of 162, and Gust et al. (2016) who estimate $\gamma = 94$. In what follows we will report results for values of γ that range from 100 to 700. A complete description of our estimation strategy and estimates is reported in Section F of the Online Appendix.

3.2. The extent of nominal rigidity

Previous research has found that weak nominal rigidities are associated with large fiscal multipliers at the ZLB (see for instance, Christiano et al. 2011). Section 4 shows that the extent of nominal rigidity is even more important when using the NL equilibrium conditions. Supply-side fiscal stimulus is expansionary in a new region of the parameter space and the size of this region depends on the magnitude of γ.

One way to assess the plausibility of the extent of nominal rigidities in our model is to compare the implied value of the slope of the loglinearized New Keynesian Phillips curve ($slope(NKPC)$) with other estimates from the literature. Schorfheide (2008) surveys previous estimates of $slope(NKPC)$ and finds that the most sharply identified estimates range from 0.01 to 0.03. In our model $slope(NKPC)$ is given by $\theta(\sigma+\nu)/\gamma$. Since γ and ν are estimated their posteriors can be used to derive the posterior for $slope(NKPC)$.[4] Pursuing this strategy results in a posterior mean of $slope(NKPC) = 0.021$, which is in the middle of the range of estimates reported by Schorfheide (2008), and a 90% credible interval of $[0.015, 0.032]$, which is close to his range.

A second way to assess the extent of nominal rigidities is to directly measure the resource cost of price adjustment, which is increasing in γ. In the analysis that follows we target a -1% annual inflation rate in the ZLB state. At this inflation rate the resource cost of price

[4]For consistency these statistics and all subsequent statistics in the paper are computed using the same 10,000 draws from the posterior distribution of $\{\gamma, \nu, \phi_\pi, \phi_y\}$.

adjustment ranges from 0.10% to 0.21% of output using the 90% credible interval for γ and has a posterior mean of 0.15%. Although it is difficult to directly measure the resource cost of price adjustment, Levy et al. (1997) provide a rough idea of the potential magnitude of this cost. They find that menu costs constitute 0.7% of revenues of supermarket chains.

Overall, both the mean extent of nominal rigidities implied by our parameterization and the range of nominal rigidities using the posterior distribution of our estimates are in good accord with the previous literature.[5]

3.3. The amplitude and duration of the ZLB shock

As explained in the introduction a number of recent papers have found that demand-side fiscal stimulus is particularly effective at the ZLB and that supply-side stimulus is contractionary. These results have been used by Eggertsson and Krugman (2012) and others to argue that demand-side fiscal measures are the appropriate way for policy to respond to the GR. In order to assess this policy recommendation it is desirable to discipline the analysis that follows by focusing on parameterizations of the model that capture the main features of the GR. The GR was associated with a large 7% decline in GDP but only a 1% decline in the annualized inflation rate.[6] Suitable adjustments to the shocks to preferences and technology $\{d^L, z^L\}$ insures that each parameterization of the model reproduces these two observations.[7]

This strategy for calibrating the model to reproduce the GR declines in GDP and inflation is rough but has a number of other merits. First, the large output decline in conjunction with the small decline in the inflation rate suggests that the aggregate supply

[5]As pointed out above to facilitate comparison with the previous literature we will sometimes report results for values of γ as low as 100. When $\gamma = 100$, $slope(NKPC) = 0.105$ and the resource costs of price adjustment are 0.03% of output.

[6]These targets are taken from Christiano et al. (2011). They provide empirical evidence that the U.S. financial crisis that ensued after the collapse of Lehman Brothers in the third quarter of 2008 produced a 7% decline in GDP which they measure as the sum of private consumption, investment and government consumption and a 1% decline in the annualized inflation rate.

[7]Section G of the Online Appendix shows that, for a given value of p, there is a unique mapping from these two targets to $\{d^L, z^L\}$.

curve is relatively flat. This has implications for the size of the government purchase multiplier. Second, in nonlinear models the size and sign of the fiscal multipliers depends on the size of the shocks. This calibration scheme provides a way to focus on choices of shocks that are empirically relevant. Third, when multiple ZLB equilibria occur this calibration strategy selects the one that is empirically relevant.

The expected duration of zero interest rates is governed by p, which plays a central role in determining the size and signs of the fiscal multiplier. One way to calibrate p is to use evidence from recent zero interest rate episodes. Interest rates were effectively zero for about seven years in the U.S. starting from the fourth quarter of 2008 and ending in the fourth quarter of 2015. Interest rates have been at zero for long periods in other countries as well. Japan, Switzerland and the U.K. have also experienced long episodes of very low interest rates that date back to late 2008 and Japan had a previous episode of very low interest rates that extended from March 1999 through June 2006. Hence we would like to entertain values of p that are large enough to assign positive mass to ZLB episodes of seven years. To give the reader a point of reference, p must exceed 0.92 if the probability of seven years of zero interest rates is to rise above 0.10. On the other hand, results based on structural models suggest that the expected duration of the ZLB may be much shorter. Recent papers by Gust et al. (2016) or Fernandez-Villaverde et al. (2015) have expected durations of zero interest rates that range from three to five quarters. And, the estimates of Aruoba et al. (2016) imply expected durations of only one quarter in most quarters after 2008 using U.S. data.

It is our view that recent events are more consistent with large values of p and in what follows the baseline value of $p = 0.92$. However, research with medium scale NK models does not rule out the possibility that the expected duration of zero interest rates is moderate or even very short. To entertain these possibilities results are reported for values of p that range from 0.05 to 0.95 or alternatively for expected durations of the ZLB that range from about one quarter to eight years. For each choice of p, $\{d^L, z^L\}$ will be adjusted so that the model reproduces the GDP and inflation targets.

4. Characterization of equilibrium

The NK model has a surprisingly rich set of dynamics at the ZLB. The AD and AS schedules can have conventional local slopes at the ZLB and multiple ZLB equilibria can occur. These properties of the NK model are lost when the standard LL solution is used.

4.1. Two kinds of ZLB equilibria found in the literature

Two types of ZLB equilibria have been the focus of the previous literature. Since both of these cases may also arise using the NL equilibrium conditions, it makes sense to discuss them first.

4.1.1. The Conventional ZLB equilibrium

The ZLB equilibrium that has received the most attention in the literature has the property that the slope of the AD schedule is positive and steeper than the AS schedule: $slope(AD) > slope(AS) > 0$. We refer to this equilibrium as the Conventional ZLB equilibrium. The left panel of Figure 1 illustrates this equilibrium using our baseline parameterization of the model and setting $p = 0.8$. The solid lines show the configuration of the AD and AS schedules in state L with no shocks, L_{ns}, and the dashed lines show the equilibrium in state L with shocks to preferences and technology, L.

The figure has two noteworthy features. First, the AD schedule has a kink. Above the kink the nominal interest rate is positive and the Taylor rule obtains. Below the kink the nominal interest rate is zero. Second, the AD and AS schedules only cross once when the shocks are active. In other words, there is a unique ZLB Markov equilibrium. When the fundamental shocks $\{d^L, z^L\} = \{1, 1\}$, the equilibrium is also unique and corresponds to the zero inflation steady state.

The LL solution is shown in the right panel of Figure 1. The AD schedule also has kink and the LL solution is unique for this configuration of parameters.[8]

Previous research such as Christiano et al. (2011) and Woodford (2011) has pointed out that demand-side fiscal stimulus is particularly effective in a Conventional ZLB equilibrium. The first order effect of an increase in government spending is to shift the AD schedule rightward along an essentially stable AS schedule. Due to the presence of nominal rigidities expected inflation increases. Since the nominal interest rate is constrained by the ZLB, the monetary authority is not able to stabilize expectations for inflation and employment and output increase by more than one-for-one with the increase in the AD.

It follows that the key determinant of the magnitude of the output multiplier is $slope(AS)$. In the neighborhood of $\{\pi^L, h^L\}$ it is given by

$$slope(AS) \equiv \left[\frac{1-p\beta d^L}{slope(NKPC)}\frac{1+2\pi^L}{mc^L} + \frac{\sigma}{\sigma+\nu}\frac{(\kappa^L)'}{1-\kappa^L-\eta}\right]^{-1} \tag{13}$$

where marginal cost in state L is given by $mc^L = \pi^L(1+\pi^L)(1-p\beta d^L)\gamma/\theta + 1$.[9]

Our first result is that the LL solution for the slope of the AS schedule, $slope(AS^{LL}) \equiv slope(NKPC)/(1-p\beta)$, is biased. To illustrate why this is the case observe that the first term in the right-hand side of (13) simplifies to $slope(AS^{LL})^{-1}$ when evaluated at $(d^L, \pi^L) = (1, 0)$. The second term is negative in a deflationary ZLB equilibrium but disappears at $\pi^L = 0$. It reflects the fact that the resource cost of price adjustment constrains the supply of goods when there is deflation. Less deflation creates a positive wealth effect that puts downward pressure on labor supply because fewer resources are absorbed by costly price adjustment. It follows that the approximation bias in the LL solution will be large when the second term in equation (13) is relatively large. This occurs either when p is large and/or when γ is large. Thus, even in the Conventional ZLB

[8] The other noteworthy feature of this figure is that AS^{LL} is independent of d^L and z^L and does not shift. This is due to the fact that σ has been set to 1. See Section C of the Online Appendix for details.

[9] This formula is derived in Section D.1 of the Online Appendix.

equilibrium, using LL solutions may bias inference about the magnitude of government purchase multipliers.

A second property of the Conventional ZLB equilibrium is that supply-side fiscal stimulus is contractionary. A labor tax cut, for instance, shifts the AS schedule to the right along a stable AD schedule. In this equilibrium lower labor taxes reduce inflation and both employment and output fall. Eggertsson (2011) refers to this property of the Conventional ZLB equilibrium as the paradox of toil.

4.1.2. The Sunspot ZLB equilibrium

A second type of ZLB equilibrium has been considered in the previous literature. We refer to it as the Sunspot ZLB equilibrium because there are two equilibria and thus sunspots are possible. These two equilibria are depicted in the left panel of Figure 2 for the case where $p = 0.92$. The ZLB equilibrium has the property that $slope(AS) > slope(AD) > 0$ and the second equilibrium has a positive interest rate. Employment and inflation are depressed in the ZLB equilibrium and the two equilibria also exist when the shocks are set to their steady-state values.[10]

Bullard (2010) and Mertens and Ravn (2014) introduce sunspot variables and explore the possibility that the contractionary ZLB equilibrium is induced by a confidence shock with no change in fundamentals. The ZLB equilibrium with shocks depicted in Figure 2 can also be interpreted as a sunspot equilibrium where the confidence shock is perfectly correlated with fundamentals. We will limit attention to this type of sunspot equilibrium in the analysis that follows. The right panel of Figure 2 shows that the LL solution shares these properties of equilibrium for this configuration of parameters.[11]

Demand-side stimulus, such as an increase in government spending, is less effective in the

[10] When p is set to one and the shocks are set to their steady-state values, the ZLB equilibrium corresponds to the zero interest rate steady state in Benhabib et al. (2001).

[11] Under weak conditions described in Section C of the Online Appendix it can also be shown that there is a single $R = 0$ crossing point of AD^{LL} and AS^{LL}.

Sunspot ZLB equilibrium because the inflation rate falls and this puts downward pressure on employment and output.[12] Supply-side stimulus, in contrast, is expansionary. A lower labor tax shifts the AS schedule right along the AD schedule and inflation, employment and output increase. The magnitude of the employment and output responses to supply-side stimulus depends on the slope of the AD schedule and inferences about this magnitude using the LL solution are biased.

This approximation bias is also due to the resource costs of price adjustment. The slope of the AD schedule in the neighborhood of $\{\pi^L, h^L\}$, which is given by

$$slope(AD) \equiv \left[\frac{1}{\sigma}\frac{p\beta d^L/(1+\pi^L)^2}{1-p\beta d^L/(1+\pi^L)} + \frac{(\kappa^L)'}{1-\kappa^L-\eta}\right]^{-1}, \tag{14}$$

also has two terms.[13] The first term in (14) simplifies to $p/\{\sigma(1-p)\} \equiv slope(AD^{LL})^{-1}$ when evaluated at $(\beta d^L, \pi^L) = (1, 0)$. The second term arises because price adjustment absorbs resources. These costs are negative in a deflationary ZLB equilibrium and act like a leaky bucket driving a wedge between what is produced and what is available for consumption. The parameters p and γ are also important determinants of the magnitude of the approximation bias. The second term is relatively large when p is small and γ is large. Thus, we see that the LL solution introduces biases in both types of equilibria that have been considered by the previous literature.

4.2. New types of ZLB equilibria in the NK model

We now turn to discuss two types of ZLB equilibria that only occur when using the NL equilibrium conditions.

[12] The overall response of output can still be positive in a Sunspot Equilibrium because the AS schedule shifts as well. Section D.2 of the Online Appendix provides the specific condition for when this occurs.

[13] This formula is derived in Section D.1 of the Online Appendix.

4.2.1. The New ZLB equilibrium

Perhaps the most novel result that emerges from analyzing the NL equilibrium conditions is that the AD schedule can be downward sloping at the ZLB. Consider equation (14) and observe that when $p = 0$ the first term in the expression disappears. The second term, however, is negative in any ZLB equilibrium with deflation and it follows that the slope of the AD schedule is also negative. More generally, $slope(AD)$ will be negative when the second term is larger than the first term. This occurs when p is small and when γ is large. The AS schedule is upward sloping when p is small and γ is large and it follows that $slope(AD) < 0 < slope(AS)$ in what we will refer to as a New ZLB equilibrium. Figure 3a shows a New ZLB equilibrium with $p = 0.4$. The ZLB equilibrium depicted in the figure is globally unique.[14] Fiscal policy in a New ZLB equilibrium works in the same way as when the nominal interest rate is positive. A rightward shift of the AD schedule moves the equilibrium along an upward-sloping AS schedule, and employment and output increase less than proportionately. It follows that the government purchase output multiplier is less than one. Second, a labor tax cut shifts the AS schedule down and results in lower inflation and *higher* employment. In other words, there is no paradox of toil in a New ZLB equilibrium.

Inspection of $slope(AD^{LL}) = \sigma(1-p)/p$ reveals that the slope of the loglinearized AD schedule is non-negative. Thus, in any New ZLB equilibrium $slope(AD^{LL})$ has the incorrect sign.

4.2.2. Multiple ZLB equilibria

The second novel property of the NK model is that the AD and AS schedules can have multiple crossing points at the ZLB. We refer to this case as Multiple ZLB equilibria. Figure 3b provides an example of this situation using the baseline parameterization with

[14] More generally, if $slope(AD) < 0 < slope(AS)$ we always find that the ZLB equilibrium is globally unique.

$p = 0.88$. In this example, there are *two ZLB equilibria* and one equilibrium with a positive nominal interest rate.

When the AD and AS schedules have multiple crossing points, the two ZLB equilibria have different local properties. In the example in Figure 3b, the ZLB equilibrium that reproduces the GR targets (targeted equilibrium) exhibits $slope(AS) > slope(AD) > 0$ and the other ZLB equilibrium (non-targeted equilibrium) exhibits $0 > slope(AD) > slope(AS)$. The second equilibrium in this example is not very interesting because the inflation rate and decline in GDP are implausibly small at -16.8% per annum and -14.8% respectively. However, if p is set to 0.862 instead, the inflation rate in the non-targeted ZLB equilibrium is also -1% and GDP declines by -7.02%. These magnitudes are virtually indiscernible from the targeted equilibrium which reproduces an inflation rate of -1% and a 7% decline in GDP. Yet the two equilibria have different implications for fiscal policy. The targeted equilibrium satisfies $slope(AS) > slope(AD) > 0$ and thus has the same local dynamics as the Sunspot ZLB equilibrium. Whereas, the non-targeted equilibrium satisfies $slope(AD) > slope(AS) > 0$ and its local dynamics are the same as the Conventional ZLB equilibrium.

These two examples illustrate that the properties of this class of equilibrium can be very sensitive to the particular choice of p. Increasing p by 0.018 results in equilibria with different local properties. The reason why the dynamics are so complicated for p in this interval is because it includes the bifurcation point where $slope(AD) = slope(AS)$.

The LL solution, in contrast, cannot produce multiple ZLB equilibria because AD^{LL} and AS^{LL} are linear at the ZLB. This results in a type of classification error. In the best case the LL solution will exhibit the same local dynamics as the targeted equilibrium. However, it is not difficult to find examples where this does not occur.

The LL solution can break down in another way. If p is sufficiently large $slope(AS)$ can be negative whereas $slope(AS^{LL})$ is always positive. In the results that follow a downward sloping AS schedule only occurs for values of p that are well above the maximum threshold of 0.95. So this case does not arise in the analysis that follows.[15]

4.3. When is the economy in which case?

The parameters γ and p play central roles in determining the case of equilibrium as is illustrated in Figure 4a. This figure reports results for $p \in [0.05, 0.95]$ and $\gamma \in [100, 700]$. The posterior mean of $\gamma = 495.8$ and the baseline value of $p = 0.92$ are denoted with black lines. However, it is worth emphasizing that each equilibrium depicted in the figure reproduces the GR declines in output and inflation and has small costs of price adjustment. Moreover, the implied value of $slope(NKPC)$ is also reasonable when $\gamma \in [276, 700]$ for the reasons given in Section 3.2.

Consider first what happens as p is varied holding γ fixed at its baseline value. Figure 4a indicates that the Sunspot ZLB equilibrium occurs when $p \geq 0.92$. As p is reduced from 0.92 the AD schedule rotates to the left and the AS to the right and the economy falls in a region with Multiple ZLB equilibria. This case obtains for $p \in [0.855, 0.905]$ and contains the bifurcation. The targeted ZLB equilibrium satisfies $slope(AD) < slope(AS)$ to the right of it and $slope(AD) > slope(AS)$ to the left of it. For $p \in [0.595, 0.85]$ the Conventional ZLB equilibrium obtains that has been the focus of the previous literature. Finally, the New ZLB equilibrium obtains when $p < 0.595$.

Figure 4a also shows how the size of the various regions varies with the magnitude of γ. As γ is increased from its posterior mean, the extent of nominal rigidities increases ($slope(NKPC)$ decreases) and the size of the region where the Sunspot ZLB equilibrium occurs shrinks. However, even at $\gamma = 700$, the Conventional ZLB does not obtain for p greater than or equal to our baseline value of 0.92. Increasing γ from its posterior mean also increases the size of the resource costs of price adjustment and this acts to increase the size of the region where the New ZLB equilibrium occurs. For instance, when $\gamma = 700$ a New ZLB equilibrium occurs for all $p \leq 0.675$.

[15] Section A.1 of the Online Appendix provides an example where this type of equilibrium occurs at smaller values of p.

Figure 4b depicts when each type of equilibrium occurs if one uses the LL solution instead. At the baseline value of γ, the LL solution misclassifies New ZLB equilibria as Conventional ZLB equilibria when $p < 0.6$. It also misclassifies most of the Multiple ZLB equilibria as Sunspot ZLB equilibria.

Next we use a statistical approach to evaluate the relative likelihood of the four cases at alternative values of p. We draw 10,000 realizations from the posterior joint distribution of $\{\gamma, \nu, \phi_\pi, \phi_y\}$, solve the model for each draw and each value of p, and determine which of the four cases obtains. The resulting frequencies of each type of equilibrium are reported in Figure 5. When $p \geq 0.925$ the posterior distribution assigns over 90% of its mass to the Sunspot ZLB equilibrium. For $p \in [0.665, 0.815]$ the posterior assigns more than 90% of its mass to the Conventional ZLB equilibrium and for all values of $p \leq 0.49$, the posterior distribution assigns over 90% of its mass to the New ZLB equilibrium. For the remaining values of p the posterior assigns more than 10% mass to two or more classes of equilibrium. The region with Multiple ZLB equilibria falls into this final category. At our baseline parameterization of the model, the Sunspot ZLB equilibrium is most likely according to the posterior distribution. Still, none of the four cases of equilibria can be ruled out because each case can reproduce the GR observations with small resource costs and moderate amounts of nominal rigidities for some choices of p.

5. Small and orthodox fiscal multipliers at the ZLB

These results have substantive implications for the conduct of fiscal policy at the zero bound. Perhaps the most striking result is that the NK model can be used to make the case for the use of supply-side fiscal stimulus in a low interest rate environment. Supply-side stimulus such as a labor tax cut increases employment for some empirically relevant parameterizations of the NK model. These same parameterizations have the property that demand-side fiscal stimulus is weak. In particular, the government purchase GDP multiplier is close to or even less than one. In the course of making these points we illustrate how the approximation biases in the LL solution documented in Section 4 affect the sign and magnitudes of the labor tax and government purchase fiscal multipliers.

In what follows the fiscal multipliers are computed by the deriving the responses of the endogenous variables to a small perturbation in either τ_w or η in state L. Analytical formulas for the two fiscal multipliers are displayed in Section D.2 of the Online Appendix.

5.1. Labor tax multiplier

The labor tax multiplier reported here measures the percentage response of employment when the labor tax is *raised* by one percentage point. For instance, a labor tax multiplier of -1 means that employment falls by one percent when the labor tax is increased from 0.2 to 0.21.

Figure 6a shows that the posterior odds of a paradox of toil are very small for a large set of plausible ZLB equilibria. The figure reports the median labor tax multiplier as well as the 10th percentile and the 90th percentile for each value of p.[16] Observe that the 90th percentile value of the labor tax multiplier is negative at the baseline value of $p = 0.92$ and all p above it. More generally, the posterior odds of a paradox of toil are less than 0.10 for all $p \geq 0.885$ and also all $p \leq 0.49$. One cannot rule out the possibility of a paradox of toil though. For $p \in [0.665, 0.83]$, the 10th percentile of the labor tax multiplier is positive.

Why are the posterior odds of the paradox of toil low for low p and high p and why are they large for $p \in [0.665, 0.83]$? Not surprisingly, these are essentially the same regions were the odds are high that the equilibrium is of a particular type. For instance, the New ZLB equilibrium also has posterior odds of more than 0.90 when $p \leq 0.49$.

Figure 6a also shows when the labor tax multiplier is large and when it is small. It is clear from the figure that it can be arbitrarily large in the neighborhood of the bifurcation.

[16]The credibility intervals were produced using the same method used to produce Figure 5. We report medians instead of means or modes because the median is the most stable measure of central tendency in the neighborhood of the bifurcation. In situations with multiple ZLB equilibria we only tabulate the targeted equilibrium.

To the right of the bifurcation the multiplier is large and negative. To the left of the bifurcation, however, it quickly becomes small. The median labor tax multiplier falls below one at $p = 0.825$ and it is less than 0.05 in absolute value for all $p \leq 0.675$.

5.2. Government purchase multiplier

Figure 6b reports the median and the 10th and 90th percentiles of the posterior distribution of government purchase GDP multipliers. The median government purchase multiplier is positive but less than one when $p \geq 0.92$.[17] In fact, the 90th percentile is also less than one for $p \geq 0.885$. This is the same interval where the posterior assigns most mass to a large and negative labor tax multiplier.

Recall that the posterior odds of a positive labor tax multiplier were very small when $p \leq 0.49$. In this interval the median value of the government purchase multiplier is less than 1.009 and the 90th percentile is less than 1.018.

A comparison with Figure 5 indicates that the government purchase multiplier is also small in regions of the parameter space where we are highly confident that the equilibrium is a Conventional ZLB equilibrium. The posterior assigns more than 90% of its mass to a Conventional ZLB when $p \in [0.665, 0.815]$, but the government purchase multiplier is less than 1.05 when $p \leq 0.78$.

Thus, we see that there are two rather large regions of the parameter space as indexed by p where the government purchase multiplier is less than 1.05 and in a large subset of these intervals the labor tax multiplier is also negative. Moreover, based on the reasoning provided in Section 3, all of these parameterizations are empirically relevant.

It should be emphasized that these results do not rule out the possibilities of large government purchase multipliers and positive labor tax multipliers. The median government purchase multiplier exceeds 1.2 and the median labor tax multiplier is positive when $p \in [0.815, 0.86]$. It is worth noting though that there is massive amounts of uncertainty

[17] The government purchase multiplier can be negative but the region where this occurs is very small. See Section D.2 of the Online Appendix for the specific condition.

about the magnitude of the government purchase multiplier in this region. The 10th percentile for the government purchase multiplier never exceeds 1.09 for $p \in [0.815, 0.83]$ and then is negative for $p \in [0.845, 0.86]$ while the 90th percentile increases monotonically from 1.72 to 4.07 when $p \in [0.815, 0.86]$. The reason for this uncertainty is that the dynamics bifurcate in this region of the parameter space and small variations in the configuration of parameters shift the location of the bifurcation.

5.3. Comparison with previous estimates based on LL

The previous literature has reached different conclusions from ours. It has concluded that supply-side fiscal stimulus is contractionary at the ZLB and has found that the government purchase multiplier is large. How do we reconcile our results with previous results?

One reason previous results about the properties of fiscal stimulus at the ZLB are at odds with our results is that the previous literature has relied on LL solutions. The breakdown of the LL solution is largest in the case of the labor tax multiplier as shown in Figures 7a and 7c. The NL solution has the property that the labor tax multiplier is negative in the leftmost region while the LL solution implies that the labor tax multiplier is positive. This distinction between the NL solution and the LL solutions arises because this is the region where a New ZLB equilibrium occurs. Even when the two solutions have the same sign, the LL solution exhibits an upward bias in the size of the labor tax multiplier. Consider, for instance, the region with a Conventional ZLB equilibrium. Using the baseline value of γ the LL solution overstates the true multiplier by 0.07 when $p = 0.7$ and by 0.19 when $p = 0.8$. These approximation biases are large given that the overall size of the NL labor tax multiplier is 0.08 at $p = 0.7$ and 0.47 at $p = 0.8$.

We also find that the government purchase multiplier is small in large regions of the parameter space whereas the previous literature has concluded that it is large. This difference is not due to the solution as can been seen by comparing Figure 7b with Figure 7d. The two solutions produce a similar pattern of government purchase multipliers everywhere except for a small range of p's that are close to the asymptote. Instead our finding is due to the fact that we entertain a large range of values of p. The expected duration of zero interest rates is crucial for the size of the government purchase multiplier. This point is easiest to see if one considers the values of p used in previous research. Christiano and Eichenbaum (2012) set $p = 0.775$ and Denes et al. (2013) use a value of $p = 0.856$. Their models are different from ours, but it is clear from Figure 6b that in our model the government purchase multiplier can also be large for p's of these magnitudes. Given the size of the credibility intervals though, it is impossible to say whether the government purchase multiplier is very large, very small or even large and negative in this region of the parameter space.

6. Objections and responses

We have found that it can matter how one solves the NK model at the ZLB. Using the NL equilibrium conditions the NK model exhibits four distinct types of ZLB equilibria when calibrated to the GR and the LL solution cannot discern two of them. Fiscal multipliers are small and orthodox over a large and plausible set of configurations of the model's parameters. Supply-side fiscal stimulus may be expansionary at the ZLB and the government purchase multiplier may be close to or even less than one. We now turn to discuss potential objections to these findings.

6.1. Calvo price adjustment

Our analysis has used Rotemberg price setting. We believe that our finding that the LL solution fails at the ZLB is not specific to the form of costly price adjustment. As described above price dispersion using Calvo price setting also reduces the resources that are available for private and public consumption. In particular, if Calvo price setting is used instead the term κ in the resource constraint becomes $\kappa_t \equiv (x_t - 1)/x_t$ where x_t summarizes the relative price dispersion described in Yun (2005).

Unfortunately, x_t is an endogenous state variable and computing ZLB equilibria in this situation is considerably more complicated. To give the reader an indication about what might happen under Calvo price adjustment Section B of the Online Appendix derives results for a stylized but tractable model with Calvo price adjustment. In this model x_t is only allowed to take on two distinct values: $x_t = x^L$ in state L and $x_t = 1$ in state H. This assumption is valid if the LL solution is accurate because x_t is constant at one when loglinearized around the zero inflation steady state.

Figure 8 compares the AD and AS schedules under this version of Calvo pricing with the baseline model using a value of $p = 0.4$. The figure shows that the two models of price adjustment are almost indistinguishable in the neighborhood of the equilibrium. In particular, the equilibrium using Calvo pricing is also a New ZLB equilibrium. Section B of the online Appendix illustrates that this version of the Calvo pricing also has very similar properties to Rotemberg at higher values of p. For instance, the equilibrium is also a Conventional ZLB equilibrium when $p = 0.8$ and a Sunspot ZLB equilibrium when $p = 0.9$.[18]

6.2. Size of the shocks

We have chosen the shocks so that the model reproduces the GR declines in output and inflation for each choice of p. The size and signs of the resulting shocks may have implications for the plausibility of a particular region of the parameter space. Figure 9 displays the values of d^L and z^L for the baseline parameterization of the model.

Notice that low values of p that are around 0.4 or less require positive technology shocks and large values of d^L to reproduce the GR targets. A positive technology shock does not play a central role in our findings. Section A.2 of the Online Appendix repeats our analysis, holding technology fixed and varies θ instead. The size of the regions with small and orthodox fiscal multipliers increases.

A second property of Figure 9 is that a very large value of the preference shock is required

[18] To draw the AS and AD schedules with Calvo pricing, the probability that a firm is unable to change its price (α) is calibrated such that $slope(NKPC)$ is the same as in the Rotemberg model with our baseline parameterization. The implied value is $\alpha = 0.88$. In the Rotemberg model, $slope(NKPC) = \theta(\sigma + \nu)/\gamma$ while in the Calvo model $slope(NKPC) = (1 - \alpha)(1 - \beta\alpha)(\sigma + \nu)/\alpha$.

to reproduce a 7% decline in GDP when the expected duration of state L is very short. For instance, at $p = 0.4$ a value of $d^L = 1.0445$ is needed using the baseline parameterization. This corresponds to a 19% annualized increase in the preference discount factor. The simple NK model considered here is not rich enough to provide a comprehensive explanation for what caused the GR and how long people expected the nominal rate to be zero. But, it is not surprising that a big shock is needed to produce a large decline in output when the expected duration of the ZLB shock is only about 2.5 quarters.

6.3. *Other parameterizations*

The analysis limited attention to preferences with log utility over consumption. A higher value of σ reduces the size of the region where a Conventional ZLB equilibrium occurs and increases the size of the region with a New ZLB equilibrium. To understand why this occurs suppose that the economy is in a Conventional ZLB equilibrium so that $slope(AD) > slope(AS) > 0$. Then note from equation (14) that increasing σ acts to rotate $slope(AD)$ to the left until it eventually turns negative. Thus, when σ is increased labor supply responses to supply-side stimulus are orthodox and government purchase multipliers are small in a larger region of the parameter space.[19]

It is also possible that values of other parameters that were held fixed matter. In Section A of the Online Appendix we report additional results that are designed to address this concern. Section A.3 reports results using parameterizations along the lines of Christiano and Eichenbaum (2012). Their parameterization implies a much larger value of $slope(NKPC)$ compared to what we have considered here. Section A.4 reports results using parameterizations along the lines of Denes et al. (2013). Their estimates of θ and ν are much larger than ours and yet their value of $slope(NKPC)$ is very small. We continue to find regions of the parameter space that have small government purchase multipliers and negative labor tax multipliers using either of these alternative parameterizations. Perhaps

[19] For instance, setting $\sigma = 2$ and using the baseline value of γ there is a ZLB equilibrium with $slope(AD) < 0$ for all $p \leq 0.725$. See Section A.1 of the Online Appendix for more details.

the most important new finding is that multiple ZLB equilibria occur can in much larger regions of the parameter space and, in particular, in regions where p is very small (see Figures 8 and 11 of the Online Appendix).

Our finding that the LL solution works well when computing the government purchase multiplier depends on the size of the shocks. The LL solution exhibits much larger approximation biases if the shocks are calibrated to observations from the Great Depression instead (see Section A.5 in the Online Appendix for more details).

6.4. Equilibrium selection

In situations with multiple equilibria, we have adopted the convention of reporting the targeted ZLB equilibrium that reproduces the GR calibration target. Is this a reasonable way to proceed? Christiano and Eichenbaum (2012) propose using an E-learning criterion to rule out multiple equilibria instead.

Our basic conclusions about the possibility of small and orthodox fiscal multipliers continue to obtain when one uses the Christiano and Eichenbaum (2012) E-learning equilibrium selection criterion. The size of the government purchase multiplier can be small even if one restricts attention to the Conventional ZLB equilibrium which is E-stable. For instance, it is less than 1.1 for $p \in [0.6, 0.8]$ for the baseline parameterization. The New ZLB equilibrium is also E-stable. Thus, we continue to find that employment may increase when the labor tax is cut at the ZLB.

Our calibration strategy can be interpreted as an equilibrium selection criterion and there are some important differences between the two criteria. Our strategy always selects a single equilibrium. However, the E-learning criterion sometimes selects two equilibria. For instance, our model has three equilibria and two of these equilibria are E-stable using our baseline parameterization with $p \in [0.855, 0.89]$.

A second difference is that our criterion always selects an empirically relevant equilibrium. The E-learning criterion, in contrast, sometimes selects the non-targeted ZLB equilibrium and rejects the targeted ZLB equilibrium. For instance, when $p = 0.89$, the E-stable ZLB equilibrium produces a 29% decline in GDP and an annualized rate of deflation of 16%.

Finally, applying the E-learning criterion to our model rules out *all* ZLB equilibria that assign more than 4.5% probability to an episode of zero interest rates that lasts seven years. Episodes of zero interest rates observed in the U.S. and other countries such as Japan, Switzerland and the U.K. are then a puzzle from the perspective of the NK model under E-learning.

7. Conclusion

How one solves the NK model matters when modeling the ZLB. We have documented a range of approximation biases in LL solutions. These biases can distort inferences about the effects of fiscal stimulus at the ZLB. Some parameterizations of our model support the conventional view that supply-side fiscal policies should be avoided in low interest rate environments and that demand-side policies should be relied on instead. However, two of the principal arguments underlying this contention (labor tax cuts are contractionary and the government purchase multiplier is large) are not robust. Other empirically relevant parameterizations of the same NK model have much smaller government purchase multipliers and also provide a rationale for supply-side measures such as a labor tax cut. Solving the NK model using nonlinear methods plays an essential role in reaching these conclusions.

Our message that supply-side stimulus can be expansionary at the ZLB has implications for other supply shocks. For instance, Christiano et al. (2011) find that the response of output to an improvement in technology is contractionary at the ZLB. This finding runs counter to empirical evidence in Wieland (2015) that suggests improvements in technology are also expansionary at the ZLB. In our model positive technology shocks are expansionary in New and Sunspot ZLB equilibria when $\sigma = 1$.[20]

[20] See Section D.2 of the Online Appendix for details.

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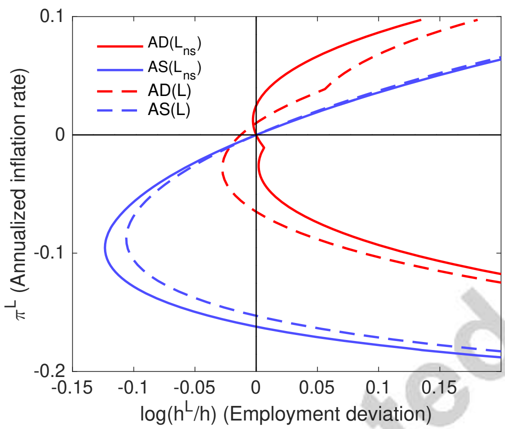
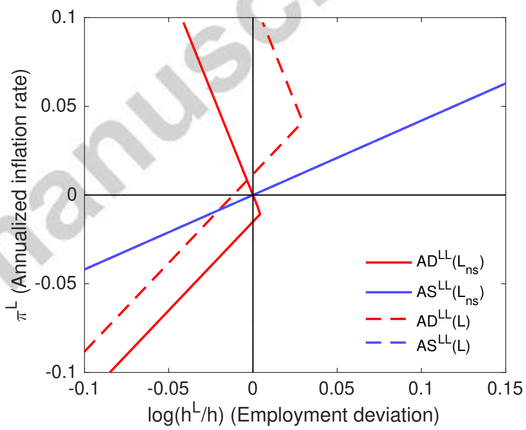
List of Figures

FIGURE 1: THE CONVENTIONAL ZLB EQUILIBRIUM.



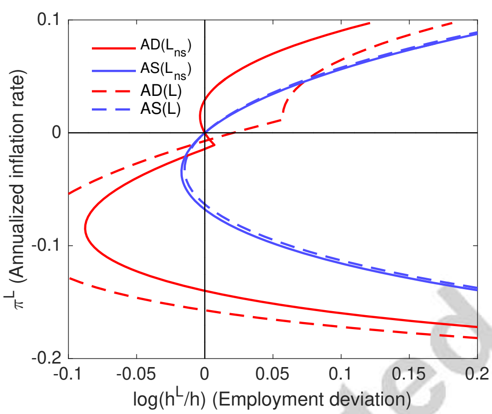
(A) NL EQUILIBRIUM



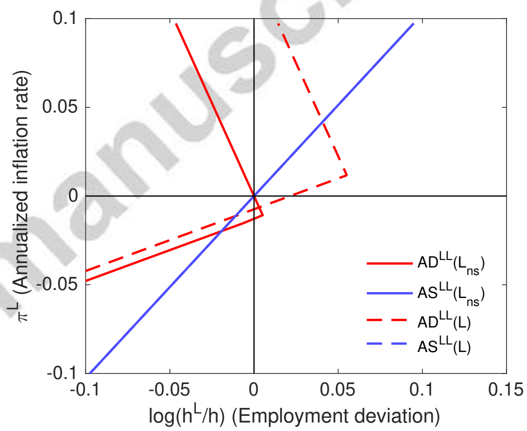
(B) LL EQUILIBRIUM

Notes: The plots are based on our baseline parameterization with $p = 0.8$. The schedules labeled L_{ns} set all shocks to their steady-state values. The schedules labeled L use shocks $\widehat{d}^L$ and $\widehat{z}^L$ that reproduce our GR targets for GDP and inflation using the nonlinear equilibrium conditions. The loglinearized AS schedule is the same in states L and L_{ns} because $\sigma = 1$.

Figure 2: The Sunspot ZLB equilibrium.



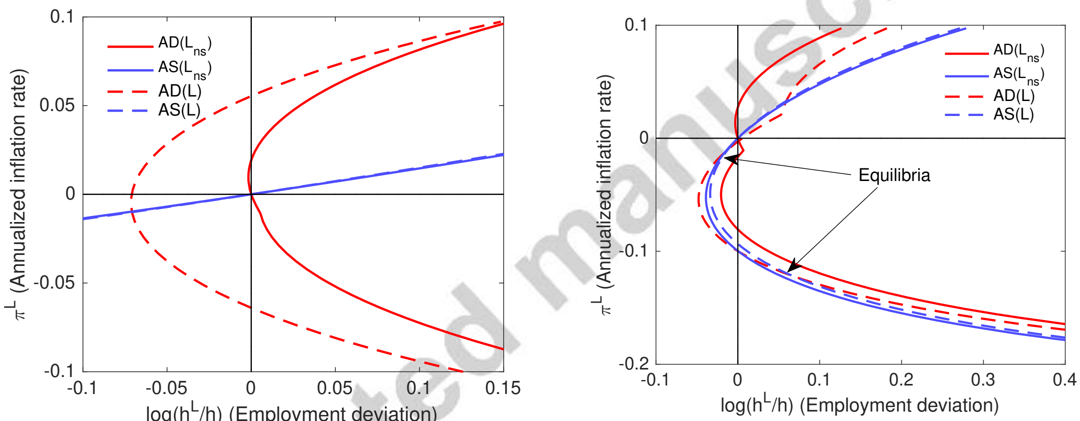
(a) NL Equilibrium



(b) LL Equilibrium

Notes: The plots are based on our baseline parameterization with $p = 0.92$. The schedules labeled L_{ns} set all shocks to their steady-state values. The schedules labeled L use shocks $\widehat{d}^L$ and $\widehat{z}^L$ that reproduce our GR targets for GDP and inflation using the nonlinear equilibrium conditions. The loglinearized AS schedule is the same in states L and L_{ns} because $\sigma = 1$.

FIGURE 3: NEW TYPES OF ZLB EQUILIBRIA

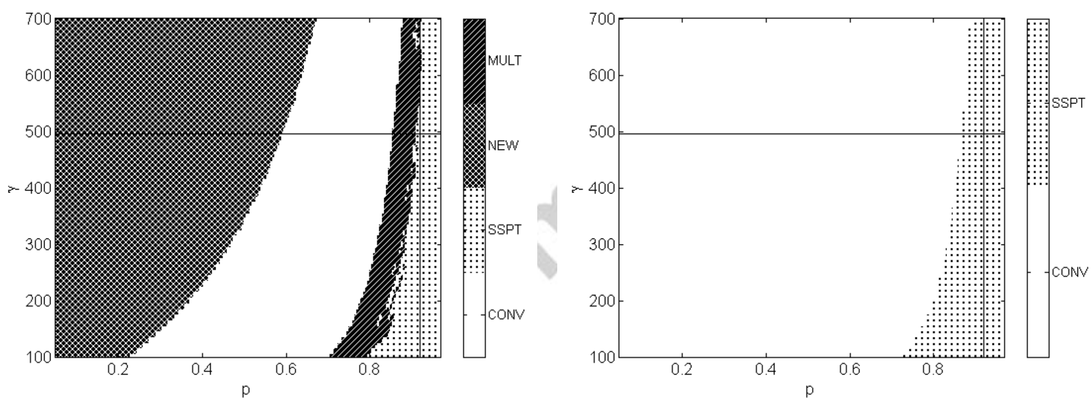


(A) NEW ZLB EQUILIBRIUM: $slope(AD) < 0 < slope(AS)$, $(p = 0.4)$

(B) MULTIPLE ZLB EQUILIBRIA, $(p = 0.88)$

Notes: The plots are based on our baseline parameterization. The schedules labeled L_{ns} set all shocks to their steady-state values. The schedules labeled L use shocks $\widehat{d}^L$ and $\widehat{z}^L$ that reproduce our GR targets for GDP and inflation using the nonlinear equilibrium conditions.

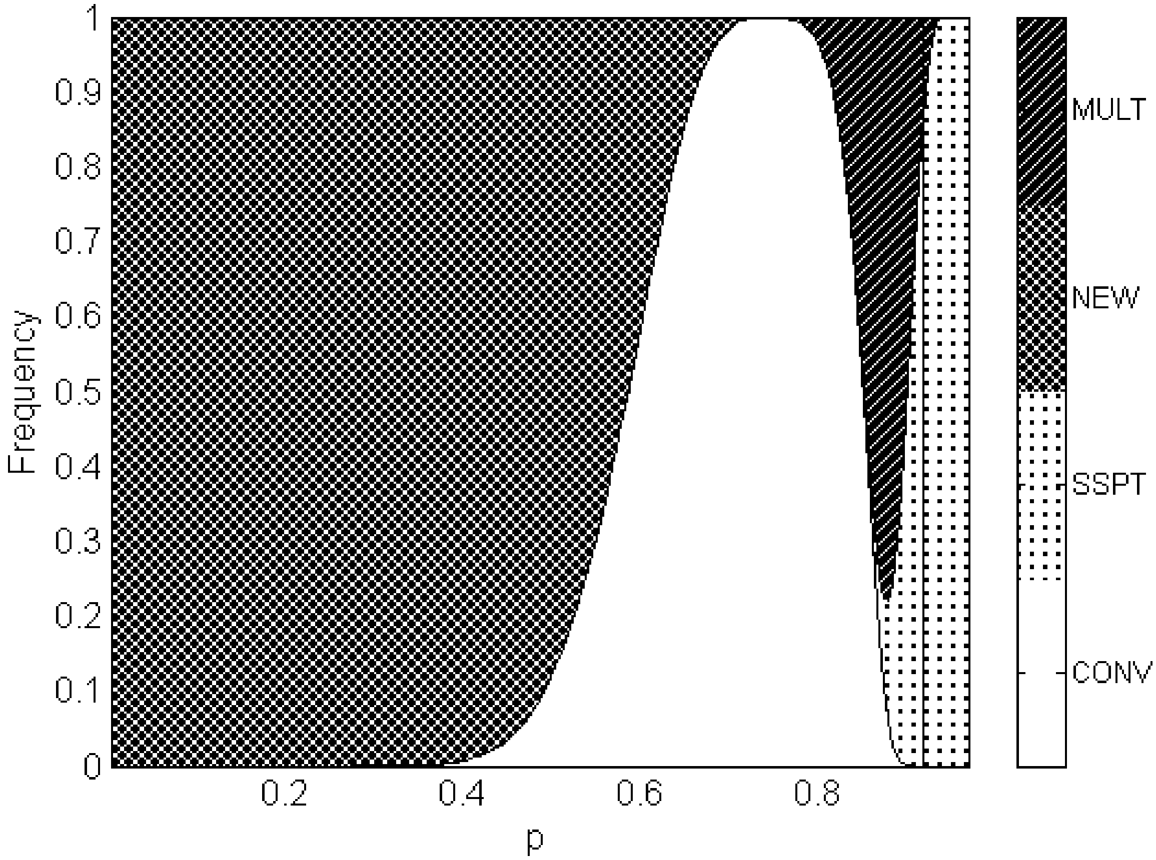
FIGURE 4: TYPES OF ZLB EQUILIBRIA FOR ALTERNATIVE VALUES OF p AND γ



(A) NL EQUILIBRIUM (B) LL EQUILIBRIUM

Notes: CONV: Conventional ZLB equilibrium (slope(AD)>slope(AS)>0); SSPT: Sunspot ZLB equilibrium (slope(AS)>slope(AD)>0); NEW: New ZLB equilibrium (slope(AS)>0>slope(AD)); MULT: Multiple ZLB equilibria).

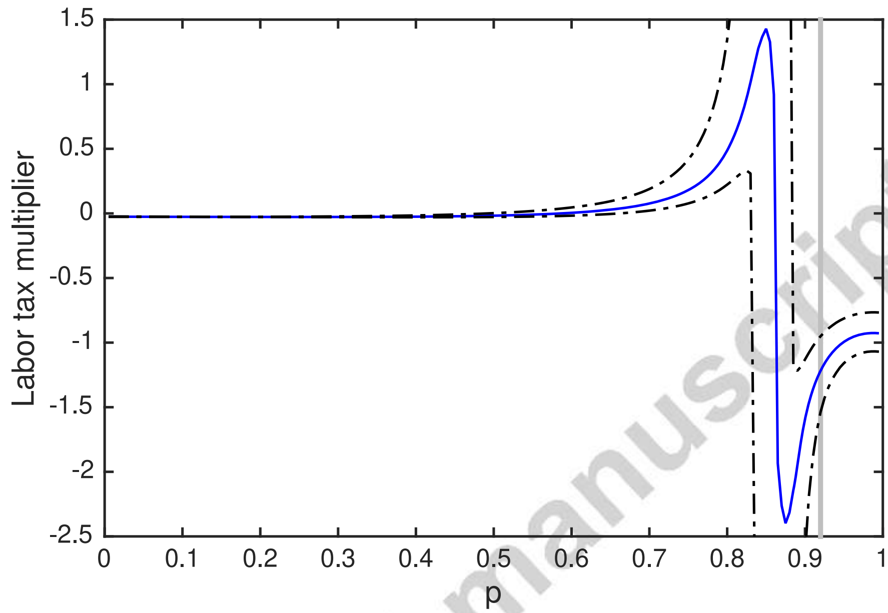
FIGURE 5: FREQUENCY OF TYPES OF ZLB EQUILIBRIA FOR ALTERNATIVE VALUES OF p.



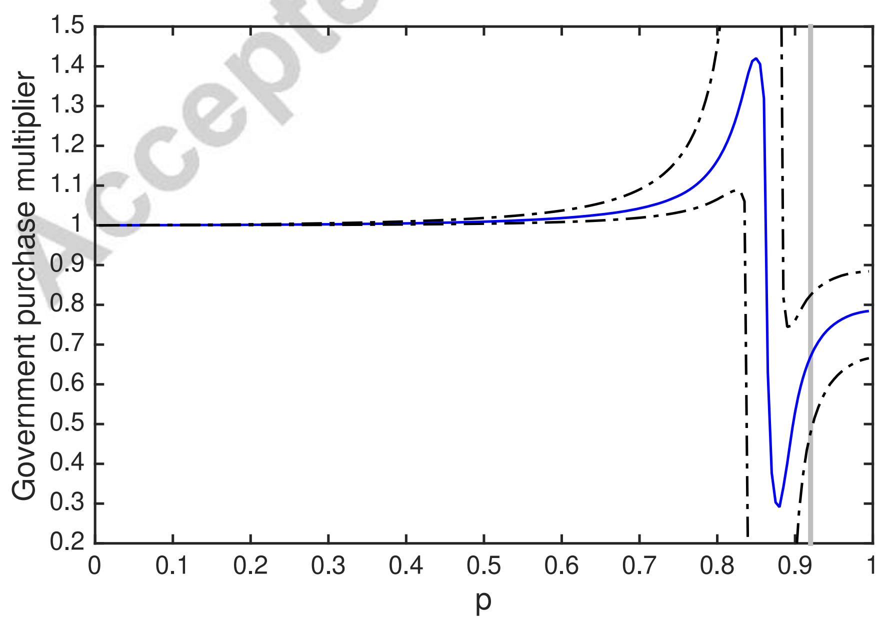
Notes: CONV: Conventional ZLB equilibrium (slope(AD)>slope(AS)>0); SSPT: Sunspot ZLB equilibrium (slope(AS)>slope(AD)>0); NEW: New ZLB equilibrium (slope(AS)>0>slope(AD)); MULT: Multiple ZLB equilibria); The baseline parameterization is denoted with a black line.

Figure 6: The median and the 10th and 90th percentiles of the posterior distribution of multipliers

(a) Labor tax employment multipliers

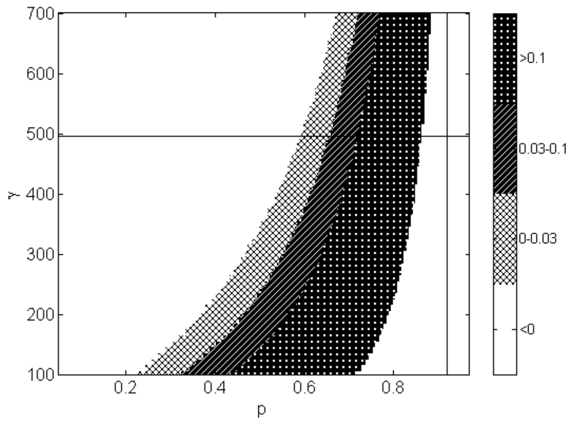


(b) Government purchase GDP multipliers

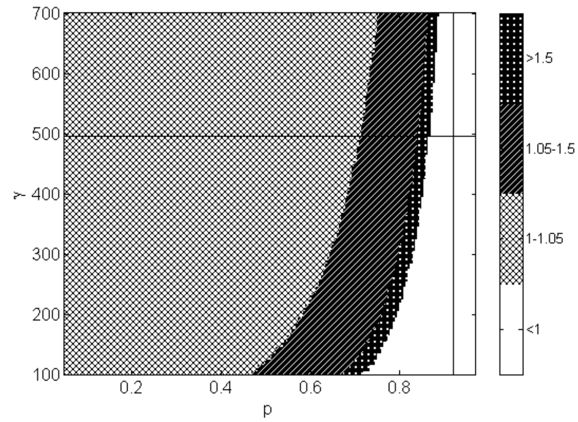


Notes: The figures report the posterior median and 90% credible intervals of each fiscal multiplier at alternative settings of p.

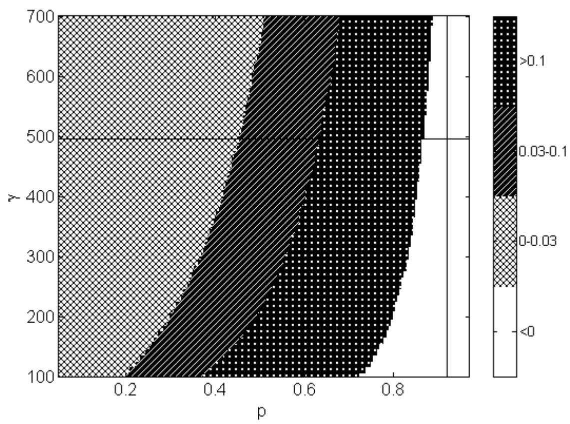
Figure 7: Fiscal multipliers using the nonlinear and loglinear equilibrium conditions at alternative values of p and γ.



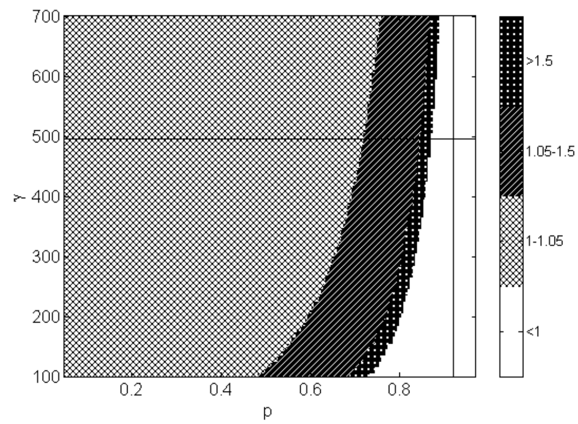
(a) NL Labor tax multiplier



(b) NL Govt. purchase multiplier



(c) LL Labor tax multiplier



(d) LL Govt. purchase multiplier

Notes: Red: Labor tax multiplier is negative (employment increases when the labor tax is cut) / the government-purchase-GDP-multiplier is less than 1; Green: Labor tax multiplier is in $[0, 0.03]$ / the government-purchase-GDP-multiplier is in $[1, 1.05]$; Yellow: labor tax multiplier is in $(0.03, 0.1]$ / the government-purchase-GDP-multiplier is in $[1.05, 1.5]$; Blue: labor tax multiplier exceeds 0.1 / the government-purchase-GDP-multiplier exceeds 1.5. The black line shows the baseline value of γ and p.

FIGURE 8: ZLB EQUILIBRIA IN THE CALVO VS. ROTEMBERG MODEL ($p = 0.4$)

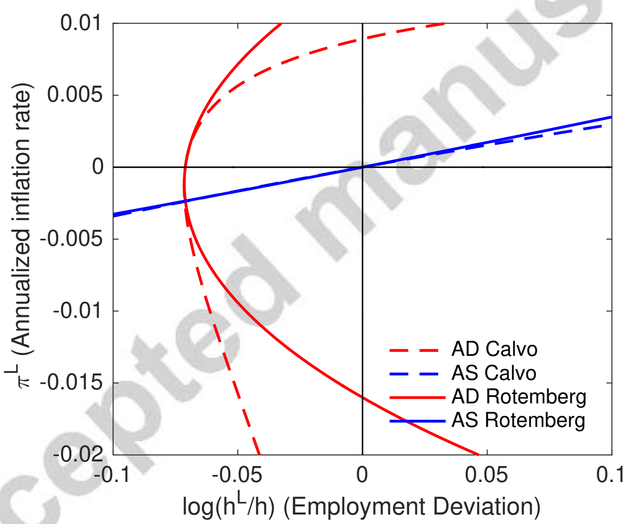
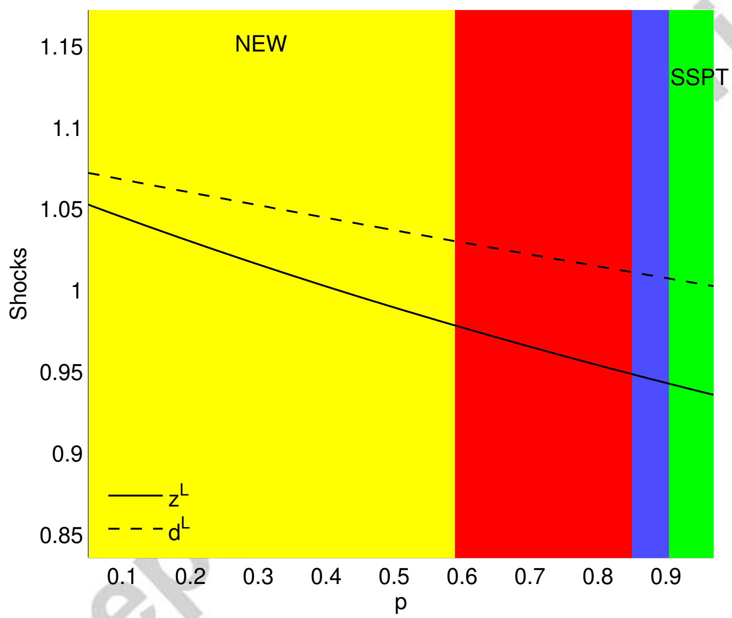


FIGURE 9: SIZE OF CALIBRATED PREFERENCE AND PRODUCTIVITY SHOCKS FOR ALTERNATIVE P



Notes: This figure shows the values of the preference shock d^L and the technology shock z^L that produce a 10% decline in GDP and a 1% decline in the inflation rate using the baseline parameterization of the model and $p \in [0.05, 0.97]$. The figure also shows the type of ZLB equilibrium; CONV: Conventional ZLB equilibrium (slope(AD)>slope(AS)>0); SSPT: Sunspot ZLB equilibrium (slope(AS)>slope(AD)>0); NEW: New ZLB equilibrium (slope(AS)>0>slope(AD)); MULT: Multiple ZLB equilibria); The baseline parameterization is denoted with a black line.