

SOME VARIANTS OF FIBRE CONTRACTION PRINCIPLE AND APPLICATIONS: FROM EXISTENCE TO THE CONVERGENCE OF SUCCESSIVE APPROXIMATIONS

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Abstract. Let (X_1, \rightarrow) and (X_2, \hookrightarrow) be two L -spaces, U be a nonempty subset of $X_1 \times X_2$ such that $U_{x_1} := \{x_2 \in X_2 \mid (x_1, x_2) \in U\}$ is nonempty, for each $x_1 \in X_1$. Let $T_1 : X_1 \rightarrow X_1$, $T_2 : U \rightarrow X_2$ be two operators and $T : U \rightarrow X_1 \times X_2$ defined by $T(x_1, x_2) := (T_1(x_1), T_2(x_1, x_2))$. If we suppose that $T(U) \subset U$, $F_{T_1} \neq \emptyset$ and $F_{T_2(x_1, \cdot)} \neq \emptyset$ for each $x_1 \in X_1$, the problem is in which additional conditions T is a weakly Picard operator? In this paper we study this problem in the case when the convergence structures on X_1 and X_2 are defined by metrics. Some applications to the fixed point equations on spaces of continuous functions are also given.

Key Words and Phrases: Triangular operator, fibre contraction, weakly Picard operator, generalized metric space, generalized contraction, well-posedness, Ostrowki property, Ulam-Hyers stability, Volterra operator, functional differential equation, functional integral equation.

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