

Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems

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Abstract

The single valued triangular neutrosophic number (SVTrN-number) is simply an ordinary number whose precise value is somewhat uncertain from a philosophical point of view, which is a generalization of triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. Also, SVTrN-number may express more abundant and flexible information as compared with the triangular fuzzy numbers and triangular intuitionistic fuzzy numbers. This article introduces an approach to handle multi-criteria decision making (MCDM) problems under the SVTrN-numbers. Therefore, we first proposed some new geometric operator is called SVTrN weighted geometric operator, SVTrN ordered weighted geometric operator, SVTrN ordered hybrid weighted geometric operator. Also we studied some desirable properties of the geometric operators. And then, an approach based on the SVTrN ordered hybrid weighted geometric operator is developed to solve multi-criteria decision making problems with SVTrN-number. Finally, a numerical example is used to demonstrate how to apply the proposed approach.

Keyword 0.1 *Neutrosophic set, single valued neutrosophic numbers, triangular neutrosophic numbers, geometric operators, decision making.*

1 Introduction

Zadeh [44] proposed the notation of fuzzy set X on a fixed set E characterized by a membership function denoted by μ_X such that $\mu_X : E \rightarrow [0, 1]$ which are the powerful tools to deal with imperfect and imprecise information. Then, by adding non-membership function to fuzzy sets, Atanassov [1] presented the notation of intuitionistic fuzzy set K on a fixed set E characterized by a membership function $\mu_K : E \rightarrow [0, 1]$ and a non-membership function $\gamma_K : E \rightarrow [0, 1]$ such that $0 \leq \mu_K(x) + \gamma_K(x) \leq 1$ for any $x \in E$, which is a generalization of fuzzy set [44]. By Smarandache [24], intuitionistic fuzzy set was extended to develop the notation of neutrosophic set A on a fixed set E characterized by a truth-membership function T_A , a indeterminacy-membership function I_A and a falsity-membership function F_A such that $T_A(x), I_A(x), F_A(x) \in]-0, 1[+$ which is a generalization of fuzzy set and intuitionistic fuzzy set. The neutrosophic sets may express more abundant and flexible information as compared with the fuzzy sets and intuitionistic fuzzy sets. Recently, neutrosophic sets have been researched by many scholars in different fields. For example; on neutrosophic similarity clustering [4, 6, 7, 39, 40, 43], on multi-criteria decision making problems [38, 41] etc. Also the notations such as fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets has been applied to some different fields in [3, 5, 8, 9, 11, 12, 13, 18, 25, 28, 36, 42].

Aggregation operators, which is an important research topic in decision-making theory, have been researched by many scholars such as; intuitionistic fuzzy sets [14, 19, 21, 26], intuitionistic fuzzy numbers [14, 15, 17, 22, 27, 29, 30, 31, 32, 33, 35], neutrosophic sets [2, 20, 23], neutrosophic number [41], and so on. Especially, Xu and Yager [26], introduced some new geometric aggregation operators, is called intuitionistic fuzzy weighted geometric operator, intuitionistic fuzzy ordered weighted geometric operator, and intuitionistic fuzzy hybrid geometric operator. Also, Wu and Cao [32] presented some geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers.

Since neutrosophic numbers [10] are a special case of neutrosophic sets, the neutrosophic numbers are importance for neutrosophic multi criteria decision making (MCDM) problems. As a generalization of fuzzy numbers and intuitionistic fuzzy number, a neutrosophic number seems to suitably describe an ill-known quantity. To the our knowledge, existing approaches are not suitable for dealing with MCDM problems under SVTrN-numbers. Therefore, the remainder of this paper is organized as follows: In section 2, some basic definitions of fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, single valued neutrosophic sets and single valued triangular neutrosophic number are briefly reviewed. In section 3, some new geometric operator is called SVTrN weighted geometric operator, SVTrN ordered weighted geometric operator, SVTrN ordered hybrid weighted geometric operator are defined(adapted from [41, 14]. In section 4, an approach based on the SVTrN ordered hybrid weighted geometric operator is developed to solve multi-criteria decision making problems with single valued triangular neutrosophic number is developed. In section 5, a numerical example is given to demonstrate how to apply the proposed approach. In section 7, the study is concluded.

2 Preliminary

In this section, we recall some basic notions of fuzzy sets [44], intuitionistic fuzzy sets [1], intuitionistic fuzzy numbers [14] and neutrosophic sets [24]. For more details, the reader could refer to [1, 14, 24, 25, 44]. From now on we use $I_n = \{1, 2, \dots, n\}$ and $I_m = \{1, 2, \dots, m\}$ as an index set for $n \in N$ and $m \in N$, respectively.

Definition 2.1 [44] *Let E be a universe. Then a fuzzy set X over E is a function defined as follows:*

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where $\mu_X : E \rightarrow [0,1]$.

Here, μ_X called membership function of X , and the value $\mu_X(x)$ is called the grade of membership of $x \in E$. The value represents the degree of x belonging to the fuzzy set X .

Definition 2.2 [1] *Let E be a universe. An intuitionistic fuzzy set K on E can be defined as follows:*

$$K = \{< x, \mu_K(x), \gamma_K(x) > : x \in E\}$$

where, $\mu_K : E \rightarrow [0, 1]$ and $\gamma_K : E \rightarrow [0, 1]$ such that $0 \leq \mu_K(x) + \gamma_K(x) \leq 1$ for any $x \in E$.

Here, $\mu_K(x)$ and $\gamma_K(x)$ is the degree of membership and degree of non-membership of the element x , respectively.

Definition 2.3 [24] *Let E be a universe. A neutrosophic sets(NS) A in E is characterized by a truth-membership function T_A , a indeterminacy-membership function I_A and a falsity-membership function F_A . $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. It can be written as*

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) > : x \in E, T_A(x), I_A(x), F_A(x) \in]^{-}0, 1[^{+}\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.4 [28] *Let E be a universe. A single valued neutrosophic sets(SVNS) A , which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A , a indeterminacy-membership function I_A and a falsity-membership function F_A . $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. It can be written as*

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) > : x \in E, T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.5 [10] Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0, 1]$ and $a_1, b_1, c_1 \in R$ such that $a_1 \leq b_1 \leq c_1$. Then, a single valued triangular neutrosophic number (SVTrN-number)

$$\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$$

is a special neutrosophic set on the real number set R , whose truth-membership indeterminacy-membership and falsity-membership functions are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)w_{\tilde{a}}/(b_1 - a_1) & (a_1 \leq x < b_1) \\ w_{\tilde{a}} & (x = b_1) \\ (c_1 - x)w_{\tilde{a}}/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1 - x + u_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ u_{\tilde{a}} & (x = b_1) \\ (x - b_1 + u_{\tilde{a}}(c_1 - x))/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 1 & \text{otherwise,} \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1 - x + y_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ y_{\tilde{a}} & (x = b_1) \\ (x - b_1 + y_{\tilde{a}}(c_1 - x))/(c_1 - b_1) & (b_1 < x \leq c_1) \\ 1 & \text{otherwise,} \end{cases}$$

respectively.

If $a_1 \geq 0$ and at least $c_1 > 0$ then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a positive SVTrN-numbers, denoted by $\tilde{a} > 0$. Likewise, if $c_1 \leq 0$ and at least $a_1 < 0$, then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a negative SVTrN-numbers, denoted by $\tilde{a} < 0$. A SVTrN-numbers $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ may express an ill-known quantity about a , which is approximately equal to a .

Note that the set of all SVTrN-numbers on R will be denoted by Δ .

Definition 2.6 [10] Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle \in \Delta$ and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$
2. $\tilde{a} - \tilde{b} = \langle (a_1 - a_2, b_1 - b_2, c_1 - c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$
3. $\tilde{a}\tilde{b} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (a_1c_2, b_1b_2, c_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1c_2, b_1b_2, a_1a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$
4. $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/c_2, b_1/b_2, c_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1/a_2, b_1/b_2, a_1/c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$
5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$
6. $\tilde{a}^\gamma = \begin{cases} \langle (a_1^\gamma, b_1^\gamma, c_1^\gamma); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (c_1^\gamma, b_1^\gamma, a_1^\gamma); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$

Likewise, it is easily proven that the results obtained by multiplication and division of two SVTrN-numbers are not always SVTrN-numbers. However, we often use SVTrN-numbers to express these computational results approximately.

Example 2.7 Let $\tilde{a} = \langle (4, 5, 6); 0.7, 0.5, 0.3 \rangle$ and $\tilde{b} = \langle (2, 3, 4); 0.6, 0.1, 0.4 \rangle$ be two SVTrN-numbers then,

1. $\tilde{a} + \tilde{b} = \langle (6, 8, 10); 0.6, 0.5, 0.4 \rangle$
2. $\tilde{a} - \tilde{b} = \langle (0, 2, 4); 0.6, 0.5, 0.4 \rangle$
3. $\tilde{a}\tilde{b} = \langle (8, 15, 24); 0.6, 0.5, 0.4 \rangle$
4. $\tilde{a}/\tilde{b} = \langle (1, \frac{5}{3}, 3); 0.6, 0.5, 0.4 \rangle$
5. $2\tilde{a} = \langle (8, 10, 12); 0.7, 0.5, 0.3 \rangle$
6. $\tilde{b}^2 = \langle (4, 9, 16); 0.6, 0.1, 0.4 \rangle$

Definition 2.8 [10] We defined a method to compare any two SVTrN-numbers which is based on the score function and the accuracy function. Let $\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \in \Delta$, then

$$S(\tilde{a}) = \frac{1}{8}[a + b + c] \times (2 + \mu_{\tilde{a}} - \nu_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (1)$$

and

$$A(\tilde{a}) = \frac{1}{8}[a + b + c] \times (2 + \mu_{\tilde{a}} - \nu_{\tilde{a}} + \gamma_{\tilde{a}})$$

is called the score and accuracy degrees of \tilde{a} , respectively.

Definition 2.9 [10] Let $\tilde{a}_1, \tilde{a}_2 \in \Delta$. Then,

1. If $S(\tilde{a}_1) < S(\tilde{a}_2)$, then \tilde{a}_1 is smaller than \tilde{a}_2 , denoted by $\tilde{a}_1 < \tilde{a}_2$
2. If $S(\tilde{a}_1) > S(\tilde{a}_2)$, then \tilde{a}_1 is bigger than \tilde{a}_2 , denoted by $\tilde{a}_1 > \tilde{a}_2$
3. If $S(\tilde{a}_1) = S(\tilde{a}_2)$;
 - (a) If $A(\tilde{a}_1) < A(\tilde{a}_2)$, then \tilde{a}_1 is smaller than \tilde{a}_2 , denoted by $\tilde{a}_1 < \tilde{a}_2$
 - (b) If $A(\tilde{a}_1) > A(\tilde{a}_2)$, then \tilde{a}_1 is bigger than \tilde{a}_2 , denoted by $\tilde{a}_1 > \tilde{a}_2$
 - (c) If $A(\tilde{a}_1) = A(\tilde{a}_2)$, then \tilde{a}_1 and \tilde{a}_2 are the same, denoted by $\tilde{a}_1 = \tilde{a}_2$

3 Geometric operators of the SVTrN-number

In this section, three SVTrN weighted geometric operator of SVTrN-numbers is called SVTrN weighted geometric operator, SVTrN ordered weighted geometric operator, SVTrN ordered hybrid weighted geometric operator is given. Some of it is quoted from application in [10, 14, 15, 16, 17, 18, 26, 31, 32, 37].

Definition 3.1 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Delta$ ($j \in I_n$). Then SVTrN weighted geometric operator, denoted by G_{go} , is defined as;

$$G_{go} : \Delta^n \rightarrow \Delta, \quad G_{go}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{i=1}^n \tilde{a}_i^{w_i}$$

where, $w = (w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the G_{go} operator, for every $j \in I_n$ such that, $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Theorem 3.2 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Gamma$ ($j \in I$), $w = (w_1, w_2, \dots, w_n)^T$ be a weight vector of \tilde{a}_j , for every $j \in I_n$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Then, their aggregated value by using G_{go} operator is also a SVTrN-number and

$$G_{go}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left\langle \left(\prod_{j=1}^n a_j^{w_j}, \prod_{j=1}^n b_j^{w_j}, \prod_{j=1}^n c_j^{w_j} \right); \bigwedge_{j=1}^n w_{\tilde{a}_j}, \bigvee_{j=1}^n u_{\tilde{a}_j}, \bigvee_{j=1}^n y_{\tilde{a}_j} \right\rangle$$

Proof The proof can be made by using mathematical induction on n as; Assume that,

$$\tilde{a}_1 = \langle (a_1, b_1, c_1); w_{\tilde{a}_1}, u_{\tilde{a}_1}, y_{\tilde{a}_1} \rangle$$

and

$$\tilde{a}_2 = \langle (a_2, b_2, c_2); w_{\tilde{a}_2}, u_{\tilde{a}_2}, y_{\tilde{a}_2} \rangle$$

be two SVTrN-numbers then, for $n = 2$, we have

$$G_{ogo}(\tilde{a}_1, \tilde{a}_2) = \left\langle \left(\prod_{j=1}^2 a_j^{w_j}, \prod_{j=1}^2 b_j^{w_j}, \prod_{j=1}^2 c_j^{w_j} \right); \bigwedge_{j=1}^2 w_{\tilde{a}_j}, \bigvee_{j=1}^2 u_{\tilde{a}_j}, \bigvee_{j=1}^2 y_{\tilde{a}_j} \right\rangle$$

If holds for $n = k$, that is

$$G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k) = \left\langle \left(\prod_{j=1}^k a_j^{w_j}, \prod_{j=1}^k b_j^{w_j}, \prod_{j=1}^k c_j^{w_j} \right); \bigwedge_{j=1}^k w_{\tilde{a}_j}, \bigvee_{j=1}^k u_{\tilde{a}_j}, \bigvee_{j=1}^k y_{\tilde{a}_j} \right\rangle$$

then, when $n = k + 1$, by the operational laws in Definition 2.6, I have

$$\begin{aligned} G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k, \tilde{a}_{k+1}) &= \left\langle \left(\prod_{j=1}^k a_j^{w_j}, \prod_{j=1}^k b_j^{w_j}, \prod_{j=1}^k c_j^{w_j} \right); \bigwedge_{j=1}^k w_{\tilde{a}_j}, \bigvee_{j=1}^k u_{\tilde{a}_j}, \bigvee_{j=1}^k y_{\tilde{a}_j} \right\rangle \\ &\quad \times \left\langle \left(w_{k+1} a_{k+1}, w_{k+1} b_{k+1}, w_{k+1} c_{k+1} \right); w_{\tilde{a}_{k+1}}, u_{\tilde{a}_{k+1}}, y_{\tilde{a}_{k+1}} \right\rangle \\ &= \left\langle \left(\prod_{j=1}^{k+1} a_j^{w_j}, \prod_{j=1}^{k+1} b_j^{w_j}, \prod_{j=1}^{k+1} c_j^{w_j} \right); \bigwedge_{j=1}^{k+1} w_{\tilde{a}_j}, \bigvee_{j=1}^{k+1} u_{\tilde{a}_j}, \bigvee_{j=1}^{k+1} y_{\tilde{a}_j} \right\rangle \end{aligned}$$

therefore proof is valid.

Definition 3.3 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Delta$ ($j \in I_n$). Then SVTrN ordered weighted geometric operator denoted by G_{ogo} , is defined as;

$$G_{ogo} : \Delta^n \rightarrow \Delta, \quad G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{k=1}^n \tilde{b}_k^{w_k}$$

where $w = (w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the mapping G_{ogo} , which satisfies the normalized conditions: $w_k \in [0, 1]$ and $\sum_{k=1}^n w_k = 1$; $\tilde{b}_k = \langle (a_k, b_k, c_k); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle$ is the k -th largest of the n SVTrN-numbers \tilde{a}_j ($j \in I_n$) which is determined through using ranking method in Definition 2.8.

It is not difficult to follows from Definition 3.3 that

$$\begin{aligned} G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) &= \prod_{k=1}^n \tilde{b}_k^{w_k} \\ &= \prod_{k=1}^n \left(\langle (a_k, b_k, c_k); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle \right)^{w_k} \\ &= \prod_{k=1}^n \langle (a_k^{w_k}, b_k^{w_k}, c_k^{w_k}); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle \\ &= \langle (\prod_{k=1}^n a_k^{w_k}, \prod_{k=1}^n b_k^{w_k}, \prod_{k=1}^n c_k^{w_k}); \bigwedge_{k=1}^n w_{\tilde{a}_k}, \bigvee_{k=1}^n u_{\tilde{a}_k}, \bigvee_{k=1}^n y_{\tilde{a}_k} \rangle \end{aligned}$$

which is summarized as in Theorem 3.4.

Theorem 3.4 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Delta$ ($j \in I$). Then SVTrN ordered weighted geometric operator denoted by G_{ogo} , is defined as;

$$G_{ogo} : \Delta^n \rightarrow \Delta, \quad G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left\langle \left(\prod_{k=1}^n a_k^{w_k}, \prod_{k=1}^n b_k^{w_k}, \prod_{k=1}^n c_k^{w_k} \right); \bigwedge_{k=1}^n w_{\tilde{a}_k}, \bigvee_{k=1}^n u_{\tilde{a}_k}, \bigvee_{k=1}^n y_{\tilde{a}_k} \right\rangle \quad (2)$$

where $w_k \in [0, 1]$, $\sum_{k=1}^n w_k = 1$; $\tilde{b}_k = \langle (a_k, b_k, c_k); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle$ is the k -th largest of the n neutrosophic sets \tilde{a}_j ($j \in I_n$) which is determined through using some ranking method in Definition 2.8.

Proof The proof can be made by using mathematical induction on n as;

for $n = 2$, we have

$$G_{ogo}(\tilde{a}_1, \tilde{a}_2) = \left\langle \left(\prod_{j=1}^2 a_j^{w_j}, \prod_{j=1}^2 b_j^{w_j}, \prod_{j=1}^2 c_j^{w_j} \right); \bigwedge_{j=1}^2 w_{\tilde{a}_j}, \bigvee_{j=1}^2 u_{\tilde{a}_j}, \bigvee_{j=1}^2 y_{\tilde{a}_j} \right\rangle$$

If holds for $n = k$, that is

$$G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k) = \left\langle \left(\prod_{j=1}^k a_j^{w_j}, \prod_{j=1}^k b_j^{w_j}, \prod_{j=1}^k c_j^{w_j} \right); \bigwedge_{j=1}^k w_{\tilde{a}_j}, \bigvee_{j=1}^k u_{\tilde{a}_j}, \bigvee_{j=1}^k y_{\tilde{a}_j} \right\rangle$$

then, when $n = k + 1$, by the operational laws in Definition 2.6, I have

$$\begin{aligned} G_{ogo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_k, \tilde{a}_{k+1}) &= \left\langle \left(\prod_{j=1}^k a_j^{w_j}, \prod_{j=1}^k b_j^{w_j}, \prod_{j=1}^k c_j^{w_j} \right); \bigwedge_{j=1}^k w_{\tilde{a}_j}, \bigvee_{j=1}^k u_{\tilde{a}_j}, \bigvee_{j=1}^k y_{\tilde{a}_j} \right\rangle \\ &\quad \times \left\langle \left(a_{k+1}^{w_{k+1}}, b_{k+1}^{w_{k+1}}, c_{k+1}^{w_{k+1}} \right); w_{\tilde{a}_{k+1}}, u_{\tilde{a}_{k+1}}, y_{\tilde{a}_{k+1}} \right\rangle \\ &= \left\langle \left(\prod_{j=1}^{k+1} a_j^{w_j}, \prod_{j=1}^{k+1} b_j^{w_j}, \prod_{j=1}^{k+1} c_j^{w_j} \right); \bigwedge_{j=1}^{k+1} w_{\tilde{a}_j}, \bigvee_{j=1}^{k+1} u_{\tilde{a}_j}, \bigvee_{j=1}^{k+1} y_{\tilde{a}_j} \right\rangle \end{aligned}$$

therefore proof is valid.

Now, we give an example (is adapted from [14].)

Example 3.5 *There are four experts who are invited to evaluate some enterprise. Their evaluations are expressed with the single valued neutrosophic sets*

$$\begin{aligned} \tilde{a}_1 &= \langle (0.123, 0.234, 0.325); 0.4, 0.5, 0.7 \rangle, \\ \tilde{a}_2 &= \langle (0.234, 0.354, 0.451); 0.3, 0.6, 0.6 \rangle, \\ \tilde{a}_3 &= \langle (0.125, 0.365, 0.465); 0.2, 0.7, 0.5 \rangle, \\ \tilde{a}_4 &= \langle (0.215, 0.345, 0.435); 0.1, 0.8, 0.4 \rangle, \end{aligned}$$

respectively. To eliminate effect of individual bias on comprehensive evaluation, the unduly high evaluation and the unduly low evaluation are punished through giving a smaller weight. Assume that the position weight vector is $w = (0.15, 0.35, 0.35, 0.15)$. Compute the comprehensive evaluation of the four experts on the enterprise though using the neutrosophic ordered weighted averaging operator.

Solving According to Eq. (3.2), the scores of the neutrosophic sets \tilde{a}_j ($j = 1, 2, 3, 4$) are obtained as follows:

$$\begin{aligned} S(\tilde{a}_1) &= \frac{1}{8}[0.123 + 0.234 + 0.325] = 0.102 \times (2 + 0.4 - 0.5 - 0.7), \\ S(\tilde{a}_2) &= \frac{1}{8}[0.234 + 0.354 + 0.451] = 0.143 \times (2 + 0.3 - 0.6 - 0.6), \\ S(\tilde{a}_3) &= \frac{1}{8}[0.125 + 0.365 + 0.465] = 0.119 \times (2 + 0.2 - 0.7 - 0.5), \\ S(\tilde{a}_4) &= \frac{1}{8}[0.215 + 0.345 + 0.435] = 0.112 \times (2 + 0.1 - 0.8 - 0.4), \end{aligned}$$

respectively. It is obvious that $S(\tilde{a}_2) > S(\tilde{a}_3) > S(\tilde{a}_4) > S(\tilde{a}_1)$. Hence according to the above scoring function ranking method, its follows that $\tilde{a}_2 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_1$. Hence, we have:

$$\begin{aligned} \tilde{b}_1 &= \tilde{a}_2 = \langle (0.234, 0.354, 0.451); 0.3, 0.6, 0.6 \rangle, \\ \tilde{b}_2 &= \tilde{a}_3 = \langle (0.125, 0.365, 0.465); 0.2, 0.7, 0.5 \rangle, \\ \tilde{b}_3 &= \tilde{a}_4 = \langle (0.215, 0.345, 0.435); 0.1, 0.8, 0.4 \rangle, \end{aligned}$$

$$\tilde{b}_4 = \tilde{a}_1 = \langle (0.123, 0.234, 0.325); 0.4, 0.5, 0.7 \rangle,$$

Using Eq. (2), we obtain:

$$\begin{aligned} G_{ogo}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4) &= \langle (0.234^{0.15} \times 0.125^{0.35} \times 0.215^{0.35} \times 0.123^{0.15}, \\ &\quad 0.354^{0.15} \times 0.365^{0.35} \times 0.345^{0.35} \times 0.234^{0.15}, \\ &\quad 0.451^{0.15} \times 0.465^{0.35} \times 0.435^{0.35} \times 0.325^{0.15}); 0.1, 0.8, 0.7 \rangle \\ &= \langle (0.166, 0.333, 0.429); 0.1, 0.8, 0.7 \rangle \end{aligned}$$

Definition 3.6 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Delta$ ($j \in I$). Then SVTrN ordered hybrid weighted geometric operator denoted by G_{hgo} , is defined as;

$$G_{hgo} : \Delta^n \rightarrow \Delta, \quad G_{hgo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \prod_{k=1}^n \hat{b}_k^{w_k}$$

where $w = (w_1, w_2, \dots, w_n)^T$. $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$ is a weight vector associated with the mapping G_{hgo} , $a_j \in \Delta$ a weight with $n\omega$ ($j \in I_n$) is denoted by \tilde{A}_j i.e., $\tilde{A}_j = n\omega\tilde{a}_j$, here n is regarded as a balance factor $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is a weight vector of the $a_j \in \Delta$ ($j \in I_n$); \hat{b}_k is the k -th largest of the n SVTrN-numbers $\tilde{A}_j \in \Delta$ ($j \in I_n$) which are determined through using some ranking method such as the above scoring function ranking method.

Note that if $\omega = (1/n, 1/n, \dots, 1/n)^T$, then G_{hgo} degenerates to the G_{ogo} .

Example 3.7 $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in \Delta$. Their evaluations are expressed with the G_{go} .

$$\tilde{a}_1 = \langle (0.123, 0.234, 0.325); 0.4, 0.5, 0.7 \rangle,$$

$$\tilde{a}_2 = \langle (0.234, 0.354, 0.451); 0.3, 0.6, 0.6 \rangle,$$

$$\tilde{a}_3 = \langle (0.125, 0.365, 0.465); 0.2, 0.7, 0.5 \rangle,$$

$$\tilde{a}_4 = \langle (0.215, 0.345, 0.435); 0.1, 0.8, 0.4 \rangle$$

respectively. Assume that the weight vector of the three experts is $\omega = (0.2, 0.3, 0.3, 0.2)^T$ and the position weight vector is $w = (0.4, 0.1, 0.1, 0.4)^T$. Compute the comprehensive evaluation of the three experts on the decision alternative through using the G_{hgo} .

Solving

$$\tilde{A}_1 = 4 \times 0.2 \times \tilde{a}_1 = 4 \times 0.2 \times \langle (0.123, 0.234, 0.325); 0.4, 0.5, 0.7 \rangle = \langle (0.098, 0.187, 0.260); 0.4, 0.5, 0.7 \rangle$$

$$\tilde{A}_2 = 4 \times 0.3 \times \tilde{a}_2 = 4 \times 0.3 \times \langle (0.234, 0.354, 0.451); 0.3, 0.6, 0.6 \rangle = \langle (0.281, 0.425, 0.541); 0.3, 0.6, 0.6 \rangle$$

$$\tilde{A}_3 = 4 \times 0.3 \times \tilde{a}_3 = 4 \times 0.3 \times \langle (0.125, 0.365, 0.465); 0.2, 0.7, 0.5 \rangle = \langle (0.150, 0.438, 0.558); 0.2, 0.7, 0.5 \rangle$$

$$\tilde{A}_4 = 4 \times 0.2 \times \tilde{a}_4 = 4 \times 0.2 \times \langle (0.215, 0.345, 0.435); 0.1, 0.8, 0.4 \rangle = \langle (0.172, 0.276, 0.348); 0.1, 0.8, 0.4 \rangle$$

we obtain the scores of the SVTrN-numbers \tilde{A}_j ($j = 1, 2, 3$) as follows:

$$S(\tilde{A}_1) = \frac{1}{8}[0.098 + 0.187 + 0.260] \times (2 + 0.4 - 0.5 - 0.7) = 0.082,$$

$$S(\tilde{A}_2) = \frac{1}{8}[0.281 + 0.425 + 0.541] \times (2 + 0.3 - 0.6 - 0.6) = 0.171,$$

$$S(\tilde{A}_3) = \frac{1}{8}[0.150 + 0.438 + 0.558] \times (2 + 0.2 - 0.7 - 0.5) = 0.143,$$

$$S(\tilde{A}_4) = \frac{1}{8}[0.172 + 0.276 + 0.348] \times (2 + 0.1 - 0.8 - 0.4) = 0.090.$$

Obviously, $S(\tilde{A}_2) > S(\tilde{A}_3) > S(\tilde{A}_1) > S(\tilde{A}_4)$. Thereby, according to the above scoring function ranking method, we have

$$\begin{aligned}\hat{b}_1 &= \tilde{A}_2 = \langle (0.281, 0.425, 0.541); 0.3, 0.6, 0.6 \rangle \\ \hat{b}_2 &= \tilde{A}_3 = \langle (0.150, 0.438, 0.558); 0.2, 0.7, 0.5 \rangle \\ \hat{b}_3 &= \tilde{A}_4 = \langle (0.172, 0.276, 0.348); 0.1, 0.8, 0.4 \rangle \\ \hat{b}_4 &= \tilde{A}_1 = \langle (0.098, 0.187, 0.260); 0.4, 0.5, 0.7 \rangle\end{aligned}$$

It follows from (3.2) that

$$\begin{aligned}G_{hgo}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4) &= \langle (0.281 \times 0.4 + 0.150 \times 0.1 + 0.172 \times 0.1 + 0.098 \times 0.4, \\ &\quad 0.425 \times 0.4 + 0.438 \times 0.1 + 0.276 \times 0.1 + 0.187 \times 0.4, \\ &\quad 0.541 \times 0.4 + 0.558 \times 0.1 + 0.348 \times 0.1 + 0.260 \times 0.4); 0.1, 0.8, 0.7 \rangle \\ &= \langle (0.184, 0.316, 0.411); 0.1, 0.8, 0.7 \rangle\end{aligned}$$

Theorem 3.8 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Delta$ ($j \in I_n$), $w = (w_1, w_2, \dots, w_n)^T$ be a weight vector of \tilde{a}_j with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Then their aggregated value by using G_{hgo} operator is also a SVTrN-number and

$$G_{hgo} : \Delta^n \rightarrow \Delta, \quad G_{hgo}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \left\langle \left(\prod_{k=1}^n a_k^{w_k}, \prod_{k=1}^n b_k^{w_k}, \prod_{k=1}^n c_k^{w_k} \right); \bigwedge_{k=1}^n w_{\tilde{a}_k}, \bigvee_{k=1}^n u_{\tilde{a}_k}, \bigvee_{k=1}^n y_{\tilde{a}_k} \right\rangle \quad (3)$$

where $\hat{b}_k = \langle (a_k, b_k, c_k); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle$ is the k -th largest of the n SVTrN-numbers $\hat{A}_j = n\omega_j \tilde{a}_j$ ($j \in I_n$) which is determined through using some ranking method such as the above scoring function ranking method.

Proof Theorem 3.8 can be proven in a similar way to that of Theorem 3.4 (omitted).

4 Multi-criteria decision making based on SVTrN-numbers

In this section, we define a multi-criteria decision making method, so called SVTrN-multi-criteria decision-making method, by using the G_{hgo} operator. Some of it is quoted from application in [10, 14, 17, 18, 37].

There is a panel with four possible alternatives to invest the money (adapted from [18]): (1) x_1 is a car company; (2) x_2 is a food company; (3) x_3 is a computer company; (4) x_4 is a television company. The investment company must take a decision according to the following three criteria: (1) u_1 is the risk analysis; (2) u_2 is the growth analysis; (3) u_3 is the environmental impact analysis; (4) u_4 social political impact analysis. The four possible alternatives are to be evaluated under the above three criteria by corresponding to linguistic values of SVTrN-numbers for linguistic terms (adapted from [37]), as shown in Table 1.

Linguistic terms	Linguistic values of SVTrN-numbers
Absolutely low	$\langle (0.1, 0.2, 0.3); 0.1, 0.2, 0.3 \rangle$
Low	$\langle (0.2, 0.3, 0.4); 0.2, 0.3, 0.4 \rangle$
Fairly low	$\langle (0.3, 0.4, 0.5); 0.3, 0.4, 0.5 \rangle$
Medium	$\langle (0.4, 0.5, 0.6); 0.4, 0.5, 0.6 \rangle$
Fairly high	$\langle (0.5, 0.6, 0.7); 0.5, 0.6, 0.7 \rangle$
High	$\langle (0.6, 0.7, 0.8); 0.6, 0.7, 0.8 \rangle$
Absolutely high	$\langle (0.7, 0.8, 0.9); 0.7, 0.8, 0.9 \rangle$

Table 1: SVTrN-numbers for linguistic terms

Definition 4.1 Let $X = (x_1, x_2, \dots, x_m)$ be a set of alternatives, $U = (u_1, u_2, \dots, u_n)$ be the set of attributes. If $\tilde{a}_{ij} = \langle (a_{ij}, b_{ij}, c_{ij}); w_{ij}, u_{ij}, y_{ij} \rangle \in \Delta$, then

$$[\tilde{a}_{ij}]_{m \times n} = \begin{matrix} & u_1 & u_2 & \cdots & u_n \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix} \end{matrix}$$

is called an SVTrN-multi-criteria decision-making matrix of the decision maker.

Now, we can give an algorithm of the SVTrN-multi-criteria decision-making method as follows;

Algorithm:

- Step 1.* Construct the decision-making matrix $[\tilde{a}_{ij}]_{m \times n}$ for decision;
- Step 2.* Compute the SVTrN-numbers $\tilde{A}_{ij} = n\omega_i\tilde{a}_{ij}$ ($i \in I_m; j \in I_n$) and write the decision-making matrix $[\tilde{A}_{ij}]_{m \times n}$;
- Step 3.* Obtain the scores of the SVTrN-numbers \tilde{A}_{ij} ($i \in I_m; j \in I_n$);
- Step 4.* Rank all SVTrN-numbers \tilde{A}_{ij} ($i \in I_m; j \in I_n$) by using the ranking method of SVTrN-numbers and determine the SVTrN-numbers $[b_i]_{1 \times n} = \tilde{b}_{ik}$ ($i \in I_m; k \in I_n$) where \tilde{b}_{ik} is k-th largest of \tilde{A}_{ij} for $j \in I_n$;
- Step 5.* Give the decision matrix $[b_i]_{1 \times n}$ for $i = 1, 2, 3, 4$;
- Step 6.* Compute $G_{hgo}(\tilde{b}_{i1}, \tilde{b}_{i2}, \dots, \tilde{b}_{in})$ for $i \in I_m$;
- Step 7.* Rank all alternatives x_i by using the ranking method of SVTrN-numbers and determine the best alternative.

5 Application

In this section, we give an application for the SVTrN-multi-criteria decision-making method, by using the G_{hgo} operator. Some of it is quoted from application in [10, 14, 18, 37].

Example 5.1 Let us consider the decision-making problem adapted from [41]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with the set of the four alternatives is denoted by $X = \{x_1 = \text{car company}, x_2 = \text{food company}, x_3 = \text{computer company}, x_4 = \text{television company}\}$ to invest the money. The investment company must take a decision according to the set of the four attributes is denoted by $U = \{u_1 = \text{risk analysis}, u_2 = \text{growth analysis}, u_3 = \text{environmental impact analysis}, u_4 = \text{social political impact analysis}\}$. Then, the weight vector of the attributes is $\omega = (0.1, 0.2, 0.3, 0.4)^T$ and the position weight vector is $w = (0.24, 0.26, 0.26, 0.24)^T$ by using the weight determination based on the normal distribution. For the evaluation of an alternative x_i ($i = 1, 2, 3, 4$) with respect to a criterion u_j ($j = 1, 2, 3, 4$), it is obtained from the questionnaire of a domain expert. Then,

- Step 1.* The decision maker construct the decision matrix $[\tilde{a}_{ij}]_{4 \times 4}$ as follows:

$$\begin{matrix} & u_1 & u_2 & u_3 & u_4 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} & \begin{pmatrix} (0.1, 0.2, 0.3); 0.1, 0.2, 0.3 & (0.3, 0.4, 0.5); 0.3, 0.4, 0.5 & (0.6, 0.7, 0.8); 0.6, 0.7, 0.8 & (0.1, 0.3, 0.9); 0.1, 0.3, 0.9 \\ (0.3, 0.6, 0.9); 0.3, 0.6, 0.9 & (0.1, 0.6, 0.9); 0.1, 0.6, 0.9 & (0.4, 0.5, 0.6); 0.4, 0.5, 0.6 & (0.1, 0.6, 0.9); 0.1, 0.6, 0.9 \\ (0.2, 0.3, 0.4); 0.2, 0.3, 0.4 & (0.5, 0.6, 0.7); 0.5, 0.6, 0.7 & (0.7, 0.8, 0.9); 0.7, 0.8, 0.9 & (0.2, 0.4, 0.8); 0.2, 0.4, 0.8 \\ (0.6, 0.7, 0.8); 0.6, 0.7, 0.8 & (0.2, 0.3, 0.8); 0.2, 0.3, 0.8 & (0.2, 0.7, 0.8); 0.2, 0.7, 0.8 & (0.2, 0.7, 0.8); 0.2, 0.7, 0.8 \end{pmatrix} \end{matrix}$$

Step 2. Compute $\tilde{A}_{ij} = n\omega_i\tilde{a}_{ij}$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) as follows:

$$\begin{aligned}\tilde{A}_{11} &= 4 \times 0.1 \times \tilde{a}_{11} \\ &= \langle (0.1^{0.4}, 0.2^{0.4}, 0.3^{0.4}); 0.1, 0.2, 0.3 \rangle \\ &= \langle (0.398, 0.525, 0.618); 0.1, 0.2, 0.3 \rangle\end{aligned}$$

Likewise, we can obtain other SVTrN-numbers $\tilde{A}_{ij} = n\omega_i\tilde{a}_{ij}$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) which are given by the SVTrN-decision matrix $[\tilde{A}_{ij}]_{4 \times 4}$ as follows:

$$[\tilde{A}_{ij}]_{4 \times 4} =$$

$$\begin{array}{cccc} & \begin{matrix} u_1 \\ (0.398, 0.525, 0.618); 0.1, 0.2, 0.3 \\ (0.618, 0.815, 0.959); 0.3, 0.6, 0.9 \\ (0.525, 0.618, 0.693); 0.2, 0.3, 0.4 \\ (0.815, 0.867, 0.915); 0.6, 0.7, 0.8 \end{matrix} & \begin{matrix} u_2 \\ (0.382, 0.480, 0.574); 0.3, 0.4, 0.5 \\ (0.158, 0.665, 0.919); 0.1, 0.6, 0.9 \\ (0.574, 0.665, 0.754); 0.5, 0.6, 0.7 \\ (0.276, 0.382, 0.837); 0.2, 0.3, 0.8 \end{matrix} & \begin{matrix} u_3 \\ (0.542, 0.652, 0.765); 0.6, 0.7, 0.8 \\ (0.333, 0.435, 0.542); 0.4, 0.5, 0.6 \\ (0.652, 0.765, 0.881); 0.7, 0.8, 0.9 \\ (0.145, 0.652, 0.765); 0.2, 0.7, 0.8 \end{matrix} & \begin{matrix} u_4 \\ (0.025, 0.146, 0.845); 0.1, 0.3, 0.9 \\ (0.025, 0.442, 0.845); 0.1, 0.6, 0.9 \\ (0.076, 0.231, 0.700); 0.2, 0.4, 0.8 \\ (0.076, 0.565, 0.700); 0.2, 0.7, 0.8 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} & \left(\begin{array}{cccc} \end{array} \right) & & & \end{array}$$

Step 3. We can obtain the scores of the SVTrN-numbers \tilde{A}_{ij} of the alternatives x_j ($j = 1, 2, 3, 4$) on the four attributes u_i ($i = 1, 2, 3, 4$) as follows:

$$\begin{aligned}S(\tilde{A}_{11}) &= 0.308 & S(\tilde{A}_{12}) &= 0.251 & S(\tilde{A}_{13}) &= 0.269 & S(\tilde{A}_{14}) &= 0.114 \\ S(\tilde{A}_{21}) &= 0.239 & S(\tilde{A}_{22}) &= 0.131 & S(\tilde{A}_{23}) &= 0.213 & S(\tilde{A}_{24}) &= 0.098 \\ S(\tilde{A}_{31}) &= 0.344 & S(\tilde{A}_{32}) &= 0.299 & S(\tilde{A}_{33}) &= 0.137 & S(\tilde{A}_{34}) &= 0.126 \\ S(\tilde{A}_{41}) &= 0.357 & S(\tilde{A}_{42}) &= 0.205 & S(\tilde{A}_{43}) &= 0.287 & S(\tilde{A}_{44}) &= 0.117\end{aligned}$$

Step 4. The ranking order of all SVTrN-numbers \tilde{A}_{ij} ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) as follows;

$$\begin{aligned}\tilde{A}_{11} &> \tilde{A}_{13} > \tilde{A}_{12} > \tilde{A}_{14} \\ \tilde{A}_{21} &> \tilde{A}_{23} > \tilde{A}_{22} > \tilde{A}_{24} \\ \tilde{A}_{31} &> \tilde{A}_{32} > \tilde{A}_{33} > \tilde{A}_{34} \\ \tilde{A}_{41} &> \tilde{A}_{43} > \tilde{A}_{42} > \tilde{A}_{44}\end{aligned}$$

Thus, we have:

$$\begin{aligned}\tilde{b}_{11} &= \tilde{A}_{11}, \tilde{b}_{12} = \tilde{A}_{13}, \tilde{b}_{13} = \tilde{A}_{12}, \tilde{b}_{14} = \tilde{A}_{14} \\ \tilde{b}_{21} &= \tilde{A}_{21}, \tilde{b}_{22} = \tilde{A}_{23}, \tilde{b}_{23} = \tilde{A}_{22}, \tilde{b}_{24} = \tilde{A}_{24} \\ \tilde{b}_{31} &= \tilde{A}_{31}, \tilde{b}_{32} = \tilde{A}_{32}, \tilde{b}_{33} = \tilde{A}_{33}, \tilde{b}_{34} = \tilde{A}_{34} \\ \tilde{b}_{41} &= \tilde{A}_{41}, \tilde{b}_{42} = \tilde{A}_{43}, \tilde{b}_{43} = \tilde{A}_{42}, \tilde{b}_{44} = \tilde{A}_{44}\end{aligned}$$

Step 5. The decision matrix $[b_i]_{1 \times n}$ for $i = 1, 2, 3, 4$ are given by;

$$\begin{aligned}b_1 &= (0.398, 0.525, 0.618); 0.1, 0.2, 0.3, (0.542, 0.652, 0.765); 0.6, 0.7, 0.8, (0.382, 0.480, 0.574); 0.3, 0.4, 0.5, (0.025, 0.146, 0.845); 0.1, 0.3, 0.9 \\ b_2 &= (0.618, 0.815, 0.959); 0.3, 0.6, 0.9, (0.333, 0.435, 0.542); 0.4, 0.5, 0.6, (0.158, 0.665, 0.919); 0.1, 0.6, 0.9, (0.025, 0.442, 0.845); 0.1, 0.6, 0.9 \\ b_3 &= (0.525, 0.618, 0.693); 0.2, 0.3, 0.4, (0.574, 0.665, 0.754); 0.5, 0.6, 0.7, (0.652, 0.765, 0.881); 0.7, 0.8, 0.9, (0.076, 0.231, 0.700); 0.2, 0.4, 0.8 \\ b_4 &= (0.815, 0.867, 0.915); 0.6, 0.7, 0.8, (0.145, 0.652, 0.765); 0.2, 0.7, 0.8, (0.276, 0.382, 0.837); 0.2, 0.3, 0.8, (0.076, 0.565, 0.700); 0.2, 0.7, 0.8\end{aligned}$$

Step 6. We can calculate the SVTrN-numbers $G_{hgo}(b_i) = G_{hgo}(\tilde{b}_{i1}, \tilde{b}_{i2}, \tilde{b}_{i3}, \tilde{b}_{i4})$ for $i = 1, 2, 3, 4$ as follows:

$$\begin{aligned}G_{hgo}(b_1) &= G_{hgo}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14}) \\ &= \langle (0.398^{0.24} \times 0.618^{0.26} \times 0.525^{0.26} \times 0.815^{0.24}, \\ &\quad 0.525^{0.24} \times 0.815^{0.26} \times 0.618^{0.26} \times 0.867^{0.24}, \\ &\quad 0.618^{0.24} \times 0.959^{0.26} \times 0.693^{0.26} \times 0.915^{0.24}); 0.1, 0.7, 0.9 \rangle \\ &= \langle (0.570, 0.693, 0.784); 0.1, 0.7, 0.9 \rangle\end{aligned}$$

$$\begin{aligned}
G_{hgo}(b_2) &= G_{hgo}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14}) \\
&= \langle (0.542^{0.24} \times 0.333^{0.26} \times 0.574^{0.26} \times 0.145^{0.24}, \\
&\quad 0.652^{0.24} \times 0.435^{0.26} \times 0.665^{0.26} \times 0.652^{0.24}, \\
&\quad 0.765^{0.24} \times 0.542^{0.26} \times 0.754^{0.26} \times 0.765^{0.24}); 0.2, 0.7, 0.8 \rangle \\
&= \langle (0.312, 0.538, 0.762); 0.2, 0.7, 0.8 \rangle
\end{aligned}$$

$$\begin{aligned}
G_{hgo}(b_3) &= G_{hgo}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14}) \\
&= \langle (0.382^{0.24} \times 0.158^{0.26} \times 0.652^{0.26} \times 0.276^{0.24}, \\
&\quad 0.480^{0.24} \times 0.665^{0.26} \times 0.765^{0.26} \times 0.382^{0.24}, \\
&\quad 0.574^{0.24} \times 0.919^{0.26} \times 0.881^{0.26} \times 0.837^{0.24}); 0.1, 0.8, 0.9 \rangle \\
&= \langle (0.365, 0.612, 0.726); 0.1, 0.8, 0.9 \rangle
\end{aligned}$$

$$\begin{aligned}
G_{hgo}(b_4) &= G_{hgo}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14}) \\
&= \langle (0.025^{0.24} \times 0.025^{0.26} \times 0.076^{0.26} \times 0.076^{0.24}, \\
&\quad 0.146^{0.24} \times 0.442^{0.26} \times 0.231^{0.26} \times 0.565^{0.24}, \\
&\quad 0.845^{0.24} \times 0.845^{0.26} \times 0.700^{0.26} \times 0.700^{0.24}); 0.1, 0.7, 0.9 \rangle \\
&= \langle (0.044, 0.303, 0.769); 0.1, 0.7, 0.9 \rangle
\end{aligned}$$

Step 7. The scores of $G_{hgo}(\tilde{b}_i)$ for $i = 1, 2, 3, 4$ can be obtained as follows:

$$S(G_{hgo}(b_1)) = 0.128$$

$$S(G_{hgo}(b_2)) = 0.121$$

$$S(G_{hgo}(b_3)) = 0.106$$

$$S(G_{hgo}(b_4)) = 0.070$$

It is obvious that

$$G_{hgo}(b_1) > G_{hgo}(b_2) > G_{hgo}(b_3) > G_{hgo}(b_4)$$

Therefore, the ranking order of the alternatives x_j ($j = 1, 2, 3, 4$) is generated as follows:

$$x_1 \succ x_2 \succ x_3 \succ x_4$$

The best supplier for the enterprise is x_1 .

6 Conclusion

This paper proposes three geometric operator is called SVTrN weighted geometric operator, SVTrN ordered weighted geometric operator, SVTrN ordered hybrid weighted geometric operator. Then, a approach is developed to solve multi-criteria decision making problems. It is easily seen that the proposed approach can be extended to solve more general multi-criteria decision making problems in a straightforward manner. Due to the fact that a SVTrN-number is a generalization of a triangular fuzzy number and triangular intuitionistic fuzzy number, the other existing approaches of triangular fuzzy number and triangular intuitionistic fuzzy number may be extended to SVTrN-numbers. Therefore,

1. More effective approaches for SVTrN-numbers,
2. How to determine the weight vectors for SVTrN-numbers,
3. An approach of multi-criteria decision-making with weights expressed by single valued neutrosophic sets,

will be investigated in the near future.

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