Sorting It Out: International Trade with Heterogeneous Workers

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Each worker brings a bundle of skills to the workplace, for example, quantitative and communication skills. Since employers must take this bundle as a package deal, they choose workers with just the right mix of skills. We show that international differences in the distribution of worker skill bundles—for example, Japan's abundance of workers with a modest mix of both quantitative and teamwork skills—have important implications for international trade, industrial structure, and domestic income distribution. Formally, we model two-dimensional worker heterogeneity and show that the second moments of the distribution of skills are critical, as in the Roy model.

I. Introduction

A prominent assessment of India's engineering schools found that only one in four graduates has the mix of skills needed for employability in

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the information technology sector (*New York Times*, October 17, 2006). Surprisingly, the report emphasized not so much the lack of graduates' technical skills, but the lack of teamwork and presentation skills. Indian IT employers are looking for workers with a particular bundle of skills, and the desired bundle is scarce. The fact that workers are endowed with a bundle of skills—in fancier terminology, that workers are heterogeneous in multiple dimensions—has important consequences for the way in which labor markets operate. In particular, Roy (1951), Heckman and Sedlacek (1985), Heckman and Honoré (1990), and others have shown that two-dimensional heterogeneity has profound implications for income distribution and industrial structure. The corresponding implications for international trade, however, have never been explored.

To fill this gap, we model labor markets by extending Heckman and Sedlacek (1985) to allow for a continuum of industries. The model describes the sorting behavior of heterogeneous workers endowed with two attributes, for example, quantitative and communication skills. Workers sort across industries on the basis of Ricardian comparative advantage. Industries differ by skill requirements, and each worker sorts into the industry that pays the most for the worker's particular bundle of skills.[1] The model generates two useful features: (1) As in our Indian engineering example, what matters is not just how much of each attribute is available, but *how these attributes are bundled together in workers.* (2) Although workers are perfectly mobile, each worker is *endogenously specific* to the industry that pays the most for the worker's particular bundle of attributes. Our model thus weds the perfect mobility of the Heckscher-Ohlin model with the factor specificity of the Mayer (1974) and Mussa (1974) models. Feature 1 is entirely new to the international trade literature. Feature 2 was first modeled by Mussa (1982, sec. 4) and appears in Matsuyama (1992), Leamer (1999), and Ruffin (2001).

Turning to details, we start with the first of our two features. While the effects on trade of international differences in endowments are well known, nothing is known about the effects of other moments of the distribution of worker types. For example, since capital-to-labor and other endowment ratios are similar across rich countries, it is often argued that the distribution of endowments cannot explain North-North trade (e.g., Leamer 1993, 439). However, it is also often argued that differences between Japanese and U.S. workers influence production patterns and comparative advantage. For example, Japanese comparative advantage in goods involving long chains of production and re-

[1] Leamer (1999), Grossman and Maggi (2000), and Grossman (2004) are recent prominent studies that feature trade models of worker sorting, but the sorting and hence the predictions are very different from our own. This will become clear shortly.

quiring reliability is often ascribed to a more frequent bundling of technical skills with the ability to communicate in worker circles. We model this in terms of higher moments of the distribution of worker characteristics, for example, the correlation between technical and communication skills. The idea that higher moments matter was put forward by Grossman and Maggi (2000) and Grossman (2004) and has strongly influenced our thinking. In their work, workers are endowed with talent, and it is the second moment or dispersion of talent that is a source of international comparative advantage. We extend their insights using a very different model of labor markets.[2]

Two-dimensional worker heterogeneity has many implications for domestic income distribution, implications that are very different from either the Stolper-Samuelson theorem or the predictions of the specific factors model (Mayer 1974; Mussa 1974). Even though workers are perfectly mobile, their earnings will differ across industries and *within* industries. This allows us to describe the impact of international trade on within-industry inequality, between-industry inequality, and economy-wide inequality. An earlier version of this paper (Ohnsorge and Trefler 2004) offers additional related results dealing with "skill price equalization," the political economy of protection, and skill-biased technical change.

With regard to our second feature (endogenous specificity), issues of factor immobility and heterogeneity have frequently appeared in the trade literature. In the Heckscher-Ohlin model, factors are homogeneous and perfectly mobile. In the specific factors model, capital is perfectly immobile, which means that it is heterogeneous: the productivity of sector-specific capital is high in its own sector and zero in all other sectors. There are two problems with tying heterogeneity to perfect immobility. First, the assumption of perfect immobility is too strong (Leamer 1980; Grossman 1983). Second, there is no reason to link immobility with heterogeneity. This link is broken by Mussa (1982),

[2] In Grossman and Maggi (2000), machines are produced in long chains of production involving many workers. The machine is reliable only if each worker's input is reliable. This "supermodularity" means that in equilibrium workers will be paired with others having similar levels of talent. In contrast, software output depends on the input of the most talented worker. This "submodularity" means that the most talented worker is paired with the least talented, the second most talented with the second least talented, and so on. Their main prediction is that the country with the greater dispersion in worker talents will have a comparative advantage in software. In our model there is no teamwork between workers, but there is "teamwork" between the two skills that a worker brings to the workplace. This leads to our trade and dispersion result in Sec. VII. In Grossman (2004), machinery requires teamwork and software does not. Teamwork is subject to costly monitoring and incomplete contracting, which encourages talented workers to sort into software. Trade causes the country with greater dispersion in talent to increase software production. This resolves the contracting problem for talented workers, thus raising inequality. In contrast, our inequality and dispersion result in Sec. VII is driven by sorting rather than by incomplete contracting.

Matsuyama (1992), Leamer (1999), and Ruffin (2001), all of whom assume that workers are perfectly mobile across industries but heterogeneous in terms of their productivities. For concreteness, let there be two industries and let t_i be the productivity of a worker in industry i. Worker heterogeneity means that different workers have different pairs (t_1, t_2). A worker with a high t_1/t_2 heeds Ricardo's advice to sort into industry 1 and earn quasi rents there. Thus in these papers as well as ours, heterogeneity generates specificity even when workers are perfectly mobile.

Leamer (1999) peels back the onion on heterogeneity by endogenizing the t_i. He introduces variable worker effort and assumes that effort complements capital. A worker with a low disutility of effort chooses a high level of effort, thus explaining why the worker is more productive in the capital-intensive industry. We "unpack" the t_i in a different way, by making them depend on workers' two-dimensional skill bundles.

The paper is organized as follows. Sections II and III set up the model. Section IV presents the key insight from two-dimensional worker heterogeneity. Section V dispenses with some formalities. Sections VI, VII, and VIII develop multigood Rybczynski, Heckscher-Ohlin, and income distribution theorems for higher-order moments of the distribution of endowments. Section IX presents conclusions.

II. The Model

Each worker brings two attributes to the workplace, H and L. While human capital and brawn are obvious and familiar attributes, in describing trade among rich countries we also have in mind subtler attributes such as quantitative abilities, communication skills, and teamwork skills. A type (H, L) worker employed in industry i produces a task level of $T(H, L, i)$. An employer cannot unbundle a worker's attributes and thus cares only about $T(H, L, i)$. This "bundling" assumption is central to what follows and is the core assumption of a large class of Roy-like (1950, 1951) models. The particular formulation used here is a generalization of Heckman and Sedlacek (1985) to allow for a continuum of industries (see also Rosen 1972, 1978; Sattinger 1975; Willis and Rosen 1979; Heckman and Scheinkman 1987; Heckman and Honoré 1990).

To abstract from other sources of comparative advantage that might affect worker sorting and international trade flows, we assume that industry output is the sum of the tasks performed by workers in the industry. This is a common simplification in the literature (e.g., Mussa 1982; Ruffin 2001). It implies that $T(H, L, i)$ is also a worker's marginal product. A worker is paid the value of her marginal product. We assume that T is subject to constant returns to scale in H and L so that the earnings of a type (H, L) worker in industry i are $W(H, L, i) = P(i)T(H/L, 1, i)L$, where $P(i)$ is the producer price and we have used constant returns to scale.

The analysis is greatly simplified by working with attributes $(H/L, L)$ rather than (H, L) and by log-linearizing earnings. To this end, define

$$\begin{aligned} l &\equiv \ln L, \\ s &\equiv \ln(H/L), \\ p(i) &\equiv \ln P(i), \\ t(s, i) &\equiv \ln T(H/L, 1, i) \end{aligned} \tag{1}$$

so that log earnings can be written as

$$w(s, l, i) = p(i) + t(s, i) + l. \tag{2}$$

Equation (2) is our first of two core equations. As will be explained below, it is useful to think of s as determining a worker's comparative advantage—s for sorting. Also, it is useful to think of l as determining a worker's absolute advantage: l shifts $w(s, l, i)$ up and down by the same amount for all industries i.[3]

There is a continuum of industries indexed by $i \in [0, 1]$. A type (s, l) worker chooses the industry that maximizes $w(s, l, i)$. Note that the optimal choice of industry, $i(s)$, depends on comparative advantage s, not on absolute advantage l.

With regard to trade issues, the H-intensity of industry i is defined in the usual way on the basis of the production function $T(H, L, i)$. See Appendix A for details. We assume that there are no factor intensity reversals. Then we can choose the ordering of industries so that larger values of i correspond to more H-intensive industries. This is a choice of ordering, not an assumption. The log of the task function in (s, l) space is $t(s, i) + l$. We define the s-intensity of industry i in the usual way on the basis of the log production function $t(s, i) + l$. The following lemma shows the equivalence of four familiar concepts. Throughout this paper subscripts denote derivatives, for example, $t_{si} = \partial^2 t(s, i)/\partial s \partial i$.

LEMMA 1. The following statements are equivalent. (1) The larger i is, the more H-intensive the industry is. (2) The larger i is, the more

[3] We assume that T is twice differentiable in its arguments and increasing in H and L. This implies that t is twice differentiable in (s, i) with $t_s > 0$ (t_s is the derivative of t with respect to s). We will use $t_s > 0$ repeatedly.

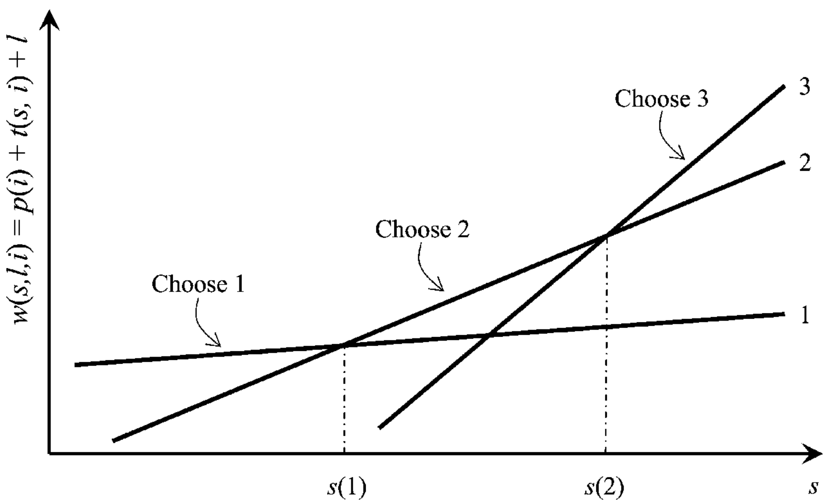

FIG. 1.—Worker sorting

s-intensive the industry is. (3) The cross-partial $t_{si} > 0$. (4) If $H'/L' > H/L$ and $i' > i$, then

$$\frac{T(H', L', i')}{T(H', L', i)} > \frac{T(H, L, i')}{T(H, L, i)}.$$

The proof appears in Appendix A. Part 1 is an assumption-free ordering of industries. Part 2 states that the orderings of industries by H-intensity and s-intensity are equivalent. Part 4 is the textbook Ricardian comparison of four marginal products: when two workers are being compared, the one with the higher H/L is said to have a comparative advantage in the H-intensive industry. Part 3 is a more convenient way of stating comparative advantage that is sometimes used in the sorting literature (e.g., Sattinger 1975).

For concreteness and in order to provide a graphical treatment of sorting that will be useful later, suppose that the task function is Cobb-Douglas: $T = (H/L)^{\beta(i)}L$. Part 1 of lemma 1 states that β is increasing in i. Equation (1) implies that $t(s, i) = \beta(i)s$. Figure 1 plots $w(s, l, i) = p(i) + \beta(i)s + l$ against s for three industries. Although i is a continuous index on the unit interval, in the figure we abuse notation by denoting the three industries as $i = 1, 2, 3$. With subscripts denoting derivatives, the slope of the log earnings function is $w_s = t_s = \beta > 0$. Part 3 of lemma 1 states that $t_{si} = \beta_i > 0$; that is, the β are increasing in i just as in part 1. Graphically, $t_{si} > 0$ means that the higher i is, the steeper the slope of the log earnings function.

The sorting rule is also illustrated in the figure. A worker with an s between $s(1)$ and $s(2)$ chooses industry 2. The key sorting result is that high-s workers sort into s-intensive industries. This graphical result generalizes to a continuum of industries with non-Cobb-Douglas task functions. In particular, $i(s)$ is a nondecreasing correspondence. (Recall that $i(s)$ is the optimal choice of industry for a type s worker.) See Appendix E for a proof.

Nondecreasingness is all we need to know about $i(s)$ in order to prove the results of this paper. However, for the sake of clarity alone we add two assumptions that ensure that $i(s)$ is strictly increasing: $w(s, l, i)$ is twice differentiable in i and has a unique maximum. Then from equation (2), a type (s, l) worker chooses the industry $i(s)$ that satisfies the first-order condition $w_i = p_i(i(s)) + t_i(s, i(s)) = 0$. By uniqueness, the second-order condition $w_{ii} = p_{ii} + t_{ii} < 0$ is satisfied at $i = i(s)$. Implicit differentiation of the first-order condition yields[4]

$$\frac{\partial i(s)}{\partial s} = -\frac{t_{si}}{p_{ii} + t_{ii}} > 0.$$

Thus part 3 of lemma 1 ($t_{si} > 0$) implies that $i(s)$ is strictly increasing in s.[5]

What does figure 1 look like for a continuum of industries? There will be an infinite number of curves (lines in the Cobb-Douglas case) whose upper envelope $w(s, l, i(s))$ is increasing in s.[6]

Figure 1 highlights a key feature of models of heterogeneous worker sorting. Suppose that firms in industry 2 offer slightly higher earnings so that the $w(s, l, 2)$ profile shifts up. Then industry 2 firms will attract slightly more workers (those with an s near $s(1)$ or $s(2)$). That is, the partial equilibrium supply of workers to the industry is upward-sloping. This implication of worker heterogeneity differs from the Heckscher-Ohlin and specific factors models in which perfect mobility of homogeneous workers leads to earnings equalization across industries.

[4] To prove the following equation, note that $w_i(s, l, i(s)) = 0$ implies $w_{si} + w_{ii}\partial i(s)/\partial s = 0$, where $w_{ii} = p_{ii} + t_{ii}$ and $w_{si} = t_{si}$.

[5] Without the two assumptions just made, $i(s)$ may be nondecreasing; i.e., it may be independent of s in some region. This means that many different worker types choose the same industry. Any such region must be an interval, e.g., $(s(1), s(2))$ in fig. 1. But this "intervals" case is exactly the same as the finite-industry case considered in our working paper (Ohnsorge and Trefler 2004). Hence, as shown in that paper, all our results hold even if $i(s)$ is not a strictly increasing function. Unfortunately, the proofs in the working paper are much more complicated.

[6] In our working paper (Ohnsorge and Trefler 2004), we established all our results for the case with a finite number of industries. Moving to the continuum dramatically shortens the proofs. As an example of this, under our differentiability assumptions the slope of $w(s, l, i(s))$ is $w_s + w_i i_s$. By the first-order condition, $w_i = 0$. By eq. (2), $w_s = t_s > 0$. Hence the upper envelope slopes upward.

III. The Distribution of Worker Types: Endowments

Let $F_{sl}(s, l)$ be the measure or number of type (s, l) workers in the economy.[7] Since there are no scale effects in the economy, we take F_{sl} to be a cumulative distribution function and assume that it has a density function f_{sl}. For many of the results of this paper we follow the time-honored tradition in the Roy literature of assuming that s and l are bivariate normal:

$$\begin{bmatrix} s \\ l \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_l \\ \rho\sigma_s\sigma_l & \sigma_l^2 \end{bmatrix}\right), \tag{3}$$

where ρ is the correlation between s and l. We set $\sigma_s = \sigma_l = 1$ whenever they play no role.[8]

IV. The Role of Two Attributes: Income Distribution

The obvious thing about figure 1 is that workers are sorting on the basis of the single attribute s just as in Mussa (1982), Matsuyama (1992), Leamer (1999), and Ruffin (2001). What then is the role of two attributes? It turns out that both attributes are needed to discuss production, earnings, and earnings inequality. In particular, the correlation ρ between l and s is crucial. To explain this we use Cobb-Douglas task functions, though this plays only a temporary expository role.

Under normality the expectation of l given s is

$$\mathrm{E}(l|s) = \rho(s - \mu). \tag{4}$$

This is the second of our two core equations. As before, in the Cobb-Douglas case we have $w(s, l, i) = p(i) + \beta(i)s + l$. Taking expectations over l conditional on s yields

$$\mathrm{E}(w(s, l, i(s))|s) = p(i) - \rho\mu + [\beta(i) + \rho]s. \tag{5}$$

This is the average log wage of type s workers who have sorted into industry i. An average is needed because not all workers who sort into $i(s)$ have the same l.

Figure 2a plots $\mathrm{E}(l|s)$ against s for the case in which $\rho < 0$. It also plots the conditional distribution of l given s, $f_{l|s}(l|s)$. The larger s is, the more

[7] When $F_{HL}(H, L)$ is the measure of (H, L) workers in the economy, $F_{sl}(s, l)$ is derived trivially from $F_{HL}(H, L)$ together with eq. (1).

[8] Defining $\sigma_h^2 \equiv \sigma_s^2 + \sigma_l^2 + 2\rho\sigma_s\sigma_l$, we get

$$\begin{bmatrix} \ln H \\ \ln L \end{bmatrix} \sim N\left(\begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_h^2 & (\sigma_l + \rho\sigma_s)/\sigma_h \\ (\sigma_l + \rho\sigma_s)/\sigma_h & \sigma_l^2 \end{bmatrix}\right).$$

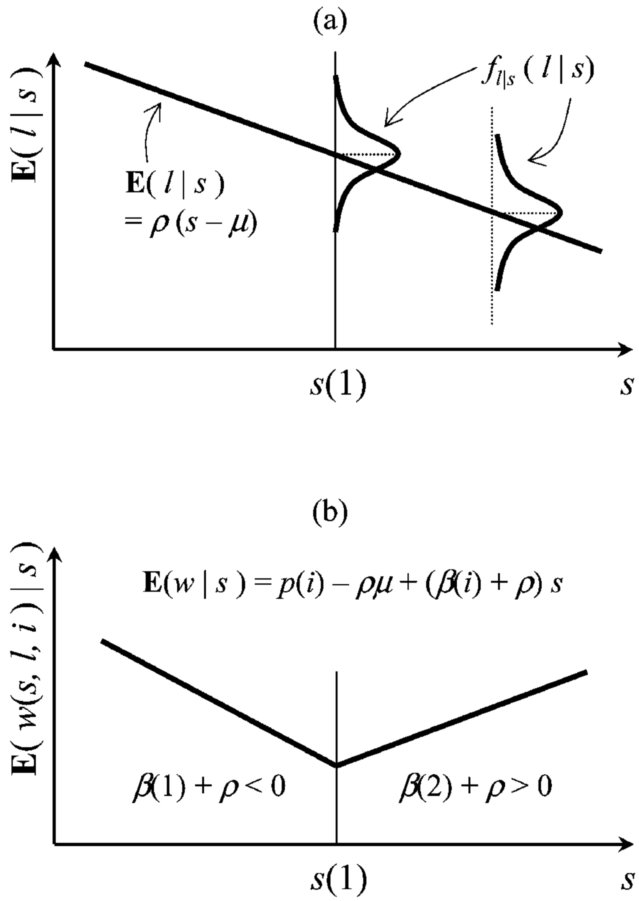

Fig. 2.—Income distribution

the conditional distribution of l is shifted toward smaller values of l. Figure 2*b* plots $E(w(s, l, i(s))|s)$ against s for the case of two industries. This wage profile is piecewise linear with slope $\beta(i) + \rho$. The key is that the β and ρ terms correspond to productivity and two-attribute selection effects, respectively. With l held constant, $w_s = \partial[p(i) + \beta(i)s + l]/\partial s = \beta(i)$. That is, workers with more s produce more task and hence earn more. This is the productivity effect. Further, $\partial E(l|s)/\partial s = \rho$; that is, the average amount of l that an s type has depends on ρ. If ρ is negative, then higher s is associated on average with lower l and hence with lower productivity and earnings. This is the two-attribute selection effect. *It does not appear anywhere in the international trade literature.*[9] To recap, as s increases, (a) average earnings rise because s is a productive input and (b) average earnings may rise or fall depending on whether the average l rises or falls, that is, depending on the two-factor selection effect ρ.

Figure 2 displays the case in which $\beta(1) + \rho < 0 < \beta(2) + \rho$ so that the two-attribute selection effect dominates in industry 1 and the productivity effect dominates in industry 2. The average earnings profile illustrated in figure 2 can also be downward-sloping throughout (i.e., $\beta(i) + \rho < 0$ for all i) or upward-sloping throughout (i.e., $\beta(i) + \rho > 0$ for all i). The message to be taken from this is that while s determines worker sorting, it does not determine the amount of the other productive asset l that workers bring to the workplace. Thus s alone does not determine output, earnings, or inequality.

V. Industry Output and Equilibrium

Let $Y(i)$ be output in industry i. It is the sum of the tasks performed by workers who choose i. A type (H, L) worker produces task level $T(H, L, i)$. Equivalently, a type (s, l) worker produces task level $e^{t(s,i)+l}$.[10] Hence the sum of tasks in industry $i = i(s)$ is

$$Y(i) = \int_{-\infty}^{\infty} e^{t(s,i)+l} f_{sl}(s, l)\,dl \quad \text{for } i = i(s). \tag{6}$$

Appendix B provides an expression for output under the assumption of normality.

With regard to the definition of equilibrium, labor market equilibrium is described by the set of earnings functions $w(s, l, i)$ that satisfy profit-maximizing *demand* for tasks (eq. [2]) and a set of earnings-maximizing *supply* of tasks $i(s)$. Product markets are perfectly competitive. They clear at the international level, and there are no barriers to trade so that all countries face the same product prices $p(i)$. Equilibrium in product markets is described by a set of prices $p(i)$ that (1) equate industry supplies $Y(i)$ with as yet unspecified industry demands and (2) balance international trade.

[9] We have correctly ignored a third effect that is not apparent in figures with a finite number of industries. With a continuum of industries, an increase in s increases $i(s)$, which in turn raises average earnings by $w_i i_s$. However, this effect is zero because the first-order condition is $w_i = 0$.

[10] From eq. (1),

$$\exp[t(s, i) + l] = \exp[\ln T(H/L, 1, i) + \ln L] = T(H/L, 1, i)L = T(H, L, i).$$

VI. The Role of ρ

The remainder of this paper describes how the distribution of endowments influences industrial structure, international trade, and income distribution. The endowments of a country are completely described by the parameters of the distribution of (s, l), which under normality are μ, ρ, σ_s, and σ_l. In this section we examine the influence of ρ. For concreteness in interpreting ρ, let H be quantitative skills, let L be communication skills, and recall that $s \equiv \ln H/L$. A worker with a large s has a *comparative* advantage in quantitative-intensive industries; that is, we showed that high-s workers sort into quantitative-intensive industries. Given s, a worker with a large l has an *absolute* advantage in all industries, that is, is productive in all industries. To see this, let $h \equiv \ln H$. For a given $s = h - l$, a large l implies a large h and hence an abundance of both skills. Another way of making this point is that in equation (2), l shifts up the earnings function by the same amount for all i. Under this interpretation of s and l, ρ is the correlation across workers between comparative advantage and absolute advantage. In a country with a large positive ρ, workers with an absolute advantage in all industries tend to be found in quantitative-intensive industries and workers with an absolute *dis*advantage in all industries tend to be found in communication-intensive industries. The reverse is true in a country with a large negative ρ. Finally, when ρ is close to zero, workers with an absolute advantage in all industries are spread out randomly across quantitative- and communication-intensive industries.

To our knowledge there are no studies of international differences in ρ. We thus made a few rudimentary calculations of our own using the International Adult Literacy Survey. The survey was conducted in 15 western European and English-speaking countries and has a sample size of about 3,000 adults per country. The survey scores adults on their ability to understand text-based instructions (e.g., dosage labels on over-the-counter drugs) and to do basic mathematical operations (e.g., calculating regional temperature differentials using a newspaper's weather page). Adults received two summary scores, one for text-based understanding and one for quantitative skills. We interpret these two scores as measures of L and H, respectively. For each country we calculated the correlation across 3,000 adults of s with l. We interpret this correlation as a measure of ρ.

Table 1 reports the results. Column 1 shows that l and h are highly correlated in all countries. Adults who have one skill tend to have both skills, that is, tend to have an absolute advantage in both skills. Column 2 shows that ρ tends to be negative: *absolute* advantage in text-based understanding is negatively correlated with *comparative* advantage in quantitative skills. Columns 3 and 4 show that there is large and statis-

TABLE 1
International Differences in ρ

Country	corr(h, l) (1)	ρ (2)	$\rho - \rho_{US}$ (3)	t-Statistic (4)
Germany	.77	−.52	−.25	9.57
Sweden	.77	−.39	−.13	5.36
Finland	.77	−.33	−.07	2.81
Denmark	.76	−.33	−.07	2.71
Italy	.81	−.32	−.05	2.17
Switzerland	.82	−.31	−.05	2.06
Netherlands	.80	−.31	−.05	1.88
Norway	.78	−.31	−.04	1.78
Canada	.84	−.28	−.02	.83
Belgium	.81	−.28	−.01	.50
New Zealand	.82	−.28	−.01	.48
United States	.84	−.27	.00	.00
Great Britain	.83	−.24	.03	−1.06
Northern Ireland	.86	−.24	.03	−1.06
Ireland	.82	−.24	.03	−1.05

tically significant cross-country variation in the estimated ρ. Column 3 reports the difference between a country's ρ and the ρ of the United States (i.e., $\rho - \rho_{US}$), and column 4 reports the *t*-statistic for this difference. Interestingly, northern European countries such as Sweden and Germany have the lowest correlations, whereas English-speaking countries such as the United States and Great Britain have the highest correlations. This is, of course, a very cursory assessment of cross-country differences in ρ.

A. *Production: A "Correlate" of Rybczynski*

In this subsection we examine how changes in ρ lead to changes in industrial structure. This is a variant of a Rybczynski theorem, that is, of a theorem that describes the effect of endowment changes holding product prices $p(i)$ and factor prices constant. (Prices will be endogenous in the next subsection.) To get a flavor of things, let us return to the two industries of figure 2. Figure 2 is reproduced on the right side of figure 3. We continue to abuse notation by naming industries $i = 1, 2$. Figures 3*a* and *c* illustrate the case of $\rho > 0$. A rise in ρ causes $E(l|s) = \rho(s - \mu)$ to pivot around the point $(\mu, 0)$. To keep figure 3 simple, we have drawn it for the case in which $s(1) = \mu$. When prices are held fixed, a rise in ρ does not alter the sorting rule.[11] In terms of figure 3, a rise in ρ does not alter $s(1)$. Figures 3*c* and *d* plot E(w(s, l,

[11] The sorting rule is derived from maximizing $w(s, l, i) = p(i) + t(s, i) + l$ with respect to i. The rule thus depends on ρ only via equilibrium prices $p(i)$, which are being held fixed in this section.

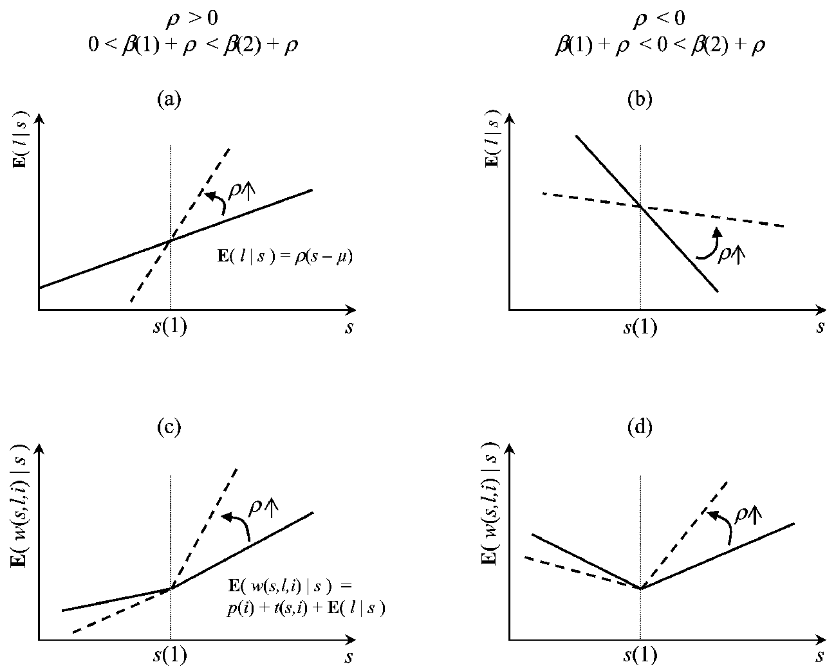

FIG. 3.—The impact of ρ

$i)|s)$. From equation (2), the expected log of tasks is just $E(w(s, l, i)|s) - p(i)$. Hence the wage profiles in figures 3*c* and *d* are similar to output profiles.

As ρ rises, the average level of l falls for workers with $s < s(1)$ so that output of industry 1 falls. For workers with $s > s(1)$, the opposite is true so that output of industry 2 rises. This is a Rybczynski-style result: as ρ rises, industry 2 expands and industry 1 contracts. Rybczynski results typically hold in clear form only when there are two industries. Our figure 3 result generalizes to any number of industries.

THEOREM 1 (Industrial structure and ρ). Define $s^{\rho} \equiv \mu + \rho$ and consider an increase in ρ, the correlation between s and l. With product prices held constant, all industries with $i > i(s^{\rho})$ expand and all industries with $i < i(s^{\rho})$ contract.

Proof. Under normality $\ln Y(i)$ is given by Appendix equation (B2). Differentiating this equation with respect to ρ and setting $\sigma_s = \sigma_l = 1$ yields

$$\frac{d \ln Y(i)}{d\rho} = s - \mu - \rho = s - s^{\rho}, \tag{7}$$

where i is evaluated at $i = i(s)$. Since $i(s)$ is strictly increasing, theorem 1 follows immediately.

Equation (7) also implies a "magnification" effect: the more extreme an industry's s-intensity as measured by $|s - s^{\rho}|$, the greater the log output change $|d\ln Y(i)/d\rho|$.

B. *Trade: A "Correlate" of Heckscher-Ohlin*

The conventional wisdom is that the similarity of endowments among northern countries makes the Heckscher-Ohlin model irrelevant for describing North-North trade. This view has been challenged by Davis (1997) and by Davis and Weinstein (2001), who argue that the factor content of intraindustry trade is determined by endowments. Our Rybczynski result suggests that subtler aspects of international endowment differences may matter for both North-North and North-South trade. To model the implications of international differences in ρ for trade patterns, we make the usual Heckscher-Ohlin similarity assumptions.

ASSUMPTION 1. (*a*) Preferences are homothetic and identical internationally. (*b*) The task functions $t(s, i)$ are identical internationally. (*c*) There are no barriers to trade so that consumers in both countries face the same prices $p(i)$. (*d*) Trade is balanced.

In addition, we remind the reader that in this section we are assuming that μ, σ_s, and σ_l are the same across countries. Restated, other than ρ, all parameters of the distribution of endowments are the same across countries. Thus differences in ρ are the only source of differences across countries and the only reason for international trade.

Consider first the standard trade theorem in which comparative advantage is defined in terms of autarky price differences. Let ρ and ρ^* be the correlations in the home and foreign countries, respectively. We start with two identical economies ($\rho = \rho^*$) and consider two goods, $i > i(s^{\rho})$ and $i' < i(s^{\rho})$. Let q^a (q^{a*}) be the home (foreign) country's autarky price of i relative to i'. Now let ρ^* fall so that $\rho > \rho^*$. With $q^a = q^{a*}$ held constant, theorem 1 states that the home country will have the larger output of i and the smaller output of i'. Since demand is homothetic and internationally identical, the home country will have an excess supply of i and prices will adjust. In particular, i will be relatively cheaper at home than abroad: $q^a < q^{a*}$. That is, autarky relative prices reveal the high-ρ home country to have a comparative advantage in the s-intensive good. We therefore expect that under free trade the home country will export the more s-intensive good (i) and import the more l-intensive good (i'). The next theorem shows this.

THEOREM 2 (International trade and ρ). Consider a world with two countries that are identical except for the correlation between s and l. More specifically, let assumption 1 hold and assume that μ, σ_s, and σ_l

are the same in both countries. (1) There exists an equilibrium. (2) In equilibrium there is a cutoff industry i^{ρ} such that the high-ρ country exports all relatively s-intensive goods ($i > i^{\rho}$) and imports all relatively l-intensive goods ($i < i^{\rho}$).

The proof follows from our Rybczynski theorem and appears in Appendix C.[12]

The Heckscher-Ohlin theorem predicts trade on the basis of levels of s and l. Theorem 2 predicts trade on the basis of a higher moment of the distribution of endowments.[13]

C. *Average Earnings and Earnings Inequality*

We have already seen in figures 3*c* and *d* that as ρ rises, average earnings fall in l-intensive industries ($i < i^{\rho}$) and rise in s-intensive industries ($i > i^{\rho}$). This is a general result. From equations (2) and (4), average earnings are

$$\mathrm{E}(w(s,\ l,\ i)|s) = p(i) + t(s,\ i) + \rho(s - \mu), \tag{8}$$

where $i = i(s)$. The derivative of this with respect to ρ is $s - \mu$ so that the high-ρ economy will have high average wages in s-intensive industries ($s > \mu$) and low average wages in l-intensive industries ($s < \mu$). Obviously this differs substantially from the Heckscher-Ohlin and specific factors models, where earnings of mobile factors are the same in all industries.

International differences in ρ have implications for earnings inequality. We first consider between-industry inequality. (One may think of this as between-group inequality where groups are defined by s.) From figure 3, if $\rho > 0$, then a rise in ρ steepens the economywide wage profile, thus raising between-industry inequality. This generalizes to a continuum of industries. From equation (8), the slope of $\mathrm{E}(w(s,\ l,\ i(s))|s)$ with respect to s is $t_s(s,\ i(s)) + \rho$. (In the Cobb-Douglas case, $t_s(s,\ i) + \rho = \beta(i) + \rho$ as in fig. 3.) When $\rho > 0$, this is positive and increasing in ρ, just as in figure 3. Thus the high-ρ country will have the steeper earnings profile and the greater inequality. When $\rho < 0$, changes in ρ have somewhat more complicated effects on between-industry inequality.

Next we consider within-industry inequality. This arises from the fact

[12] The generalization to many countries is straightforward: Rather than a single i^{ρ}, each country k has its own cutoff industry $i^{\rho}(k)$.

[13] A rise in the correlation between s and l (i.e., a rise in ρ) is different from a rise in the correlation between $\ln H$ and $\ln L$. The former is the correlation between comparative and absolute advantage. The latter is about endowment inequality: a high correlation between $\ln H$ and $\ln L$ implies that a worker with a lot of H also has a lot of L. We discuss endowment inequality in the next section. The results there are identical to results based on international differences in the correlation between $\ln H$ and $\ln L$. We therefore do not report separate comparative static results based on the correlation between $\ln H$ and $\ln L$.

that even within industries workers have different amounts of l. We measure within-industry inequality by the variance of log wages given s, $\mathrm{Var}\,(w(s,\ l,\ i(s))|s) = \mathrm{Var}\,(l|s) = 1-\rho^2$. If $\rho < 0$, then the high-ρ country has greater within-industry inequality. In summary, the model has many implications for average earnings and earnings inequality that do not appear in the Heckscher-Ohlin and specific factors models. We have only skimmed the surface here.

VII. Endowment Inequality

We next turn to the role of endowment inequality. Consider two economies, one of which has more mass in the tails of its bivariate density $f_{sl}(s,\ l)$ and hence has more workers with extreme values of either s or l. This will have implications for trade flows and inequality that are related to those discussed in Grossman and Maggi (2000) and Grossman (2004). As noted in the introduction, our trade mechanism is related to the Grossman and Maggi supermodularity mechanism, and our inequality mechanism is very different from the Grossman incomplete-contracts mechanism. To focus ideas, consider first a simple example with three goods: movies, information technologies, and machinery. Suppose that the production of movies is intensive in communication skills l and the production of Silicon Valley information technologies is intensive in quantitative reasoning s. In contrast, machinery is an "O-ring" reliable good whose production involves many components and whose overall reliability is the reliability of the least reliable component. Reliability therefore depends on a mix of worker skills in the sense of requiring both l and s. If, say, the United States has a more unequal distribution of endowments than Germany, then the United States will export movies and information technologies to Germany and Germany will export machinery to the United States.

Formalizing this is tricky because there is no consensus on how to define "greater inequality" for bivariate distributions. We proceed by defining a form of mean-preserving spread for bivariate distributions. To ease notation we have been setting the variance of s (σ_s) and the variance of l (σ_l) to unity. In this section we reintroduce σ_s and σ_l explicitly. We define a bivariate mean-preserving spread as an increase in σ_s and σ_l that does not change any means ($\mathrm{E}(s) \equiv \mu$ and $\mathrm{E}(l) \equiv 0$) or any conditional means ($\mathrm{E}(l|s) \equiv \rho\sigma_l(s-\mu)/\sigma_s$ and $\mathrm{E}(s|l) \equiv \mu + (\rho\sigma_s l/\sigma_l)$). This is satisfied by increases in σ_s and σ_l that leave σ_s/σ_l unchanged. Let γ be an index of endowment inequality and let $\sigma_s(\gamma)$ and $\sigma_l(\gamma)$ be increasing unit-elastic functions ($\partial \ln \sigma_k/\partial\gamma = 1,\ k = s,\ l$) so that $d\ln(\sigma_s/\sigma_l)/d\ln\gamma = 0$. Then an increase in γ raises variances σ_s and σ_l

without affecting means or conditional means. We associate an increase in endowment inequality with an increase in γ.[14]

A. *Production: A Rybczynski "Variant"*

We start with a Rybczynski-like theorem, that is, a theorem that holds product prices $p(i)$ constant. When $p(i)$ is held constant, an increase in γ has no effect on a worker's earnings (eq. [2]) and hence no effect on the sorting rule $i(s)$. It thus has no effect on our diagrams. What an increase in γ does is redistribute the mass of f_{sl} away from its middle toward its tails. Therefore, the most l-intensive and s-intensive industries attract more workers and experience a relative rise in output. Differentiating Appendix equation (B2) with respect to γ and then setting $\sigma_l = \sigma_s = 1$ yields

$$\frac{\partial \ln Y(i(s))}{\partial \gamma} = (s-\mu)^2 - \rho^2. \tag{9}$$

This is a quadratic equation in $s-\mu$ with roots at $\pm\rho$ (see fig. 4). To restate, the roots occur at $s = \mu \pm \rho$. Thus the country with the greater endowment inequality will have greater output in industries $i < i(\mu - \rho)$ and $i > i(\mu + \rho)$ and lesser output in the middle industries $i \in (i(\mu-\rho),\ i(\mu+\rho))$.[15]

[14] This parameterization of inequality satisfies Atkinson and Bourguignon's (1982) multivariate generalization of second-order stochastic dominance, which they use to measure bivariate inequality.

In the International Adult Literacy Survey, there are large international differences in the variance of test scores. As before, let H and L be (roughly speaking) the quantitative and communication scores, respectively. Consider $l = \ln L$ and $s = \ln H/L$. The United States has the largest values of σ_s and σ_l among the 15 countries. For example, σ_s is 50 percent higher in the United States than in Germany and σ_l is 100 percent higher in the United States than in Germany. Thus, by adult literacy measures, the United States has much greater endowment inequality than Germany.

[15] If σ_l and σ_s are not set to unity, then $\partial \ln Y(i(s))/\partial\gamma = [(s-\mu)/\sigma_s]^2 + k$, where $k \equiv \sigma_l^2(1-\rho^2) - 1$. If $k > 0$, then $\partial \ln Y(i(s))/\partial\gamma$ is everywhere above zero; i.e., all industries expand. This does not affect our Heckscher-Ohlin result since the result depends on the sign of *relative* output effects:

$$\frac{\partial \ln Y(i(s))}{\partial\gamma} - \frac{\partial \ln Y(i(s'))}{\partial\gamma} = \sigma_s^2[(s-\mu)^2 - (s'-\mu)^2].$$

Thus the sign of relative effects is independent of σ_l and σ_s. Why would output expand in all industries? As the tails of the distribution of l fatten, by Jensen's inequality, $\mathrm{E}(e^l|s)$ of eq. (B2) rises for all s, thus driving up output in all industries. Unlike other results in this paper, the all-industries-expanding result is functional-form dependent (our log specification) and is important only when σ_l is large.

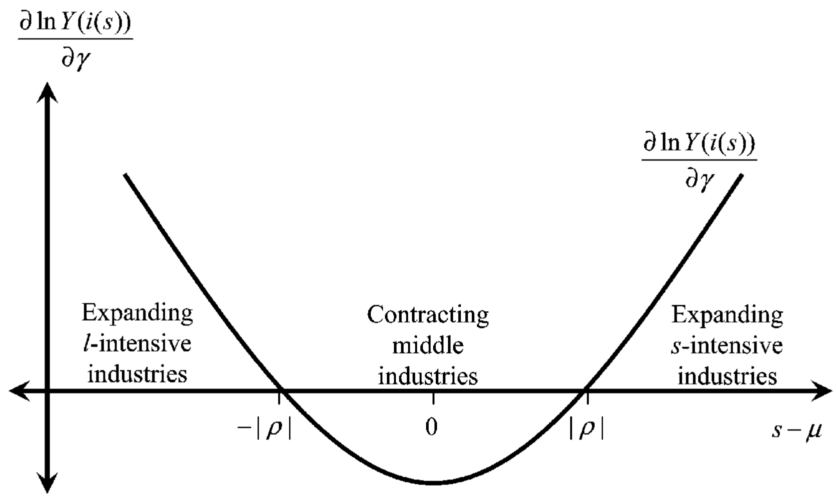

FIG. 4.—The Rybczynski theorem for rising endowment inequality

B. *Trade: A Heckscher-Ohlin "Variant"*

The previous subsection showed that international differences in production patterns can be driven by endowment inequality. This has immediate implications for trade, including North-North trade. First consider a comparison of autarky with free trade. Let assumption 1 hold and suppose that there are only two countries, home and foreign, with home having the higher level of endowment inequality γ. Our production structure result implies that in autarky the home country produces more of the most l- and s-intensive goods than the foreign country does. Hence the home country has the lower autarky relative price for these extreme goods. That is, autarky prices reveal the home country to have a comparative advantage in the most l- and s-intensive goods. We therefore expect the home country to export these goods and import the middle goods. The next theorem confirms this prediction. Recall that μ, ρ, σ_s, and σ_l completely characterize the distribution of endowments (s, l).

THEOREM 3 (International trade and γ). Let the variances $\sigma_s(\gamma)$ and $\sigma_l(\gamma)$ be increasing and unit-elastic functions of γ so that γ is an index of endowment inequality. Consider a world with two countries that are identical except for endowment inequality. More specifically, let assumption 1 hold and assume that μ and ρ are the same in both countries, but allow γ (and hence σ_s and σ_l) to differ across countries. (1) There exists an equilibrium. (2) In equilibrium there exist industries $\underline{i}$ and $\bar{i}$ such that the high-inequality country exports both the most s-intensive goods ($i > \bar{i}$) and the most *l*-intensive goods ($i < \underline{i}$) while importing the middle goods ($\underline{i} < i < \bar{i}$).

The proof appears in Appendix D.

Theorem 3 demonstrates that differences in endowment inequality can provide a coherent account of trade between rich countries. To return to our example above, if the United States had greater endowment inequality than Germany, the United States would have a comparative advantage in making movies, which intensively use communication skills *l*. The United States would also have a comparative advantage in Silicon Valley information technologies, which intensively use quantitative reasoning *s*. On the other hand, Germany would have a comparative advantage in machinery and other O-ring reliable goods produced using long chains of production and for which each link requires moderate levels of both *s* and *l* in order to ensure reliability.

Our model also provides implications for inequality. For example, the within-industry variance in log earnings is $\sigma_l^2(\gamma)(1-\rho^2)$. Thus within-industry inequality is increasing in γ. Space precludes further discussion of inequality.

VIII. Average Endowments: Heckscher-Ohlin Revisited

We finish up our discussion of endowments and trade with the role of the average level of endowments. This is a standard Heckscher-Ohlin exercise. Intuitively, a country with a high average *s* per worker (i.e., a high μ) should have production patterns that are skewed toward *s*-intensive goods. This in turn should lead the country to export *s*-intensive goods. Ruffin (1988) was the first to describe such a mechanism.

Theorem 4 (Rybczynski with μ). Define $s^\mu \equiv \mu + \rho$ and consider an increase in μ, the mean of *s*. With product prices held constant, all industries with $i > i(s^\mu)$ expand and all industries with $i < i(s^\mu)$ contract.

Proof. Differentiating Appendix equation (B2) with respect to μ yields $\partial \ln Y(i(s))/\partial\mu = s - (\mu+\rho) = s - s^\mu$. Theorem 4 follows immediately. QED

Our Rybczynski theorem has immediate implications for trade flows.

Theorem 5 (Heckscher-Ohlin with μ). Consider a world with two countries that are identical except that one is better endowed with *s* (i.e., has a higher mean μ). More specifically, let assumption 1 hold and assume that ρ, σ_s, and σ_l are the same in both countries. (1) There exists an equilibrium. (2) In equilibrium there is an industry i^μ such that the high-μ country exports all relatively *s*-intensive goods ($i > i^\mu$) and imports all relatively *l*-intensive goods ($i < i^\mu$).

The proof follows the proof of theorem 2 almost exactly and is therefore omitted.

The basic insight—due to Ruffin (1988)—is simple. A country full of workers with a comparative advantage in *H*-intensive industries is a country that will export *H*-intensive goods. This is our continuum counterpart to the Heckscher-Ohlin theorem.

IX. Conclusions

We examined the implications of skill bundling (two-dimensional worker heterogeneity) and worker sorting for industrial structure, international trade, and domestic income distribution. Our model featured heterogeneous workers who differ in two dimensions, for example, quantitative skills and communication skills. We started off by showing in figures 2 and 3 that two-dimensional heterogeneity leads to a rich set of predictions about production, earnings, and inequality. We then described how higher moments of the bivariate distribution of skills are interesting predictors of trade, including North-North trade. For example, high-ρ economies will have a comparative advantage in s-intensive goods. They will also have high levels of within- and between-industry inequality. For another example, economies with high levels of endowment inequality will export goods that intensively use either skill, but not both skills. We used this to explain U.S. dominance in industries such as film and information technologies and to explain German dominance in machinery and other O-ring reliable goods involving long chains of production.

A feature of our model is that it yields sharp and easily characterized predictions about international patterns of production and trade, even in the case of a continuum of goods. In the Heckscher-Ohlin model, these predictions are sharp and are easily characterized only in the two-good, two-factor case. The Heckscher-Ohlin predictions fall flat in all other cases.[16] In the specific factors model the patterns of production and trade depend on impossibly detailed factor demand elasticities (Jones and Neary 1984, 24). For example, an increase in the mobile factor increases output in both industries, but more so in the industry with the more elastic labor demand. Thus our model improves our ability to concisely and intuitively predict international patterns of production and trade.

The model presented offers additional insights into a range of questions that have not been explored here but that appear in an earlier version of this paper (Ohnsorge and Trefler 2004). These include the political economy of protection, "skill" price equalization or lack

[16] (*a*) With equal numbers of goods and factors—but more than two of both—predictions depend on the complex inverse of the technology matrix and so have no intuitive appeal. (*b*) With more goods than factors the predictions are indeterminate. (*c*) With more factors than goods the model is simply not interesting.

thereof, specialization due to Ricardian technology differences, and economic development.

Appendix A

Proof of Lemma 1

The definition of H-intensity.—Totally differentiating $T(H, L, i) = T(H/L, 1, i)L$ yields the slope of an isoquant in (L, H) space:

$$\frac{dH}{dL}(i) = \frac{H}{L} - \frac{T(H/L, 1, i)}{T_H(H/L, 1, i)}. \tag{A1}$$

Ordering industries by H-intensity means ordering industries so that $(dH/dL)(i)$ is increasing in i. If this is unclear, the reader should draw a diagram of two intersecting isoquants in (L, H) space and compare slopes at the intersection point.

Equivalence of parts 1 and 2.—Since $ds = d\ln H - d\ln L$, an isoquant in (l, s) space is

$$\frac{ds}{dl}(i) = \frac{d(\ln H - \ln L)}{d\ln L} = \frac{L}{H}\frac{dH}{dL}(i) - 1.$$

It follows that $(dH/dL)(i)$ is increasing in i if and only if $(ds/dl)(i)$ is increasing in i.

Equivalence of parts 1 and 3.—From equation (A1),

$$\frac{d}{di}\left(\frac{dH}{dL}\right) = (T_{Hi}T - T_H T_i)T_H^{-2}.$$

From equation (1),

$$t_s = \frac{\partial \ln T(H/L, 1, i)}{\partial \ln H/L} = \frac{H/L}{T}T_H.$$

Hence

$$t_{si} = (T_{Hi}T - T_H T_i)T^{-2}H/L = \frac{d}{di}\left(\frac{dH}{dL}\right)T_H^2 T^{-2}H/L.$$

Since $T_H^2 T^{-2}H/L > 0$, t_{si} and $(d/di)(dH/dL)$ have the same sign. By part 1 and the definition of H-intensity, this sign is positive.

Equivalence of parts 3 and 4.—By constant returns to scale, the inequality in part 4 can be rewritten as $T(H'/L', 1, i')/T(H'/L', 1, i) > T(H/L, 1, i')/T(H/L, 1, i)$. Taking logs and applying equation (1) yields $t(s', i') - t(s', i) > t(s, i') - t(s, i)$, where $s' \equiv \ln H'/L'$ and $s \equiv \ln H/L$. Dividing both sides by $i' - i > 0$ and taking limits yields $t_i(s', i) > t_i(s, i)$. Dividing both sides by $s' - s > 0$ and taking limits yields $t_{si}(s, i) > 0$. The argument is reversible so that part 3 implies part 4.

Appendix B

Output $Y(i)$ under Normality

Let $f_s(s)$ be the marginal density of s and let $f_{l|s}(l|s) \equiv f_{sl}(s, l)/f_s(s)$ be the density of l conditional on s. Then equation (6) can be rewritten as

$$Y(i) = \int_{-\infty}^{\infty} e^l f_{l|s}(l|s)dl \; e^{t(s,i)} f_s(s) = \mathrm{E}(e^l|s)e^{t(s,i)}f_s(s), \tag{B1}$$

where i is evaluated at $i = i(s)$. Define $s^* = (s - \mu)/\sigma_s$. Under normality,[17]

$$f_{l|s}(l|s) = [2\pi\sigma_l^2(1-\rho^2)]^{-1/2}\exp\left\{-\frac{[(l/\sigma_l) - \rho s^*]^2}{2(1-\rho^2)}\right\}$$

and

$$\mathrm{E}(e^l|s) = \exp\left[\sigma_l\rho s^* + \frac{\sigma_l^2(1-\rho^2)}{2}\right].$$

Also under normality, $f_s(s) = (2\pi\sigma_s^2)^{-1/2}\exp[-(s^*)^2/2]$. Hence, taking logs of equation (B1) and setting $i = i(s)$ yields

$$\ln Y(i) = \left[\sigma_l\rho\frac{s-\mu}{\sigma_s} + \frac{1}{2}\sigma_l^2(1-\rho^2)\right] + t(s, i) - \frac{1}{2}\ln(2\pi\sigma_s^2) - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2. \tag{B2}$$

Appendix C

Proof of Theorem 2

We first establish the existence of an equilibrium using a standard fixed-point theorem. With identical homothetic preferences and output supply functions given by equation (B2), excess demand functions satisfy properties i–v of proposition 17.B.2 in Mas-Colell, Whinston, and Green (1995). Hence by their proposition 17.C.2, there exists an equilibrium. (See also the note on p. 589 about production economies.)

[17] Consider the exponent of e in $e^l f_{l|s}$, namely, $Q \equiv l - [(l/\sigma_l) - \rho s^*]^2/[2(1-\rho^2)]$. Tedious algebra yields

$$Q = -\frac{(l-\nu)^2}{2\sigma_l^2(1-\rho^2)} + \left[\sigma_l\rho s^* + \frac{\sigma_l^2(1-\rho^2)}{2}\right]$$

for some ν that is independent of l. Since

$$[2\pi\sigma_l^2(1-\rho^2)]^{-1/2}\int_{-\infty}^{\infty}\exp\left[-\frac{(l-\nu)^2}{2\sigma_l^2(1-\rho^2)}\right]dl = 1,$$

it follows that

$$\int_{-\infty}^{\infty} e^l f_{l|s} dl = \exp\left[\sigma_l\rho s^* + \frac{\sigma_l^2(1-\rho^2)}{2}\right].$$

.

Let $Y(i)$, $Y^*(i)$, and $Y^w(i) \equiv Y(i) + Y^*(i)$ be outputs of good i in the high-ρ country, the low-ρ country, and the world, respectively. Let $C(i)$, $C^*(i)$, and $C^w(i)$ be the corresponding values of consumption of good i. All these variables are evaluated at their free-trade equilibrium values. Homotheticity implies that a country's share of world consumption $C(i)/C^w(i)$ equals its share of world income. Thus $C(i)/C^w(i)$ is the same for all goods i. Market clearing in a free-trade equilibrium requires $Y^w(i) = C^w(i)$. Hence $C(i)/Y^w(i)$ is also the same for all i.

We next show that $Y(i)/Y^w(i)$ is increasing in i. From equation (7), $d\ln Y(i)/d\rho$ is increasing in s and hence in i, that is, in $i(s)$. Since $\rho > \rho^*$, this implies that $\ln Y(i)/Y^*(i)$ is increasing in i. It follows that $Y(i)/Y^w(i)$ is also increasing in i. This together with our previous result that $C(i)/Y^w(i)$ is the same for all i implies that $[Y(i) - C(i)]/Y^w(i)$ is increasing in i.

Let $T(i) \equiv Y(i) - C(i)$ be net exports. By balanced trade, some goods are exported and some are imported. Since $T(i)/Y^w(i)$ is increasing in i, there is a unique i^ρ such that $T(i) > 0$ if and only if $i > i^\rho$. That is, the high-ρ country exports all goods $i > i^\rho$.

Appendix D

Proof of Theorem 3

We use the same notation as in the proof of theorem 2, but with γ replacing ρ. As in that theorem, $C(i)/C^w(i)$ is the same for all i. From equation (9) or figure 4, $Y(i(s))/Y^*(i(s))$ has its minimum at $s = \mu$ or, equivalently, at $i = i(\mu)$. Hence $Y(i(s))/Y^w(i(s))$ has a minimum at $i = i(\mu)$. From the logic of the proof of theorem 2, $T(i(s))/Y^w(i(s))$ also has a minimum at $i = i(\mu)$. Since by balanced trade the home country imports some good, the home country must import good $i = i(\mu)$. From figure 4, to the right of $i(\mu)$, $Y(i)/Y^w(i)$ is increasing in i whereas $C(i)/C^w(i)$ is constant. Hence as in the proof of theorem 2, there is a cutoff industry $\bar{i}$ such that the home country imports goods $[i(\mu), \bar{i})$ and exports goods $(\bar{i}, 1]$. From figure 4, to the left of $i(\mu)$, $Y(i)/Y^w(i)$ is decreasing in i whereas $C(i)/C^w(i)$ is constant. Hence as in the proof of theorem 2, there is a cutoff industry $\underline{i}$ such that the home country exports goods $[0, \underline{i})$ and imports goods $(\underline{i}, i(\mu)]$.

Appendix E

Proof That $i(s)$ Is Nondecreasing

Consider two workers, s and s', with $s > s'$. Since $i(s)$ may be a correspondence, let $\underline{i}$ be the smallest element of $i(s)$ and let $\overline{i'}$ be the largest element of $i(s')$. We want to show that $\underline{i} \geq \overline{i'}$. Suppose not, that $\underline{i} < \overline{i'}$. Equation (2) and $t_{si} > 0$ imply that $w_{si} > 0$. This in turn implies that $w_s(s, l, \underline{i}) < w_s(s, l, \overline{i'})$ or, when these derivatives are rewritten in discrete form,

$$\frac{w(s, l, \underline{i}) - w(s', l, \underline{i})}{s - s'} < \frac{w(s, l, \overline{i'}) - w(s', l, \overline{i'})}{s - s'}$$

for s close to s'. Multiplying through by $s - s'$ and rearranging yields $w(s, l, \underline{i}) - w(s, l, \overline{i'}) < w(s', l, \underline{i}) - w(s', l, \overline{i'})$. In this last inequality, (1) the left-hand side is nonnegative because by definition $\underline{i}$ solves $\max_i w(s, l, i)$, and (2) the right-hand side is nonpositive because by definition $\overline{i}'$ solves $\max_i w(\underline{s}', l, i)$. Hence the inequality is false. It follows that $\underline{i} < \overline{i}'$ is false. To restate, $\underline{i} \geq \overline{i}'$ as required.

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