J. Fluid Mech. (1981), *vol.* 108, *pp.* 443–460
Printed in Great Britain

Sound diffraction at a trailing edge

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(Received 26 March 1980)

The diffraction of externally generated sound in a uniformly moving flow at the trailing edge of a semi-infinite flat plate is studied. In particular, the coupling of the sound field to the hydrodynamic field by way of vortex shedding from the edge is considered in detail, both in inviscid and in viscous flow.

In the inviscid model the (two-dimensional) diffracted fields of a cylindrical pulse wave, a plane harmonic wave and a plane pulse wave are calculated. The viscous process of vortex shedding is represented by an appropriate trailing-edge condition. Two specific cases are compared, in one of which the full Kutta condition is applied, and in the other no vortex shedding is permitted. The results show good agreement with Heavens' (1978) observations from his schlieren photographs, and confirm his conclusions. It is further demonstrated, by an explicit expression, that the sound power absorbed by the wake may be positive or negative, depending on Mach number and source position. So the process of vortex shedding does not necessarily imply an attenuation of the sound.

In the viscous model a high-Reynolds-number approximation is constructed, based on a triple-deck boundary-layer structure, matching the harmonic plane wave outer solution to a known incompressible inner solution near the edge, to obtain the viscous correction to the Kutta condition.

1. Introduction

In the present paper we study in a number of two-dimensional model problems various aspects of the interaction of sound with a flat-plate trailing edge in a subsonic flow. Specific consideration will be given to the acoustical implications of vortex shedding from the edge, both in an inviscid and in a viscous flow. Since this vortex shedding is a viscous process, it is included in the inviscid model by imposing an appropriate condition at the edge, for instance the Kutta condition. The incident sound arises from a source at a fixed position relative to the plate; the relevant technical problem one may think of is the aeroplane engine noise diffracted by the wing trailing edge.

The first problem we shall examine is the diffraction of a cylindrical pulse (§3). It was treated before in harmonic form by Jones (1972). He investigated the harmonic field from a line source parallel to the trailing edge of a semi-infinite plate in a subsonic inviscid uniform flow. The acoustically induced singularity at the edge (infinite velocity perturbations) is relieved by way of the shedding of vorticity from the edge into the wake. The vorticity is convected away by the main stream and is assumed to constitute an undulating vortex sheet stretching behind the plate. This vortex sheet, basically of hydrodynamic nature, corresponds to a discontinuity in the acoustic (velocity) field. Although the vortex sheet is acoustically quiet, in the sense that no pressure perturbations are emitted from it, there is still a sound field, spreading away from the edge, associated with it. The hydrodynamic field of the moving vorticity concentrations has to adjust continuously to the solid body of the plate, resulting in fluctuating pressure differences across the plate and a corresponding sound field (Crighton 1972, 1975). Obviously, without the amount of shed vorticity being prescribed, the solution of the diffraction problem is not unique, with the field of the shed vorticity as an eigensolution. For instance, with no vortex shedding, we have the continuous solution (singular at the edge), and we have the 'Kutta condition' solution when just enough vorticity shed to annihilate the singularity and to satisfy the Kutta condition of finite velocity at the edge. To estimate the effect of employing the Kutta condition, Jones (1972) compared these two solutions and concluded that the fields only differed considerably in the neighbourhood of the wake. In the Kutta condition solution in potential form he found the presence of a non-decaying wave along the surface of the wake.

For a *moving* source Howe (1976) found a more dramatic difference. He showed that the sound, radiated by a line vortex passing a trailing edge, is drastically reduced with the application of the Kutta condition, *if* the vortex is convected by the main flow with exactly the main flow velocity.

To investigate experimentally the acoustic-wave trailing-edge interaction, Heavens (1978) undertook a photographic study of the diffraction of a sound pulse at an airfoil trailing edge in a subsonic flow. The sound waves were visualized with the aid of a sensitive schlieren system. The photographs showed no significant fluctuation of the wake that could confidently be attributed to the passage of the pulse, while only the diffracted wave intensity varied significantly with the prevailing flow conditions. In the event of unsteadiness in the flow due to boundary-layer separation or other causes the diffracted wave was strongly visible; in a smooth and steady flow it was weak. Heavens suggested that the Kutta condition was connected with this difference, and concluded that his observations tend to support Howe's (1976) predictions, rather than those of Jones (1972), in spite of the better correspondence between Jones' model and his experiments.

It was, however, overlooked by Heavens that Jones' surface wave is of hydrodynamic nature, disappears with pressure, and will therefore not be visible (to first acoustic order) on schlieren photographs, which measure only density ($\sim$ pressure) gradients. We will show here, that, in fact, Jones' solution in pressure form indeed exhibits all the features observed by Heavens. For this it will be convenient to replace the harmonic source by a pulse source. Then a simple, transparent solution is available, and, furthermore, a pulse configuration better approximates Heavens' experimental conditions.

The same model of an undulating vortex sheet was shown by Davis (1975) to be also applicable to the different, but related problem of vortices spontaneously shed from a blunted trailing edge. (The mechanisms to determine frequency and amplitude are obviously not included in the model and the values of these parameters have to be taken from the experiment.) He found an approximate solution (singular at the edge), which is easily seen to approximate Jones' eigensolution. Davis compared his result with schlieren photographs made by Lawrence, Schmidt & Looschen (1951), to show a good qualitative agreement. Then he went on to construct another solution to satisfy the Kutta condition, but Howe (1978) showed this second result to be erroneous since the proposed solution does not satisfy uniformly the radiation condition of outgoing waves at infinity.

The second problem we will consider (§§4, 5) is much like the first; only the source is omitted, and instead of cylindrical waves we have plane waves incident (of both harmonic and pulse form). The harmonic problem with Kutta condition was treated previously by Candel (1973). We shall extend his work with the continuous harmonic solution, and, because of their simple and clear representation, the pulse wave counterparts. Although the results with respect to the effect of employing the Kutta condition are, of course, the same as for the first problem with a source, the simpler configuration has several additional advantages. It goes together with a simpler solution, for which it is possible to investigate the acoustic energy balance exactly (within the acoustic approximation). Furthermore, Jones' eigensolution can be identified by inspection, in contrast to Jones' original more complicated approach involving the Wiener–Hopf technique. And, finally, a quantitative comparison with future experiments will be easier, as the properties of a real plane wave are more easily determined.

Our third (and last) problem (§6) will concern a viscous model, extending the previous inviscid ones. The aim is to obtain some insight into the physical background of the central issue of this paper, namely the Kutta condition, or, more generally, the level of singularity at the edge. About this boundary condition which is needed after our neglect of viscosity in order to simplify the problem, and thus in essence representing the result of the action of viscous forces, not very much is known under practical circumstances. The full Kutta condition is often, but not always found (Heavens 1978; Archibald 1975; Fleeter 1979), depending on parameters such as Strouhal number and angle of incidence of the airfoil. There is hardly any doubt that Reynolds number, Mach number, dimensionless amplitude of perturbation, turbulence intensity and maybe other parameters are important as well. The understanding of the mechanisms involved has lately been considerably deepened by the discovery of the asymptotic laminar boundary-layer structure near a flat-plate trailing edge (the so-called 'triple deck') by Stewartson (1969) and Messiter (1970), and the application of it in several related problems by Stewartson and co-workers (for example, Brown & Stewartson 1970; Brown & Daniels 1975). As proposed by Broadbent (1977), a step towards the understanding and possible calculation of the Kutta, or related, condition in our acoustical problems would therefore be to introduce weakly viscous, laminar flow near the edge and then to make use of known results from triple deck theory. We shall do so here for the plane harmonic wave problem. We choose our problem parameters in such a way as to have a tractable problem, and then incorporate Brown & Daniels' (1975) solution for the incompressible flow around an oscillating airflow.

We have, for laminar flow at high Reynolds number, the following picture. Owing to the change in boundary conditions, the flow in the boundary layer accelerates when it passes the trailing edge. This gives rise to a singularity in the pressure of the inviscid outer flow, and this singularity is smoothed in a small region around the edge. However local this flow induced pressure may look, it is nonetheless essential for the transition from the wall boundary layer to the wake. It is shown in the papers mentioned above that the Kutta condition problem can be identified with the balance between that flow induced pressure and the externally generated pressure perturbations (i.e. diffracted sound waves in our case). When the pressure of the Kutta condition solution is of the same order of magnitude (near the edge) as the flow induced pressure, the viscous smoothing forces, prepared for the flow induced singularity, take care of both and the Kutta condition is valid to leading order. Probably this is also the case when the externally generated pressure is much lower (Daniels 1978), although Bechert & Pfizenmaier (1975) argue that for very small amplitudes the diffracted sound wave effectively only sees stagnant flow close to the edge where the pressure singularity becomes important. Hence, for decreasing pressure amplitude, the Kutta condition may then eventually not be valid. For the other case, when the external pressure dominates the flow induced pressure, separation is likely to occur (Brown & Stewartson 1970), and the Kutta condition is no longer applicable. A final remark here, is that the sound pressure singularity may be helped to overcome the smoothing forces by another external mechanism capable of generating a singularity at the edge. Examples may be a plate at incidence, or a wedge-shaped edge. The two singularities add up, and the Kutta condition is then violated earlier than if one external singularity inducing process were in action. This may provide an explanation for the observation in Heavens' (1978) experiments that the diffraction pattern of the same sound pulse varied with the angle of incidence of the airfoil.

2. Formulation of the problems

We start with the presentation of the inviscid problems.

An inviscid fluid with density ρ_0, pressure p_0 and soundspeed c_0 flows in (x^*, y^*, z^*) space with uniform subsonic velocity U_0 in the positive x^* direction along a semi-infinite plate, located at $x^* < 0$, $y^* = 0$. The problem is modelled as two-dimensional and the z^* co-ordinate will be ignored. The flow is perturbed by sound: a pulse from a line source, time harmonic plane waves with frequency f^*, or a plane pulse. The problem can be made dimensionless after introducing a length scale L. Anything can be taken for L, but we leave it unspecified yet, for convenience in the viscous problem. Now space co-ordinates are made dimensionless by $x^* = xL$, $y^* = yL$, the time co-ordinate by $t^* = tL/U_0$, velocities by $u^* = (1+u)U_0$, $v^* = vU_0$, density by $\rho^* = (1+\rho)\rho_0$, and pressure by $p^* = p_0 + \rho_0 U_0^2 p$. The co-ordinates (x,y) are written in polar form $x = r\cos\theta$, $y = r\sin\theta$ with $-\pi < \theta \leqslant \pi$. The perturbation amplitudes are considered to be small enough to justify the sound speed being taken as constant and the perturbation of the wake position as negligible. The regions containing vorticity are restricted to the wake surface, and we can introduce for the velocity perturbations (u,v) a potential ϕ, defined by $(u,v) = \nabla\phi$. The line source is positioned at $(x_0,y_0) = (r_0\cos\theta_0, r_0\sin\theta_0)$, $0 < \theta_0 < \pi$, and the direction normal to the wave fronts of the plane waves is θ_i, $0 < \theta_i < \pi$. A sketch is given in figure 1. Of the scattered wave only the diffracted part is drawn. To combine the three problems (cylindrical and plane pulse, plane harmonic wave) in one picture, θ_0 is chosen equal to $\pi - \theta_s$ (θ_s will be defined in equation (4.1)).

The restriction on θ_0 and θ_i, to lie between 0 and π, is for technical reasons and of course not important for the problems. The main flow Mach number is $M = U_0/c_0$.

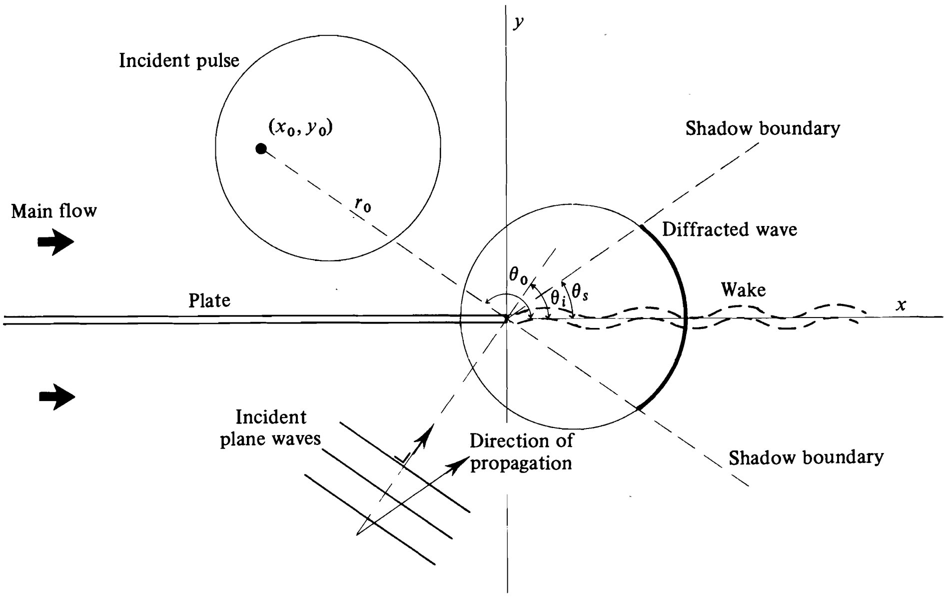

FIGURE 1. Sketch of the inviscid problems.

In the harmonic problem the Strouhal number or dimensionless radian frequency is $\omega = 2\pi f^* L/U_0$, and the Helmholtz number or dimensionless wavenumber is $k = \omega M$. The dimensionless amplitude a is small.

After the usual linearizations and other manipulations of the compressible inviscid time-dependent Navier–Stokes equations, we obtain the following equations and boundary conditions (an index x, y, t denotes differentiation with respect to that variable):

$$\left.\begin{aligned}\phi_{xx}+\phi_{yy}-M^2(\phi_{tt}+2\phi_{xt}+\phi_{xx}) &= 4\pi a\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(t)\\ &= 0 \text{ in the plane wave case;}\end{aligned}\right\} \tag{2.1}$$

$$p = -\phi_t-\phi_x; \tag{2.2}$$

$$\phi_y(x < 0, 0) = 0; \tag{2.3}$$

$$\phi_y(x > 0, +0) = \phi_y(x > 0, -0); \tag{2.4}$$

$$p(x > 0, +0) = p(x > 0, -0); \tag{2.5}$$

$$\phi \text{ and } \phi_x \text{ may have a bounded discontinuity across } y = 0, \quad x > 0; \tag{2.6}$$

$$\phi \to 0 \quad \text{as} \quad k\{x^2+y^2\}^{\frac{1}{2}} \to \infty \quad (y \neq 0); \tag{2.7}$$

the radiation condition for the pulse problems: at any time t there is a large circle outside of which the pulse field is identically zero; (2.8a)

the radiation condition for the harmonic problem: the harmonic diffracted pressure field obeys (a convective form of) Sommerfeld's radiation condition. (2.8b)

In the case of plane incident waves, the source term in equation (2.1) is identically zero, and, instead, we have to prescribe an incident wave p_i or ϕ_i. Furthermore, since the incident and associated reflected plane waves do not decay at infinity, the conditions (2.7) and (2.8a,b) only hold for the scattered field; strictly, it is necessary in that case to prescribe the reflected wave as well, but we will not do so here explicitly.

Sommerfeld's radiation condition in (2.8*b*) is necessary for uniqueness of the harmonic solution (Jones 1964) to select outgoing from ingoing waves. Usually, this condition is motivated by an energy flux argument, although the causality argument that the source is switched on long ago gives the same result. In rotational flow no satisfactory definition of acoustic energy exists (Goldstein 1976), so in the present problem including the possibility of vortex shedding, we have, in general, only causality as the physical argument to motivate the radiation condition. Indeed this condition (2.8*b*) is satisfied if the solution of the corresponding pulse problem is causal (satisfies (2.8*a*)).

The above list of boundary conditions is completed by establishing suitable edge conditions. A necessary condition, although not sufficient for uniqueness, is that the net force on any finite part of the plate is finite, which means that

$$p \text{ is integrable at } (x,y) = (0,0). \tag{2.9}$$

This is equivalent to the condition that ϕ is finite at the edge. The solution representing the field without vortex shedding is determined by the additional condition that there be no vortex sheet emanating from the edge, i.e.

$$\phi \text{ and } \phi_x \text{ continuous across } y = 0, \quad x > 0. \tag{2.10a}$$

The Kutta condition solution, in which all the vorticity possible in the present configuration is shed, is determined by requiring

$$p \text{ finite at } (x,y) = (0,0), \tag{2.10b}$$

a condition equivalent to that of finite velocity. If there is any doubt with respect to the interpretation of these edge conditions for a pulse, it is expedient to take advantage of the linearity of the problem, and to apply the condition to the corresponding Fourier transform. (Note, however, that this is not applicable to a general, amplitude dependent, edge condition, which is not Fourier transformable).

To investigate the effect of the Kutta condition, we shall compare as extremes the Kutta condition solution and the continuous solution. Intermediate solutions of the harmonic problem can be rendered unique by prescribing the amplitude of the shed vorticity. For a pulse solution this should be done for each frequency of the Fourier sum. It is obvious that the class of intermediate solutions of the pulse problem is very large, in view of the many ways the amount of shed vorticity may depend on frequency. In other words, the number of eigensolutions is infinite, and not just one as in the time harmonic case.

We conclude the description of the inviscid problem by verifying the uniqueness of the solutions. First of all, we observe the unique correspondence (at least, in the present context) between a solution and its Fourier transform, so that it is sufficient to consider harmonic solutions only. Then, we note the close relationship, via a simple transformation (Jones 1972), between the present problem and the classical half-plane diffraction problem without mean flow. The Sommerfeld condition of (2.8) is indeed to be interpreted as being modified by this transformation. By analogy the uniqueness of the continuous solution follows immediately then from the uniqueness of the related no-flow solution (see Jones 1964). For the Kutta condition solution we observe from (2.5), (2.2) with (2.4), and (2.1) that p with all its derivatives is continuous across the wake, and hence uniqueness again follows by analogy.

We proceed now with the formulation of the viscous problem.

In the problems above, we considered for convenience a semi-infinite plate. So that we can use the inviscid harmonic solution as the outer solution of the viscous problem, we maintain this geometry. However, a careless introduction of viscosity would rule out the possibility of an inviscid outer solution, the influence of the plate being felt everywhere. Therefore, we shall modify the properties of the plate a little with the rather artificial assumption that the plate is frictionless everywhere, except for a finite part of length L upstream of and including the trailing edge. (This assumption preserves the mathematical consistency of the model; of course the more practical approach would be to take a finite plate and ignore the leading-edge diffraction field.) We have co-ordinate axes x^*, y^* and a main flow in the same way as in the inviscid model. We assume, in the whole medium, the validity of the two-dimensional, time-dependent, compressible continuity, momentum and energy equations (Navier–Stokes equations) together with thermodynamic equations of state for a gas. We make quantities dimensionless as above in the inviscid case. Now however the energy equation cannot be decoupled right from the start, and we have to include temperature variations with $T^* = (1+T)T_0$, where T_0 is the main flow temperature. Introduce the Reynolds number $Re = \rho_0 U_0 L/\mu$, where μ is the viscosity coefficient. For convenience we write $Re = \epsilon^{-8}$. We assume Re large, so that ϵ is small. To make the problem tractable, we assume the Mach number $M = O(\epsilon^2)$, the Strouhal number $\omega = O(\epsilon^{-2})$ (so $k = O(1)$), and the plate temperature equal to the flow temperature. The second viscosity coefficient μ' is of the same order of magnitude as μ, i.e. $\mu'/\mu = O(1)$. The undisturbed flow is laminar. The perturbations remain harmonic and do not initiate turbulence.

With these assumptions, we can simplify the problem a little further. When we assume that the perfect gas relation for adiabatic changes is sufficient to provide order of magnitude estimates, we can deduce that the assumption $|p| \leqslant O(\sqrt{\epsilon})$ (in fact a result of the subsequent solution) reduces the energy equation to the condition of isentropy, with the solution $T = (\gamma-1)M^2p$ (where γ is the ratio of specific heats). Furthermore it follows that the pressure, density, sound speed, temperature and viscosity coefficients are, to the accuracy considered, constant (but not their derivatives of course). Thus, for example, in the continuity equation $(1+\rho)u_x$ can be taken as u_x, while in $u(1+\rho)_x$ we retain $u\rho_x$. We shall adopt this from the start to avoid unnecessary complexity. From the isentropy and constant sound speed, it follows that $\rho = M^2 p$, the acoustic approximation.

After substitution of the above, we have the following equations and boundary conditions;

$$M^2[p_t + (1+u)p_x + vp_y] + u_x + v_y = 0; \tag{2.11}$$

$$u_t + (1+u)u_x + vu_y + p_x = \epsilon^8\left[u_{xx} + u_{yy} + \frac{\mu'}{\mu}(u_{xx} + v_{xy})\right]; \tag{2.12}$$

$$v_t + (1+u)v_x + vv_y + p_y = \epsilon^8\left[v_{xx} + v_{yy} + \frac{\mu'}{\mu}(u_{xy} + v_{yy})\right]; \tag{2.13}$$

$$1 + u(-1 \leqslant x \leqslant 0, 0) = v(x \leqslant 0, 0) = 0. \tag{2.14}$$

The viscous, incompressible inner field (a variant of Brown & Daniels' 1975 solution) matches in the inviscid, compressible outer region with the diffracted field of the harmonic plane wave. In the far outer field viscosity affects the sound waves too, and should be accounted for. However, this aspect will not be considered, as it is of no relevance to the Kutta condition.

Finally, we note that various simplifying assumptions are not necessary, except for explicit calculations (Brown & Stewartson 1970, p. 584). When we rely on order-of-magnitude estimates only, it is possible to derive a more general criterion for the Kutta condition to be valid. This criterion relates incident wave amplitude and wave-length to Reynolds number.

3. The pulse from a line source

As we noted before, it is easiest for the application of the Kutta condition to consider the Fourier transform of the pulse solution. We therefore introduce

$$\hat{\phi}(x, y; \omega) = \int_{-\infty}^{\infty} \phi(x, y, t) \exp(-i\omega t)\, dt.$$

By analogy with the harmonic problem we use ω as variable of the Fourier transform.

After transformation of equations (2.1)...(2.5), we obtain a problem for $\hat{\phi}$ virtually the same as the one solved by Jones (1972). The difference is that Jones assumed *a priori* that the wake has a particular form, but it can be shown (as in Carrier 1956) that this form is necessary and would result in any case. Therefore, we can take for $\hat{\phi}$ simply Jones' solution. The continuous solution will be denoted by $\hat{\phi}_c$, the Kutta condition solution by $\hat{\phi}_k$. We write moreover

$$\hat{\phi}_k = \hat{\phi}_c + A\, \hat{\phi}_e,$$

where $A\,\hat{\phi}_e$ is a multiple of Jones' eigensolution representing shed vorticity. Following Jones we define:

$$\beta = (1-M^2)^{\frac{1}{2}}, \quad \Theta_1 = i \operatorname{arcosh} M^{-1}, \quad k = \omega M = \beta K,$$

$$x = \beta X = \beta R \cos\Theta, \quad y = R\sin\Theta \quad (\text{where } -\pi < \Theta \leqslant \pi),$$

$$x_0 = \beta X_0 = \beta R_0 \cos\Theta_0, \quad y_0 = R_0 \sin\Theta_0$$

(similar to the classical Prandtl–Glauert transformation), and introduce:

a version of Fresnel's integral

$$F(z) = \exp(iz^2) \int_z^{\infty} \exp(-it^2)\, dt,$$

and the usual Green's function for rigid half-plane diffraction

$$G(x, y; \omega) = G_1(x, y; \omega) + G_2(x, y; \omega),$$

where
$$G_{1,2}(x, y; \omega) = \int_{-\infty}^{u_{r_{1,2}}} \exp(-ikr_{1,2}\cosh u)\, du,$$

$$r_{1,2} = (r^2 + r_0^2 - 2r\,r_0 \cos(\theta \mp \theta_0))^{\frac{1}{2}},$$

$$u_{r_{1,2}} = \pm \operatorname{arsinh}\left\{2\frac{(rr_0)^{\frac{1}{2}}}{r_{1,2}} \cos\tfrac{1}{2}(\theta \mp \theta_0)\right\}.$$

The modification of $r_{1,2}$ to $R_{1,2} = (R^2 + R_0^2 - 2RR_0\cos(\Theta \mp \Theta_0))^{\frac{1}{2}}$ is obvious. The Heaviside stepfunction will be denoted by H. Furthermore we introduce

$$\Gamma_1, \bar{\Gamma}_1 = (2KR)^{\frac{1}{2}} \sin\tfrac{1}{2}(\Theta \mp \Theta_1).$$

$\hat{\phi}_H = \mathrm{sgn}(y)\exp(-\omega|y| - i\omega x)$.

Then we have
$$\hat{\phi}_c(x,\,y;\,\omega) = -\frac{a}{\beta}G\left(X,\,y;\,\frac{\omega}{\beta}\right)\exp[iKM(X-X_0)], \tag{3.1}$$

$$\hat{\phi}_e(x,\,y;\,\omega) = \frac{1}{\sqrt{\pi}}\exp(\tfrac{1}{4}\pi i)\{F(\Gamma_1)+F(\overline{\Gamma}_1)\}\exp[-iKR+iKMX] - 2H(-y)\cosh(\omega y)\exp(-i\omega x), \tag{3.2}$$

$$A(\omega) = -2\frac{a}{\beta}\left(\frac{1-M}{1+M}\right)^{\frac{1}{4}}\left(\frac{\pi}{\omega R_0}\right)^{\frac{1}{2}}\sin\tfrac{1}{2}\Theta_0\exp[-\tfrac{1}{4}\pi i - iKR_0 - iKMX_0]. \tag{3.3}$$

The transform of $\hat{\phi}_c$ back into the time domain is found by using the solution of the rigid half-plane pulse diffraction problem without flow. We have

$$\phi_c(x,\,y,\,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{\phi}(x,\,y;\,\omega)\exp(i\omega t)\,d\omega = -\frac{a}{2\pi}\int_{-\infty}^{\infty}G(X,\,y;\,\omega')\exp(i\omega' T_0)\,d\omega' = \psi(X,\,y,\,T_0),$$

where $T_0 = \beta t + M^2(X-X_0)$, and ψ is the solution of

$$\psi_{xx}+\psi_{yy}-M^2\psi_{tt} = 4\pi a\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(t).$$

The function ψ is given (with some slight alterations) by Friedlander (1958, p. 126), according to which ϕ_c is found to be

$$\phi_c(x,\,y,\,t) = -a\frac{H(T_0-MR_1)}{(T_0^2-M^2R_1^2)^{\frac{1}{2}}}\{1+H(MR+MR_0-T_0)\,\mathrm{sgn}\,(\theta-\theta_0+\pi)\} - a\frac{H(T_0-MR_2)}{(T_0^2-M^2R_2^2)^{\frac{1}{2}}}\{1+H(MR+MR_0-T_0)\,\mathrm{sgn}\,(\theta+\theta_0-\pi)\}. \tag{3.4}$$

From this expression, we obtain for the pressure of the diffracted wave (i.e. the cylindrical wave centred around the edge) the following expression:

$$p_{c,d}(x,\,y,\,t) = 2a\frac{\delta(T)}{\beta^2 M}\,\frac{1-M\cos\Theta}{\cos\Theta+\cos\Theta_0}\,\frac{\sin\frac{1}{2}\Theta\sin\frac{1}{2}\Theta_0}{(RR_0)^{\frac{1}{2}}}+\ldots, \tag{3.5}$$

where
$$T = t-\frac{M}{\beta}(R+R_0)+\frac{M^2}{\beta}(X-X_0).$$

We have ignored terms not containing δ functions and retained the front only, as only this part is changed with the Kutta condition (as we will see below).

Pressure $\hat{p}_e$ corresponding to $\hat{\phi}_e$ turns out to have a very simple form, and transformation of $A\,\hat{p}_e$ into the time domain is easily performed:

$$A(\omega)\,\hat{p}_e(x,\,y;\,\omega) = 2a\frac{\sin\frac{1}{2}\Theta\sin\frac{1}{2}\Theta_0}{\beta^2(RR_0)^{\frac{1}{2}}}\exp[-iK(R+R_0)+iKM(X-X_0)], \tag{3.6}$$

and so this eigensolution is, in the time domain,

$$p_{e,k}(x,\,y,\,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}A(\omega)\,\hat{p}_e(x,\,y;\,\omega)\exp(i\omega t)\,d\omega = 2a\frac{\delta(T)}{\beta^2}\,\frac{\sin\frac{1}{2}\Theta\sin\frac{1}{2}\Theta_0}{(RR_0)^{\frac{1}{2}}}. \tag{3.7}$$

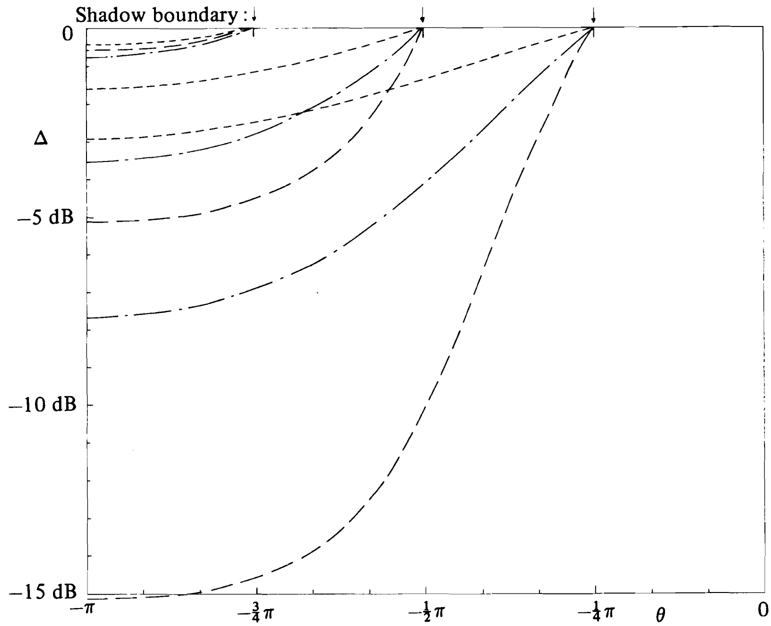

FIGURE 2. The amplification △, due to the Kutta condition, in the shadow region. ----, $M = 0{\cdot}2$; —-—, $M = 0{\cdot}5$; — —, $M = 0{\cdot}8$.

Summing $p_{c,d}$ and $p_{e,k}$ yields the front of the diffracted field in pressure form of the Kutta condition solution

$$p_{k,d} = p_{c,d} + p_{e,k} = 2a\frac{\delta(T)}{\beta^2 M}\frac{1+M\cos\Theta_0}{\cos\Theta+\cos\Theta_0}\frac{\sin\frac{1}{2}\Theta\sin\frac{1}{2}\Theta_0}{(RR_0)^{\frac{1}{2}}}+\ldots. \tag{3.8}$$

As can be expected, the Kutta condition affects only the diffracted wave, having its source at the plate edge. The effect of Kutta condition is therefore completely exposed by a comparison of $p_{c,d}$ and $p_{k,d}$. We have

$$\left.\begin{aligned}\frac{p_{k,d}}{p_{c,d}} = \frac{1+M\cos\Theta_0}{1-M\cos\Theta}+\ldots &> 1 \quad \text{when} \quad -\pi+\theta_0 < \theta < \pi-\theta_0\\ &< 1 \quad \text{elsewhere.}\end{aligned}\right\} \tag{3.9}$$

In the shadow region $-\pi < \theta < \theta_0 - \pi$ the diffracted wave is the dominant part of the sound field, and the amplification (an attenuation really) due to the Kutta condition is in decibels $\Delta = 20\log(1+M\cos\Theta_0) - 20\log(1-M\cos\Theta)$, plotted in figure 2 for $\theta_0 = \frac{1}{4}\pi, = \frac{1}{2}\pi, = \frac{3}{4}\pi$ ($\theta = \theta_0 - \pi$ is the shadow boundary), and $M = 0{\cdot}2, = 0{\cdot}5, = 0{\cdot}8$. For fixed M and θ_0 the maximal attenuation is found at $\theta = -\pi$. As a function of M (and θ_0 fixed) this maximum tends to infinity as $M \to 1$, provided the source is upstream of the edge, i.e. $\frac{1}{2}\pi \leqslant \theta_0 < \pi$. When the source is downstream, i.e. $0 < \theta_0 \leqslant \frac{1}{2}\pi$, this maximum is largest for M given by $\sin^2\theta_0 = (2M-1)/(M^3(2-M))$, and attains the value $\Delta = 20\log((2-M)/(1+M))$.

The application of the Kutta condition results in an increase of the pressure amplitude of the diffracted wave within the symmetric downstream wedge $|\theta| < \pi - \theta_0$ (the wedge 'seen' by both the source and its image), and a decrease elsewhere. Of course this increase and decrease is only for $p_{k,d}$ relative to $p_{c,d}$, and the actual difference (i.e. $p_{e,k}$) may be small. Due to the $\sin\frac{1}{2}\Theta$ term the predicted difference will be largest near the plate where $\sin\frac{1}{2}\Theta \simeq \pm 1$, and only small near the wake where $\sin\frac{1}{2}\Theta \simeq 0$. Note that this is more or less opposite to the behaviour of the velocity field; the most important change here occurs near the wake, as Jones (1972) found. So when looking for experimental evidence, we have to focus on the plate region for pressure, and on the wake region for velocity.

These qualitative conclusions are precisely what Heavens (1978) found in his photographic study. He made schlieren pictures of a sound pulse diffracted by the trailing edge of an airfoil in a subsonic flow. Relatively small variations of the main flow conditions (unsteadiness, angle of incidence of the airfoil) had a large effect on the diffracted sound wave. Heavens supposed these main flow variations to induce a corresponding trailing-edge behaviour of the sound field varying from regular (Kutta condition) to some singular stage. His main observations were: (i) in the cases where he expected the Kutta condition to apply, the pictures show near the plate a diffracted pressure wave much weaker than in the cases with more singular edge behaviour; (ii) no effect due to the incident pulse was visible in the wake region.

Although this agreement with our results is necessarily qualitative, the correspondence is sufficiently convincing for the following conclusions:

(i) The present simple mathematical model is relevant to describe interaction problems of acoustic waves with an airfoil trailing edge, provided the main flow is uniform and the acoustic source is of sufficient coherence and intensity.

(ii) Heavens' conjecture is supported with respect to the trailing edge condition in his experiments. Thus, physically, the effect of a varying edge condition is really present, particularly in the shadow region and may involve differences of several decibels.

Further and more detailed experiments would be welcome to investigate the influence of the Kutta condition quantitatively. It might become necessary then to modify expression (3.8) by using a guessed, measured or possibly calculated frequency dependent edge condition.

4. The harmonic plane wave

We have equations (2.1)...(2.10) but now without the source term of (2.1). Instead we have a plane harmonic wave incident in the direction θ_i, or, more precisely, with angle θ_i normal to the wave fronts. The real direction of propagation is θ_s, with the source position at infinity along $\theta_s - \pi$, and a shadow region $\theta_s \leqslant \theta \leqslant \pi$ behind the plate; see figure 1. Because of convection we have $\theta_i \neq \theta_s$. In experiments, θ_i will usually be easier to determine, although θ_s must be considered as the more fundamental parameter of the plane wave.

We have the following relations between θ_i and θ_s;

$$\left.\begin{aligned} \sin\theta_s = \frac{\sin\theta_i}{(1+2M\cos\theta_i+M^2)^{\frac{1}{2}}},\quad \cos\theta_s = \frac{\cos\theta_i+M}{(1+2M\cos\theta_i+M^2)^{\frac{1}{2}}},\\ \sin\theta_i = \sin\theta_s(1-M^2\sin^2\theta_s)^{\frac{1}{2}} + M\sin\theta_s\cos\theta_s,\\ \cos\theta_i = \cos\theta_s(1-M^2\sin^2\theta_s)^{\frac{1}{2}} - M\sin^2\theta_s. \end{aligned}\right\} \tag{4.1}$$

We will use the same transformation of variables as in the previous section; that of θ_s is explicitly given by

$$\sin\Theta_s = \frac{\beta\sin\theta_s}{(1-M^2\sin^2\theta_s)^{\frac{1}{2}}},\quad \cos\Theta_s = \frac{\cos\theta_s}{(1-M^2\sin^2\theta_s)^{\frac{1}{2}}}.$$

Furthermore we define

$$\Gamma_s, \bar{\Gamma}_s = (2KR)^{\frac{1}{2}}\sin\tfrac{1}{2}(\Theta\mp\Theta_s).$$

We write the flow variables such as pressure and potential in complex form. It is tacitly understood that real parts should be taken.

The basic incident wave is then

$$p_i(x,y,t) = a\exp(i\omega T_i),\quad \phi_i(x,y,t) = \frac{ia\beta^2}{\omega(1-M\cos\Theta_s)}\exp(i\omega T_i), \tag{4.2}$$

where $T_i = t-(M/\beta)R\cos(\Theta-\Theta_s)+(M^2/\beta)X$.

From the classical Sommerfeld screen diffraction solution (Jones 1964), we obtain by, again, a Prandtl–Glauert transformation the following continuous solution (not satisfying the Kutta condition)

$$\phi_c(x,y,t) = \frac{ia\beta^2}{\omega(1-M\cos\Theta_s)}\frac{1}{\sqrt{\pi}}\exp(\tfrac{1}{4}\pi i+i\omega T_d)\{F(\Gamma_s)+F(\bar{\Gamma}_s)\}, \tag{4.3}$$

where $T_d = t-(M/\beta)R+(M^2/\beta)X$, and the corresponding pressure is

$$p_c(x,y,t) = \frac{a}{\sqrt{\pi}}\exp(\tfrac{1}{4}\pi i+i\omega T_d)\left\{F(\Gamma_s)+F(\bar{\Gamma}_s)-\frac{iM\sin\frac{1}{2}\Theta\cos\frac{1}{2}\Theta_s}{1-M\cos\Theta_s}\left(\frac{2}{KR}\right)^{\frac{1}{2}}\right\}. \tag{4.4}$$

Note that the first term of (4.4) is a multiple of ϕ_c. Since both p_c and ϕ_c satisfy the governing equation (2.1), so must the remaining term in (4.4). Moreover, this term tends to zero as $r\to\infty$, is continuous across the wake, has a zero normal derivative at the plate, and has an integrable singularity in $r=0$, so it must be the eigensolution we are looking for. Indeed it is a multiple of $\hat{p}_e$, given in (3.6). As ϕ_c is regular in $r=0$, p_c becomes regular when we cancel that term. Hence we have for the Kutta condition solution

$$p_k(x,y,t) = \frac{a}{\sqrt{\pi}}\exp(\tfrac{1}{4}\pi i+i\omega T_d)\{F(\Gamma_s)+F(\bar{\Gamma}_s)\}, \tag{4.5}$$

and the corresponding potential (with $\hat{\phi}_e$ given in (3.2))

$$\phi_k(x,y,t) = \phi_c(x,y,t)-i\frac{a\beta\cos\frac{1}{2}\Theta_s}{\omega(1-M\cos\Theta_s)}(2M)^{\frac{1}{2}}(1-M)^{\frac{1}{2}}\exp(i\omega t)\,\hat{\phi}_e(x,y;\omega). \tag{4.6}$$

From the asymptotics of F for large values of the argument, we find the far field ($KR\to\infty$) of the diffracted wave in pressure form

$$p_{c,d} \simeq \frac{1-M\cos\Theta}{1-M\cos\Theta_s}\,a\frac{\sin\frac{1}{2}\Theta\cos\frac{1}{2}\Theta_s}{\cos\Theta_s-\cos\Theta}\left(\frac{2}{\pi KR}\right)^{\frac{1}{2}}\exp(-\tfrac{1}{4}\pi i+i\omega T_d), \tag{4.7}$$

$$p_{k,d} \simeq a\frac{\sin\frac{1}{2}\Theta\cos\frac{1}{2}\Theta_s}{\cos\Theta_s-\cos\Theta}\left(\frac{2}{\pi KR}\right)^{\frac{1}{2}}\exp(-\tfrac{1}{4}\pi i+i\omega T_d). \tag{4.8}$$

We see immediately the same increase and decrease as with (3.9). The effect of Kutta condition is an increase downstream in the wedge $|\theta|<\theta_s$, and a decrease elsewhere. Figure 2 applies to the present case, provided we change θ into $-\theta$ to have $\theta_s\leqslant\theta\leqslant\pi$; $\theta=\theta_s$ is the shadow boundary.

An interesting aspect, of which we can give some exact results now, is the acoustic energy balance. To generate the vorticity, energy from the acoustic and – as we will see – the main flow is converted into hydrodynamic energy of the vortices. On the other hand, at the edge the flow induced by the vortices generates sound (Crighton 1972, 1975), and so part of the hydrodynamic energy is again converted into acoustic energy. The question that arises immediately is then: what is the net result; is it a reduction or not?

The unlimited extent of the plane waves makes the total acoustic energy infinite and so a useless parameter for the problem. Therefore we shall consider as alternative the net sound power **P** (to be defined later) radiated into the wake region. When there are no sources, the acoustic energy flux of the incoming waves is equal to that of the outgoing waves, and **P** is zero. This is the case in the problem of the continuous solution, where there is no vortex shedding. For the Kutta condition solution, however, the shed vorticity in the wake acts as a source (through interaction with the edge) and **P** has (usually) a non-zero value.

We shall now define **P**. We do not want to enter the discussion on the best definition of acoustic energy and intensity in moving media. We simply take a definition, and expect others not to give qualitatively different results. The definition we adopt is the one given by Morfey (1971), or in Goldstein's book (1976, p. 41). From there we find for acoustic energy flux of irrotational flow, in our notation,

$$\mathbf{I} = (I_x, I_y) = -\rho_0 U_0^3 \phi_t(\phi_x + M^2 p, \phi_y),$$

and the acoustic power crossing a surface S into direction $\mathbf{n}$

$$\mathbf{P} = \int_S \bar{\mathbf{I}}.\mathbf{n}\, dS,$$

where $\bar{\mathbf{I}}$ (i.e. the time average of $\mathbf{I}$) denotes the acoustic intensity, and $\mathbf{n}$ is the unit normal vector on S.

Now take for S two semi-infinite planes, one just below the wake and the other just above, and connected at the plate edge. The net sound power, radiated through S into the wake, is a measure of the effect of the Kutta condition on the acoustics, since it is just the hydrodynamic energy of the vortices escaping between the two planes which is not included. (It cannot be, because the definition of **I** is only valid in an irrotational field, so we cannot close the two surfaces far downstream through the wake.) After some algebra, we find

$$\mathbf{P} = \frac{a^{*2}}{2\pi\rho_0 f^*} M \cos^2 \tfrac{1}{2}\theta_i (1 + M\cos\theta_i)(1 - M + 2M\cos\theta_i), \tag{4.9}$$

where $a^* = \rho_0 U_0^2 a$ is the dimensional amplitude of the incident pressure wave p_i. The power integral has been taken in the generalized sense, so that $\int_0^\infty \cos(\lambda x)\, dx = 0$ if $\lambda \neq 0$. Justification for this can be found by considering the energy flux of the harmonic source problem (Jones 1972) in the limit of source position and amplitude going to infinity.

In figure 3, $(a^{*2}/2\pi\rho_0 f^*)^{-1}\, \mathbf{P}$ is plotted as function of M and θ_s. The values of these parameters, corresponding to considerable deviations of $(a^{*2}/2\pi\rho_0 f^*)^{-1}\, \mathbf{P}$ from zero (somewhat arbitrarily defined as: more than 0.1), are indicated by shaded areas. Both the maximum (= 4) and the minimum (= −0.148) are found at $M = 1$.

hydrodynamic power $P_x^H = \int_{-\infty}^{\infty} \bar{I}_x^H\, dy = \frac{a^{*2}}{2\pi\rho_0 f^*} M \cos^2\tfrac{1}{2}\theta_i \ldots$

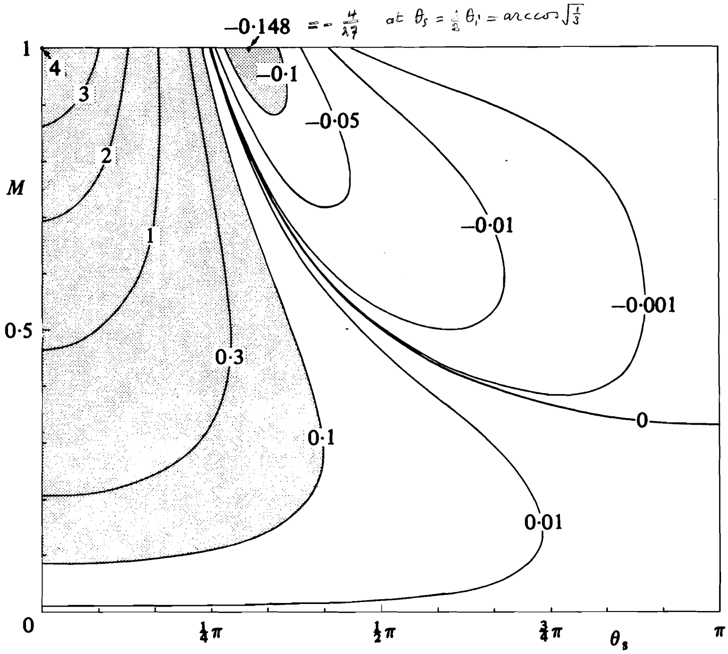

FIGURE 3. Contours of constant net sound power absorbed by the wake (plane harmonic waves with Kutta condition).

A most remarkable consequence of expression (4.9) is, that **P** can be both positive and negative, or, in other words, that the application of the Kutta condition can result in both a quieter and a noisier acoustic field. When $M > \frac{1}{3}$ and the incident wave has $\theta_s > \frac{1}{4}\pi$, **P** may be negative, but if either inequality is reversed **P** is always positive. A merely positive or zero **P** would not be surprising in view of Bechert's (1979) and Howe's (1979) sound attenuation mechanism by vortex shedding from a nozzle lip. However, a negative **P** can only be explained by assuming that energy from the main flow is, via the vortex shedding, converted into sound energy. This is clearly the case in the problem without a source (spontaneously shed vortices with a field given by $\hat{\phi}_e$; Davis 1975), where there is a strictly positive acoustic energy flux out of the wake. The only possible source of this energy lies in the mean flow, though this relation cannot be described by the linear theory. This implies that Bechert's and Howe's attenuation mechanism should not be generalized too easily, and is only valid if the main flow contributes not too much to the vortex shedding.

5. The plane pulse wave

As before, the pulse solution will be found via its Fourier transform. For the present case, this transform can be identified with the harmonic solution of the previous section. So an integration with respect to ω of this yields the pulse solution we are looking for. We find then from (4.2) the incident wave

$$p_i(x, y, t) = a\,\delta(T_i). \tag{5.1}$$

Define $\bar{T}_i = t-(M/\beta)R\cos(\Theta+\Theta_s)+(M^2/\beta)X$, the time variable of the reflected wave. Making use of the following formal identity, in a generalized sense, for real numbers λ and μ,

$$\int_{-\infty}^{\infty}\exp(\tfrac{1}{4}\pi i+i\lambda x)\,F(\mu\sqrt{x})\frac{dx}{\sqrt{\pi}} = 2\pi H(-\mu)\,\delta(\lambda+\mu^2)+\frac{H(\lambda)}{\sqrt{\lambda}}\frac{\mu}{\lambda+\mu^2}$$

where the branch of $\sqrt{x}$ with $\sqrt{-1} = -i$ is chosen to obtain a causal solution later), we find from (4.4) and (4.5) the continuous solution

$$p_c(x,y,t) = aH(\theta_s-\theta)\,\delta(T_i)+aH(-\theta_s-\theta)\,\delta(\bar{T}_i)$$
$$+\frac{a}{2\pi}\frac{H(T_d)}{\sqrt{T_d}}\left(\frac{2MR}{\beta}\right)^{\frac{1}{2}}\left[\frac{\sin\frac{1}{2}(\Theta-\Theta_s)}{T_i}+\frac{\sin\frac{1}{2}(\Theta+\Theta_s)}{\bar{T}_i}+\frac{2\beta}{R}\frac{\sin\frac{1}{2}\Theta\cos\frac{1}{2}\Theta_s}{1-M\cos\Theta_s}\right], \quad (5.2)$$

and the Kutta condition solution

$$p_k(x,y,t) = aH(\theta_s-\theta)\,\delta(T_i)+aH(-\theta_s-\theta)\,\delta(\bar{T}_i)$$
$$+\frac{a}{2\pi}\frac{H(T_d)}{\sqrt{T_d}}\left(\frac{2MR}{\beta}\right)^{\frac{1}{2}}\left[\frac{\sin\frac{1}{2}(\Theta-\Theta_s)}{T_i}+\frac{\sin\frac{1}{2}(\Theta+\Theta_s)}{\bar{T}_i}\right]. \quad (5.3)$$

Comparison of the parts, representing the diffracted waves, at the wave front, i.e. at $T_d = 0$, yields again the conclusion of an increase for $|\theta| < \theta_s$ and a decrease elsewhere with the application of the Kutta condition; see figure 2 with θ changed into $-\theta$, and $\theta = \theta_s$ the shadow boundary.

6. Application of triple-deck theory

In the present section we shall 'calculate' the Kutta condition as it results from viscous action in laminar flow. Central here is the boundary-layer structure near the trailing edge for high values of the Reynolds number, or small ϵ. The theory considering this boundary-layer structure, the 'triple deck', is developed by several authors we mentioned in the introduction, and the reader is referred to their respective papers for more and deeper information about it. We shall only apply some results relevant to the acoustic outer field.

Essential is that the interaction of the external pressure perturbation (of the Kutta condition solution) and the pressure, induced by the viscous smoothing process, takes place at a distance from the edge of order ϵ^3 (i.e. $r = O(\epsilon^3)$); here the two pressures are considered to be both of order ϵ^2 (i.e. $p \ldots O(\epsilon^2)$) for the Kutta condition to be asymptotically valid as $\epsilon \to 0$. About the other cases, of a much higher and a much lower external pressure perturbation, we can, with the present state of the art, only speculate, and we shall not consider these here.

These general thoughts from triple deck theory result already in the following estimate. From expression (4.5) we find for p_k near $r=0$,

$$p_k = \tfrac{1}{2}a\exp(i\omega t)-\frac{2a}{\sqrt{\pi}}\exp(\tfrac{1}{4}\pi i+i\omega t)\,(2KR)^{\frac{1}{2}}\sin\tfrac{1}{2}\Theta\cos\tfrac{1}{2}\Theta_s+O(R); \quad (6.1)$$

and so, for M not too close to unity and θ_s (and thus θ_i) not too close to π, the condition that the varying part of $p_k = O(\epsilon^2)$ for $r = O(\epsilon^3)$ results in the relation

$$a\sqrt{k} = O(\sqrt{\epsilon}). \quad (6.2)$$

Of course, this estimate is only an unproved hypothesis, based on other examples successfully evaluated. A first step toward a proof would be a consistent description of the flow field in the different boundary layers. This is possible if we adopt the restrictions presented in § 2 on the formulation of the viscous problem. By taking $\omega = \epsilon^{-2}\omega_0$ ($\omega_0 = O(1)$) we have the Stokes layer (the boundary layer due to viscous friction of the sound wave at the plate) of order ϵ^5 in thickness, which simplifies the way the Stokes layer enters the triple-deck region, and by taking $M = \epsilon^2 M_0 (M_0 = O(1))$ we have around the edge an incompressible region $r = O(\epsilon^2)$, which allows us to use existing results for incompressible flow. From (6.2) it follows that we need then $a = \epsilon^{\frac{1}{2}} a_0$ ($a_0 = O(1)$).

The acoustic outer field including the viscous correction from the edge region is now described by the Kutta condition solution (4.5) plus a small 'amount' of eigensolution

$$p(x, y, t) = p_k(x, y, t) + a B_0 p_e(x, y, t), \tag{6.3}$$

where we define

$$p_e(x, y, t) = \frac{\sin \frac{1}{2}\Theta}{\sqrt{R}} \exp(i\omega T_d). \tag{6.4}$$

In the triple deck region $r = \epsilon^3 r_3$, $x = \epsilon^3 x_3$, $y = \epsilon^3 y_3$ ($r_3, x_3, y_3 = O(1)$), we have after application of all the relevant approximations and neglect of the unimportant constant term

$$p = -\epsilon^2 a_0\, 2\left(\frac{2k}{\pi}\right)^{\frac{1}{2}} \cos\tfrac{1}{2}\theta_i \sin\tfrac{1}{2}\theta \exp\left(\tfrac{1}{4}\pi i + i\omega t\right)\left[\sqrt{r_3} + \frac{B_1}{\sqrt{r_3}}\right] + \dots, \tag{6.5}$$

where

$$B_0 = -\epsilon^3\, 2(2k/\pi)^{\frac{1}{2}} \cos\tfrac{1}{2}\theta_i \exp\left(\tfrac{1}{4}\pi i\right) B_1.$$

This is the form for the pressure analogous to that of Brown & Daniels (1975) given by their equation (2.7). We checked the correspondence in all detail, and it appears that the disturbances of the steady-flow boundary layers in our problem are essentially the same as in the incompressible problem of Brown & Daniels. In the region $r = O(\epsilon^2)$ the disturbances are already of incompressible nature to leading order. Outside this region, external disturbances simply govern the disturbances in the boundary layer, without being affected by any interaction, while the limit behaviour near the edge is the same as that of Brown & Daniels. Therefore, there is no need to repeat here all the calculations, and we can at once use their principal result, namely an estimate of B_0 for $\omega_0 \gg 1$ (obtained after a linearization, requiring in our notation $a_0(M_0/\omega_0)^{\frac{1}{2}} \to 0$). We only remark that in Brown & Daniels, equation (4.8) suffers from numerous misprints, while expression (4.3) does not match with expression (5.9), as it should. In (5.9) an $O(\epsilon^{\frac{3}{2}})$ term is missing, but it appears to be only a constant which does not imply any further change.

After identifying our parameters in Brown & Daniels' equation (7.32), we find

$$B_0 = \epsilon^3 \left(\frac{M_0}{\lambda\pi}\right)^{\frac{1}{2}} \cos\tfrac{1}{2}\theta_i, \tag{6.6}$$

where $\lambda = 0.3321$, a constant arising in the Blasius boundary-layer solution. Note that the linearization is valid for all fixed a_0 and M_0, as $\omega_0 \to \infty$. In the far field this viscous correction is felt in the diffracted wave in the following way:

$$p_d \simeq \epsilon^{\frac{1}{2}} a_0 \frac{\sin\frac{1}{2}\Theta \cos\frac{1}{2}\Theta_s}{(\pi K R)^{\frac{1}{2}}} \exp(i\omega T_d)\left[\frac{1-i}{\cos\Theta_s - \cos\Theta} + \epsilon^3 M_0 \left(\frac{\omega_0}{\lambda}\right)^{\frac{1}{2}}\right] \quad (R \to \infty). \tag{6.7}$$

Clearly the viscous interaction at the trailing edge yields only a small correction to the leading order outer solution (the Kutta condition solution), and hence we can conclude that the assumption of the Kutta condition is consistent with triple deck structure for this class of parameters. Nevertheless, experimental verification of the extra term in (6.7) might perhaps be possible, and would constitute an interesting indirect confirmation of the triple-deck theory, whose direct experimental verification is difficult.

7. Conclusions

The semi-infinite plate in a uniform flow is shown to provide a good model for trailing-edge, flow, sound interaction. A predicted effect of vortex shedding, by application of the Kutta condition in the model, is a decrease upstream of the dominant part of the pressure of the diffracted wave. This is argued to be in good agreement with Heavens' (1978) schlieren pictures, and supports his suggestion with respect to the role of the edge condition in his experiments.

The vortex shedding process extracts energy from both the incident acoustic wave and the main flow, while on the other hand the shed vortices produce sound by interaction with the plate. The net result may be either a noisier or a quieter acoustic field, depending on Mach number and source position (it is always quieter for $M < \frac{1}{3}$). This is shown by an exact expression for the net sound power, absorbed by the wake, in the harmonic plane wave problem with Kutta condition.

The Kutta condition, appearing in idealized inviscid problems, replaces the effect of viscosity in real flows. The physical mechanism is believed to be described in essence by the triple deck theory. In an attempt to clarify the role of viscosity in the present aero-acoustical problems, we have included results from this triple-deck theory in our acoustic outer solution. This may be of help to interpret those experiments where deviations from the Kutta condition can be expected.

The present work was prepared for publication at the National Aerospace Laboratory NLR. Part of it was taken from the author's thesis, prepared at and submitted to the Eindhoven Technical University. The first and second supervisor were Prof. G. Vossers (Eindhoven) and Prof. D. G. Crighton (Leeds), respectively, to whom the author is very grateful for their ever friendly, inspiring and stimulating cooperation. Also he is indebted to the University of Leeds for the hospitality it offered the many times he visited it.

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