J. Acoust. Soc. Jpn.(E) 6, 2 (1985)

Sound propagation in the rest atmosphere with linear sound velocity profile

Takayuki Hidaka, Kenji Kageyama, and Sadahiro Masuda

Takenaka Technical Research Laboratory,
2-5-14, Minamisuna, Koto-ku, Tokyo, 136Japan

(Received2May1984)

Sound propagation is considered in the rest atmosphere where the sound velocity varies linearly with height. With the results of the ray theory of refraction being introduced to the asymptotic solution of spherical wave propagating over a locally reacting ground surface, the approximated solution of the sound field is obtained. The results of calcula-tion by this theory explains the measurements satisfactorily, as a result quantitative prediction of excess attenuation is available.
PACS number: 43. 28. Fp

1. INTRODUCTION

Sound propagation in outdoor is strongly influ-enced by meteorological conditions and by the pres-ence of the ground surface. 1-5 However, it is not easy to analyze the sound field by means of the wave theory6) with the meteorological and the boundary conditions being simultaneously taken into account, and for many cases it is difficult to observe their rela-tive contributions to propagating sound. Thus it seems to be effective to use the ray theory7-10) in some way, when one treats of the sound field.
In this paper spherical wave propagating in the rest atmosphere, in which sound velocity varies with height, over a locally reacting11-18) ground surface is considered. This situation is the case when the wind velocity is small enough to be neglected and sound velocity is mainly determined by thermal or humidity profile of the atmosphere. First one derives path length and time in propagation of refracted wave by the ray theory when sound ve-locity varies linearly with height. Secondly they are substituted to the asymptotic solution13) of the spherical wave field above a locally reacting surface formally, and the approximated representation of the total sound field is obtained. Furthermore the effect of the sound velocity profile is investigated numeri-

cally under the actual meteorological conditions.

2.•@ ANALYSIS

2.1•@ Ray Theory
Consider a point sound source S located at a
height zs above the ground at z=0as shown in Fig.
1. The ground is assumed to be a locally reacting surface with a specific acoustic impedance ratio ƒÄ= 1/ƒÀ, where ƒÀ is the specific admittance ratio. Since the problem is two dimensional, r axis is defined in horizontal direction in the vertical plane which con-
tains S and receiver P, and horizontal distance be-
tween S and P is denoted by R.
The sound velocity c in the open air is given by

(1)

where T, e, p are the fair temperature (K), the partial pressure of water vapor and the atmospheric pressure, respectively. 8) In the surface boundary layer, the gradients of these variables are smooth and small, so the sound velocity in the atmosphere can be approximated by the linear equation

(2)
where co is the value of c at z=0and c (z) is inde-pendent of r. Then a is called the normalized sound

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Fig.1•@ Geometry of source and receiver.

Fig.2•@ Element of the sound ray.

velocity gradient and assuming the atmosphere to be rest, one can write the refractive index n(z) as

(3)

By the way each variable in Eq.(1) fluctuates general-ly, the mean value of it is considered in this paper. If above assumptions are introduced, the circular path for the ray is obt-ained. 3) The fundamental rela-tions, which are useful in later analysis, are derived on the basis of the ray theory in the following. Now the ray element in Fig.2 is given by

(4)

and using the angle of inclination of the ray, ƒµ, one can write the path length of the ray for propagating from z=0to z as

(5)

Here taking Snell's law into account

(6)

and Eq.(2), one gets the next equation after integra-tion

(7)

where ƒµ0 is the angle at z=0.
Next, introducing the time necessary for travers-
ing the element of length of the ray ds as

(8)

one obtains the time required in propagating from z=0to z from Eqs.(4) and (8),

(9)

where

(10)

Moreover defining the horizontal length r (z) as the projection of s (z) on r axis, one can write

(11)
After elementary transformation, one gets

(12)

It is seen that the ray is the arc of a circle with center at the point r=tan ƒµ0/a, z=-1/a and radius of1/|α| cos ψ0.
When sound propagates over the ground, there
are direct and reflected wave. Each ray draws circu-
lar path whose position and curvature correspond to
the value of the sound velocity gradient, then one
can determine the path of the ray from the geo-
metrical condition of source, receiver and ground as
following. Now since the atmosphere assumed to
be rest, the analysis is limited to the case when zs•…
zp. One can exchange zs and zp, if zs>zp.
2.1.1•@ Direct wave (a>0)
If the sound velocity gradient a is positive, it can
be seen that the ray bends downward from Eq.(12)
and the next equation holds with respect to the
horizontal distance R (Fig.3).

(13)

Fig.3•@ Bended ray for positive sound velocity
gradient.

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where Ro is a distance between two points of inter-section of the ray with the ground:

(14)

Here the symbol Cuu means that the source and receiver are both located on the ascending branch of the ray. The symbol Cud means that the source is located on the ascending and the receiver on the descending branches. Similarly Cdd and Cdu are defined. 14)
Now denoting Rd as the path length of direct wave, one obtains

(15)

Here zn, is the maximum height of the ray from the ground and by setting r=R0/2in Eq.(12) one gets

(16)

Therefore one has

(17)

(18)

From Eqs.(13) and (14) one obtains

(19)

and one can determine Rd by substituting ƒÕ0into
Eq.(15).
It is obvious that the two conditions, Cuu, and
Cud, is distinguished by the sign of the equation

(20)

hence Cuu holds for gd<0 and Cud holds for gd•†0.
Similar to Eq.(15), the time in propagation of the
direct wave denoted by Td is given by the equation

(21)

and is calculated by using ƒÕ0of Eq.(19). Especially
if zs=zp, one obtains

Fig.4•@ Bended ray and coordinates for nega-
tive sound velocity gradient.

(22)

and

(23)

2.1.2•@ Direct wave (a<0)
The ray bends upward in this case, then new
coordinate system,(r', z'), is introduced as shown in
Fig.4, where r' axis is defined so as to pass the
receiver P and z'axis is taken in opposite direction
of z axis.

Hence the following equation for the horizontal
distance holds:

(24)

where r'(z') is given by Eq.(11) and one writes „q=

|zs-zp| and a'=-a. Since R0is expressed by Eq.
(14), ƒÕ0which is the parameter for calculating the
path length Rd and time in propagation Td is given by

(25)

Therefore one gets Rd and Td by Eq.(25) as

(26)

(27)

where z'm is the maximum value of the ray in z' direc-tion.
It can be seen easily that, if g'd is denoted by

(28)

the condition Cui, holds for g'd<0and Cdu holds for

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By the way if z' na>zp, namely

(29)

holds under the condition Cdii, the direct wave does
not reach to the receiver. Furthermore reflected ray
does not reach to the receiver, because the angles of
incidence and reflection at the ground surface are
equal. Then the receiver is in the shadow zone, but
there exists a little sound energy in it. R. W. Morse15)
showed that the assumption that the sound velocity
gradient approaches zero in the vicinity of the ground
surface yields no shadow zone, even if the ray theory
is employed. On the other hand, the existence of
the sound energy in it is considered by the scatter-
ing from inhomogeneities outside this zone in the
wave theory. 4)
2.1.3•@ Reflected wave (a>0)
The propagating path of the wave reflected at the
ground is analyzed by similar way to direct wave.
First the equation for the horizontal distance R is
written as

(30)

where N is a natural number that means the number

of reflections at the ground and Ro is given by Eq. (14). Substituting Eq.(11) to Eq.(30) and setting
χ=tan ψ0, one gets quartic equation ofχ. In other

words the waves with successive reflections more

than two times generate under certain condition. This is a characteristic phenomenon derived by the ray theory.
But considering that the sound velocity gradient takes small value in the actual atmosphere and the asymptotic solution for zero gradient is applied in later, one limits the case of N=1in this paper. Setting N=1in Eq.(30), one gets the cubic equa-tion

(31)

(32)

The roots of this equation are calculated by Cardano's formula16) and only positive roots which satisfy Eq.(30) correspond to the reflected wave. So

the characteristics of the roots are mentioned as follows.

It can be seen that Eq.(31) always has one posi-tive root because the constant term in Eq. (31) is negative and the another two roots are real or com-plex according to the sign of the discriminant of the cubic equation. Hence the former root corresponds to the path Cud or Cdu, and there is always reflected wave of either path. By use of the sign of the equa-tion

(33)

one can decide the path of the ray so that Cud is
selected for gd>0and Cdu for gd<0.
The latter roots judged by the discriminant cor-
respond to the pathes, Cuu and Cdd. When the
geometry of the source and receiver is fixed, they
become real roots for large value of a. This can be
seen that the bending of the ray will be remarkable
as a becomes larger. In addition to this property,
the condition zm>zp must hold in this case. In this
paper, one denotes the root that satisfies the condi-
tion Cud or Cdu by ƒÕ0g, and roots that satisfy Cuu and
Cdd by/ƒÕ0u and ƒÕ0d respectively.
The path length and time in propagation of re-
flected waves are obtained by the equation of similar
form to Eq. (30), where ƒÕ0g is substituted to ƒÕ0 for
the condition Cud or Cdu, ƒÕ0u for Cuu and ƒÕ0d for
Cdd. Then the path lengths which correspond to

ψ0g

,ψ0u and ψ0d are denoted by Rg, Rgu and Rgd,

respectively, and the times in propagation, Tg, Tgu and Tgd, are defined similarly.
Now if zs=zp holds, Eq.(31) can be factorized and always three positive roots are given:

(34)

(35)

The reflected waves with the parameter of ƒÕ0u or

ψ0d can be in existence only for the case where the

content of the square in Eq.(11) is not negative.
Consider the case of a•¨0in Eq.(34), then

(36)

is derived and the ray becomes straight line. It can
be shown that the reflected wave of ƒÕ0g coincides
with that term of the asymptotic solution derived in
the case of a=0. So the reflected waves of ƒÕ0u and

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Fig.5•@ An example of ray paths when the
second reflected wave generated, where
zs=zp.

ψ0d are the peculiar ones generated in the case of a>
Obecause of the use of the ray theory. Especially
one denotes the reflected waves of ƒÕ0g, ƒÕ0u and ƒÕ0d
by „Ug, „Ugu and „Ugd, respectively, and „Ugu and „Ugd are
called the second reflected wave here after. Figure
5shows an example of ray paths when the second
reflected waves are generated.
2.1.4•@ Reflected wave (a<0)
By defining the coordinate system (r', z') similar
to section2.1.2, one gets the equation

(37)

with respect to horizontal distance. Setting ƒÔ=
tan ƒÕ0in Eq. (37), one obtains the cubic equation
of ƒÔ,

(38)

(39)

and only one positive root denoted by ƒÕ0g is given.
Since the path length Rg and time in propagation
Tg is written by

(40)

(41)

which are calculated by ƒÕ0g being substituted. Here

the condition that the receiver is not in shadow zone
is given by z'm>zp.
It should be mentioned that if zs=zp holds, from
Eq. (37), ƒÕ0g becomes

(42)

and one obtains

(43)

(44)

2.2•@ Total Field
If the sound velocity is constant (a=0), the total
sound field due to a point source at S above a ground
(see Fig.2) is expressed by

(45)

where time factor exp(-kit) is suppressed and the
first and the second term represent direct and re-
flected wave respectively. The complex reflection
coefficient Q which includes a contribution of a sur-
face wave is a function of R2, ƒÆ0, k and ƒÄ and the
expression of Q given by the Eq. (27) in Ref. 17) is
used for calculation in this paper (see Appendix I).
It can be assumed that if the path lengths and times
in propagation of direct and reflected waves, derived
in the previous chapter, are substituted to the above
equation, the total sound field, being taken the
refraction into account, is obtained approximately.
Then the reflection angle at the ground, ƒÆ0, is given
by the equation

(46)
where Snell's law is used for the case of a<0.
So denoting the reflection coefficient corresponding

φg,φgu andφgd by Q, Qu and Qd, respectively, one

can write the total field for a>0by

(47)

hence „Ud is the direct wave and Qu=Qd=0if the
second reflected waves, „Ugu and ,„Ugd, do not exist.
Similarly for a<0one has

(48)

3.•@ NUMERICAL RESULTS

In this chapter the sound field in the atmosphere
where the sound velocity is expressed by Eqs.(1) and
(2) is considered numerically. Although the sound

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Table1•@ Three years average of diurnal variation of sound velocity gradient, a (m-1)•~105, after Geiger.

velocity is given by temperature T and the relative
pressure of water vapor to the atmospheric pressure
e/p exactly, it can be seen from Eq. (1) that the sound
velocity depends mainly on the temperature.
Consider the temperature profile in the surface
boundary layer written by

(49)

where y is often referred to as the adiabatic lapse
rate and takes value about-0.01•Ž/m for the rest
atmosphere under adiabatic conditions. However
its value changes with season, hour, weather and the
state of the ground surface.
By use of Eqs. (1) and (2), the sound velocity is

(50)

and the normalized sound velocity gradient is given
by

(51)

Table1shows the diurnal variation of sound ve-
locity gradient converted by Eq. (51) after Geiger's
measurements for three years, 18) and an example of
the actual temperature profile 19) is shown in Fig.6.
It can be seen that the sound velocity gradient in
the lower layer below several tens of meters is larger
than that in the higher layer, the sound velocity
profile in the actual atmosphere can be approximated
by the linear equation of Eq.(50) if the sound velocity

gradient is effectively determined according to the
heights of the source and receiver.
The path lengths and times in propagation of each
ray are shown in Table2, when zs=zp=1.2m and
R=100m or400m. The maximum heights of the
rays, zm, and the reflection angle,ƒÆ0, are shown
simultaneously, then the former represents the maxi-
mum value in z' direction in Fig.4if a<0.
It is seen that the path lengths, Rd and Rg, increase

Fig.6•@ Diurnal variation of temperature
profile in the lowest100m of the atmo-
sphere on clear summer day after A. C.
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with the increasing absolute value of a and the path
length corresponding to+a is equal to that to-a.
This is due to the fact that c(zp)•¬co holds when a•s
1. Now, though the times in propagation, Td and
Tg, decrease with the increasing sound velocity, their
difference is proportional to a. From the approxima-
tion of the sound field in the previous section, it can
be said that, for a<0, the source and receiver would
be nearer to the ground than the case for a=0due
to increasing ƒÆ0with decreasing a. For a>0, one
can suppose the contrary result.
By the way the second reflected waves are gener-
ated as a and R increase. Because zs=zr, is assumed
in Table2, the second reflected waves,„Ugu and „Ugd,
are symmetrical each other with respect to r=R/2
(see Fig.5). It is clear that they are generated when
zni exceeds zp. In order to express these phenomena,
Fig.7shows an example of the region where the
second reflected waves and the shadow zone are

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Table2•@ Calculated values of geometrical parameters, where zs=zp=1.2m.

(a)•@ R=100m

(b)•@ R=400m

Fig.7•@ Structure of the sound field for the
case of zs=zp=1.2m, then (i) „Ud and „Ug
exist in I, (ii) „Ud, „Ug, „Ugu and „Ugd exist in
II, (iii) shadow zone generated in III.

Fig.8•@ Solid line represents the boundary of
shadow zone for the value of the sound ve-locity gradient, a, where zs=1.2m.

generated, and Fig.8shows the boundary of the shadow zone according to Eq.(28).
In order to consider the influence of the sound

velocity gradient on the total sound field, an excess attenuation Att is defined as

(52)

Hence Att is an excess attenuation without the at-

tenuation caused by geometrical divergence and the
absorption by the air, and coincides with that by the ground reflection if a=0.
Figure9shows calculated values of Att above a

grass and an asphalt surface for the change of the
sound velocity gradient, where the ground impedance
is calculated by Delany and Bazley's empirical
formula21) and flow resistivity „q is based on Ref.

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(a)

(b)

Fig.9•@ Effect of the change in sound velocity gradient for seven values of a;-2.10-4,

-10-4 ,-5・10-5,0,5・10-5,10-4,2・10-4,when zp=zp=1.2m. (a) grass(σ=300 cgs

unit), (b) asphalt („q=20000 cgs unit)

20). It can be seen that if a>0, Att diminishes and
the position of the dip of Att moves to lower fre-
quency range in comparison with the condition a=
0irrespective of the ground impedance, and, if a<
0, Att rises and that of the dip moves to heigher
range. Furthermore if a becomes smaller, then the
shadow zone is generated.
The tendency mentioned above agrees well with
the experimental results as before. 22,23) So the ap-

proximated representation of the sound field of Eqs.
(47) and (48) seems to be reasonable and now the
influences of the meteorological condition on sound
propagation can be predicted by this theory if the
atmosphere is rest.

4.•@ CONCLUSION

The sound field in the rest atmosphere where the
sound velocity varied linearly with hight above a

ground with a finite impedance was analyzed ap-proximately by means of the ray theory introduced to the asymptotic solution. As a result, quantitative prediction of sound propagation based on wave theory became possible, while only qualitative predic-tion had been available on the effect of meteorologi-cal conditions.

It should be noted that the sound velocity profile is not generally linear, one can pursue each ray numerically in the atmosphere with any profile by means of approximation by polygonal lines.

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APPENDIX I

If the ground is locally reacting, by use of the ac-curate asymptotic solution13) of the total field, the complex reflection coefficient Q can be rewritten as follows (see Fig.1).

(A.1)

(A.2)

where

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