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Source-Depth Determinations using Spectral, Pseudo-Autocorrelation and Cepstral Analysis

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Summary

Interference between P and pP (or P_n and pP_n) in a seismic signal causes the spectrum of the combined wave train to be scalloped with a period equal to the depth-phase delay time τ. Determination of the null frequency interval ($\Delta f = 1/\tau$), together with a knowledge of the overburden compressional velocity, permits an estimate of the source depth to be made. Scalloping produced by reverberations at the recording station can be reduced by forming the average of a suite of individual station spectra. The spectral bandwidth available for analysis can be broadened by removing instrument response. To determine objectively the periodicities in the average spectrum, the cepstrum and pseudo-autocorrelation are computed. This depth determination method combining spectral, pseudo-autocorrelation, and cepstral analysis is applied successfully to *Pn* or *P* data sets for three underground explosions whose depths are known *a priori*.

1. Introduction

The spectra of P waves from Long Shot have deep valleys or nulls in the 1·7 to 2·1 Hz band. Although multipath effects (including reverberations) and multiplet sources can produce nulls such as those observed, we provisionally interpreted the nulls in the Long Shot to arise from interference between P and the depth phase pP (Lambert *et al.* 1969). The reciprocal of a given spectrum's first null frequency provided an estimate of the $pP-P$ delay time, and this time, together with a measured overburden velocity, was successfully used to estimate the shot depth.

In the course of the Long Shot analysis a study was undertaken to refine the spectral-null method for depth determination (Cohen 1969). We observed that by using a network-averaged spectrum, spectral periodicities related to the source region were enhanced (for a similar technique applied to hydroacoustic signals see, Plutchok, Broome & Johnson 1967, page 5). Further, it was found that removing instrument response made a broader spectral window available for the null-frequency analysis. Finally, application of pseudo-autocorrelation and cepstral analysis to an averaged, processed spectrum was found in some instances to provide a direct determination of the pP–P delay time. The purpose of this paper is to describe the data analysis techniques employed in the determination of pP–P time delays—and hence, in the estimation of shot depth—and to demonstrate these techniques using *Pn* or *P* data from three underground nuclear explosions.

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2. Method

Let $z(t)$ be a time signal composed of an event, $y(t)$, and its undistorted echo, the latter delayed by time τ:

$$z(t) = y(t)+\alpha y(t-\tau)$$

where α is a scaling coefficient $(-1 \leqslant \alpha \leqslant 1)$. The power spectrum of $z(t)$ is then:

$$P(\omega) = \left| \int_{-\infty}^{\infty} z(t)\, e^{-i\omega t}\, dt \right|^2 = |Y(\omega)|^2 [1+\alpha^2+2\alpha . \cos(\omega\tau)], \qquad (1)$$

where $Y(\omega)$ is the spectrum of $y(t)$. From equation (1) it is apparent that the effect of the echo is to introduce a sinusoidal ripple to the term depending only on the signal spectrum. For a reflection at the free surface, α is negative, and spectral nulls produced by the interference of $y(t)$ and $y(t-\tau)$ occur at frequencies

$$f_m = m/\tau;\ m = 0, 1, 2, \ldots .$$

Short-period spectra recorded at teleseismic distances are severely band-limited by attenuation of the higher frequencies during propagation. The spectra also exhibit characteristics which derive from the reverberation of signals in the crustal layering at the recording sites, and so it is generally not possible to determine the null frequencies from individual spectra. However, if we normalize and sum the spectra of seismograms recorded at a network of stations, the resultant average spectrum will not only be enriched in the coherent scalloped components from the interference of the P and pP phases, but will also have averaged out, in large part, 'holes' in the spectra which derive from the geology at the different individual recording sites. It is necessary, of course, to be sure that the network is not so large that the echo delay time is different from one part of the network to another. Data at Pn distances, for example, should be considered separately from that recorded at P distances.

If the null frequency interval Δf (where $\Delta f = f_{m+1} - f_m$) can be determined from the average spectrum, the pP–P delay time τ is given simply by

$$\tau = 1/\Delta f. \qquad (2)$$

For a shot overburden average velocity V_1, the source depth d is then

$$d \simeq (1/2)\, V_1\, \tau. \qquad (3)$$

For pPn–Pn, the echo delay time is given by (2). However, the event depth d, overburden velocity V_1, mantle velocity V_n, and τ are better related by:

$$\tau = (2d)/(V_1 . \cos\theta) - (2d . \tan\theta)/V_n = (2d/V_1) \cos\theta \qquad (4)$$

where $\theta = \arcsin\,(V_1/V_n)$.

An important *objective* means of determining periodicities in the average spectrum is to compute the squared magnitude of the Fourier transform of the spectrum. This function, the cepstrum* can be defined as the power spectrum of the power spectrum:

$$c(\nu) = \left| \int_{-\infty}^{\infty} P(\omega)\, e^{-i\omega\nu}\, d\omega \right|^2 . \qquad (5)$$

* To help remember which domain we are talking about, we adopt the standard device of reversing the consonants of the first syllable of words referring to the domain of the spectrum-of-the-spectrum: e.g., 'spectrum' → 'cepstrum'; its argument is 'quefrency'; any filtering of the spectrum is called 'liftering', etc. This can be carried to extremes, but we have found it a helpful mnemonic device.

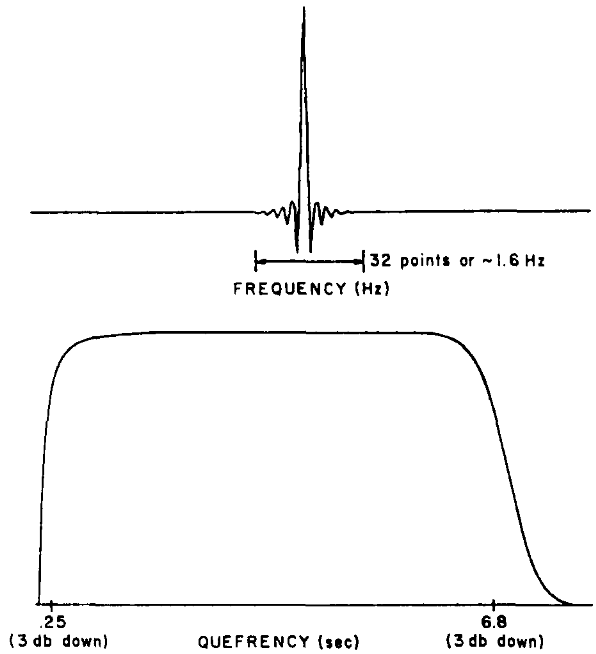

FIG. 1. Typical lifter response and pass-band. Two passes, one forward and one backward, through a long and a short pass filter result in the time bandpass characteristics shown. (From Plutchok & Stites, 1968.)

In practice, the spectrum is liftered before transformation. Liftering is multiplicative filtering in the quefrency domain. An example of a typical time-pass lifter in the quefrency domain is shown in Fig. 1.

It is important to note that since $P(\omega)$ is not made symmetric prior to transformation, the cepstrum of the unliftered, raw spectrum (equation (5)) is not simply the square of the autocorrelation. Further, just as the inverse transform of the raw power spectrum is the autocorrelation, we define the pseudo-autocorrelation as the inverse transform of the liftered spectrum. In the case of the autocorrelation and the pseudo-autocorrelation, the time series and the liftered spectrum, respectively, are made symmetric prior to transformation. Because of computational differences, the pseudo-autocorrelation and the cepstrum can, and often do, show significant differences in their presentation of the liftered spectrum's characteristics.

Bogert, Healey & Tukey (1963), who originated the idea of the cepstrum, defined the cepstrum of a data segment as the spectrum of the logarithm of the spectrum of the data; the cepstrum is then

$$c(v) = \left| \int_{-\infty}^{\infty} \log P(\omega)\, e^{-i\omega v}\, d\omega \right|^2 \tag{6}$$

where

$$\log P(\omega) = \log |Y(\omega)|^2 + \log [1+\alpha^2+2\alpha.\cos(\omega\tau)]. \tag{7}$$

When α is small, this becomes

$$\log P(\omega) \simeq \log |Y(\omega)|^2 + 2\alpha.\cos(\omega\tau). \tag{8}$$

The motivation for taking the logarithm is to convert the multiplicative effect of the echo equation (1) into an additive effect, thus avoiding a convolution in the quefrency domain (the v domain). That is, the sinusoidal modulation caused by the echo is added to the term depending only on the signal spectrum, rather than multiplying it. If the logarithm is not taken, the cepstrum contains the convolution of the transforms of the two terms, $|Y(\omega)|^2$ and $[1+\alpha^2+2\alpha.\cos(\omega\tau)]$.

If $Y(\omega)$, the spectrum of the data, is flat and band-limited, the spectrum of $\log Y(\omega)$ has the form $\sin(X)/X$, where X is proportional to the bandwidth; i.e. we would expect the cepstrum to have a peak at zero quefrency and auxiliary peaks at the side lobe quefrencies. In general, when $Y(\omega)$ is not necessarily flat but is reasonably smooth, we expect the cepstrum to have the same general form, i.e. a peak at zero quefrency and smaller peaks at regularly spaced higher quefrencies depending on the bandwidth. The zero quefrency peak is easily liftered out of the cepstrum.

Taking the logarithm of the spectrum has other consequences, however; in particular, it has the effect of whitening the spectrum and increasing the relative importance of weaker periodicities. If α is small, this can help in the determination of the echo parameters α and τ since the sinusoidal component $2\alpha.\cos(\omega\tau)$ is enhanced. But if more than one echo is present, the inherent complication of the cepstrum (it contains peaks at each combination of the interecho delay times) is further increased and may not be easily interpretable.

When the echo amplitude is large—say $|\alpha| > 0{\cdot}5$—the principal features of the spectrum may result primarily from interference of the primary phase and its echo. In such cases, taking the logarithm actually weakens the sinusoidal effect, and downgrades the cepstral peak we are seeking. In hydroacoustics, for example, Plutchok & Stites (1968) found bubble pulse periods from the spectrum of the linear spectrum, but analyses performed using the spectrum of the log spectrum gave unsatisfactory results (R. Stites 1969 personal communication).

We are thus led to omit taking the logarithm of the spectrum before computing the cepstrum, since the amplitudes of the two events composing the records used in this study are expected to be roughly the same. We expect to find that the more important cepstral peaks are enhanced, but somewhat broadened by the implicit convolution process described above.

In computing the cepstrum, the power spectrum $P(\omega)$ is treated as a time series. If $P(\omega)$ contains periodicities with periods τ_i, $c(v)$ will have peaks at times τ_i. The multiplicity of cepstral peaks which is generally observed, however, may confuse the analyst; thus, pseudo-autocorrelation analysis should always accompany cepstral analysis. Because of the negative reflection coefficient for a free-surface reflection, the *pP–P* (or *pPn–Pn*) time delay will be distinguished by the coincidence in time of cepstrum maxima (positive) and pseudo-autocorrelation minima (negative), provided that the source radiation pattern has lobes of the same sign in the two take-off directions. If the dot product of the cepstrum and pseudo-autocorrelation is formed, the resulting function should exhibit strong negative peaks at depth-phase delay times.

It was originally thought that cepstral analysis could play little part in the determination of *pP–P* delay times unless detonation depths are great enough to cause *pP* delay times greater than about 1 second (e.g. 1·75 km for $V_1 = 3{\cdot}5\,\mathrm{km\,s^{-1}}$). Teleseismic signals are almost completely attenuated by anelastic absorption above 2 Hz, and thus for $\tau \leqslant 1$ s, the cepstrum is dominated by the bandpass characteristic of the Earth. Actually, the teleseismic bandpass does not cut off completely at 2 Hz, but contains some energy, albeit not much, in the band 2–5 Hz. One means of enhancing the energy in the 2–5 Hz band, and thus providing a longer window in which to search for spectral periodicities, is to remove the instrument response from a given power spectrum, that is, to form

$$P'(\omega) = \omega^2 \, P(\omega)/|H(\omega)|^2$$

where $P'(\omega)$ is the corrected velocity spectrum, $P(\omega)$ is the signal spectrum, and $H(\omega)$ is the system displacement response. The displacement response of the instruments used in this study is shown in Fig. 2. The flat response below 0·8 Hz is incorporated in this study to prevent the modified spectra from blowing up at low frequencies. To prevent the frequency-squared function from magnifying noise at high frequencies, the spectra we compute are zeroed beyond 5 Hz.

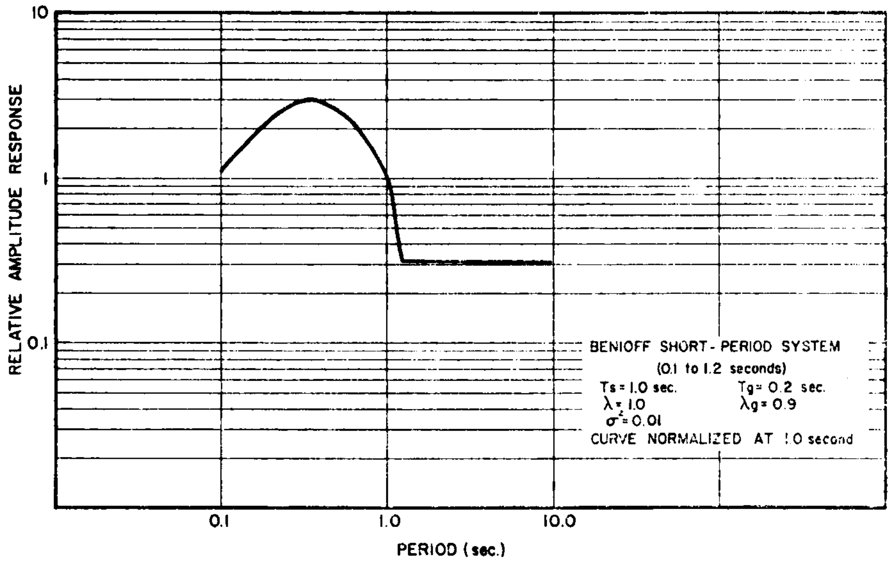

FIG 2. Modified system displacement response (LRSM).

3. Procedure

The depth-determination methods described were applied to short-period data from three underground nuclear explosions. These are: Gasbuggy, Greeley and Long Shot. All records were digitized at 20 sps, and the first 51·2 s (1024 points) recorded for each event processed. This record length was chosen so as to include in the analysis all daughter phases of the parent *P* and parent *pP* (or *Pn* and *pPn*) phases.

A time-pass lifter with 3 *db*-points at 0·1 and 35·8 s was used in this study. Our primary concern here was to remove the DC component in the raw power spectrum without significantly altering the low-quefrency components in the cepstrum. Because of the conservative lifter used, the autocorrelation and pseudo-autocorrelation are almost identical in the time-pass 0·1 to 35·8 s. For this reason, autocorrelations are not presented in the results to follow.

4. Results

Gasbuggy—Pn (*Fig.* 3)

The analysis techniques were applied to twelve records at *Pn* distances. Coincident negative and positive peaks in the pseudo-autocorrelation and cepstrum, respectively, produce a large negative peak in the dot-product function at delay time $\tau = 0{\cdot}75$ s.

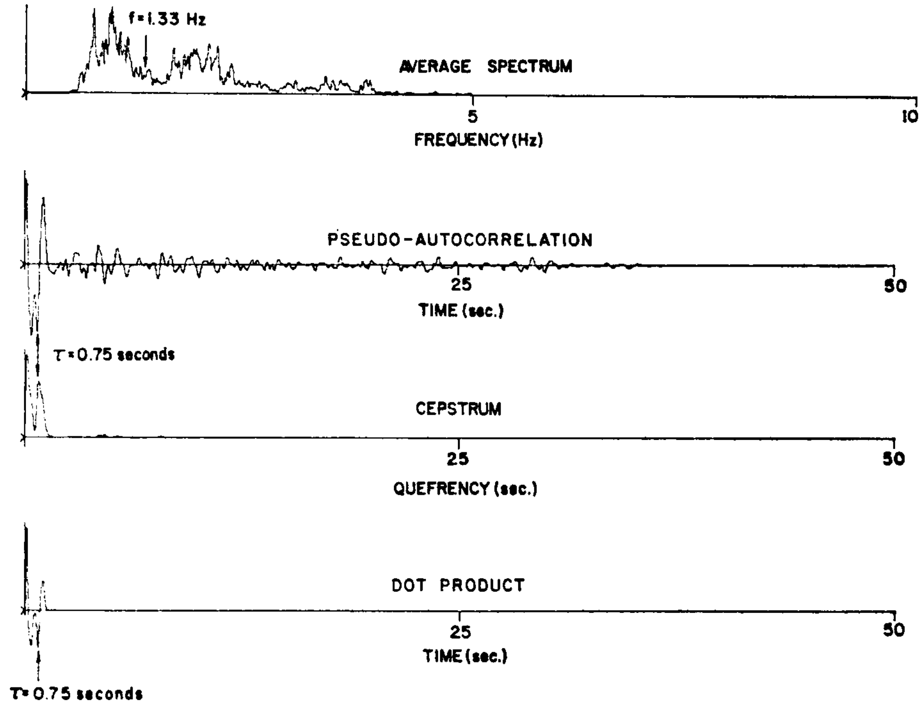

FIG 3. Analysis results, Gasbuggy–*Pn*.

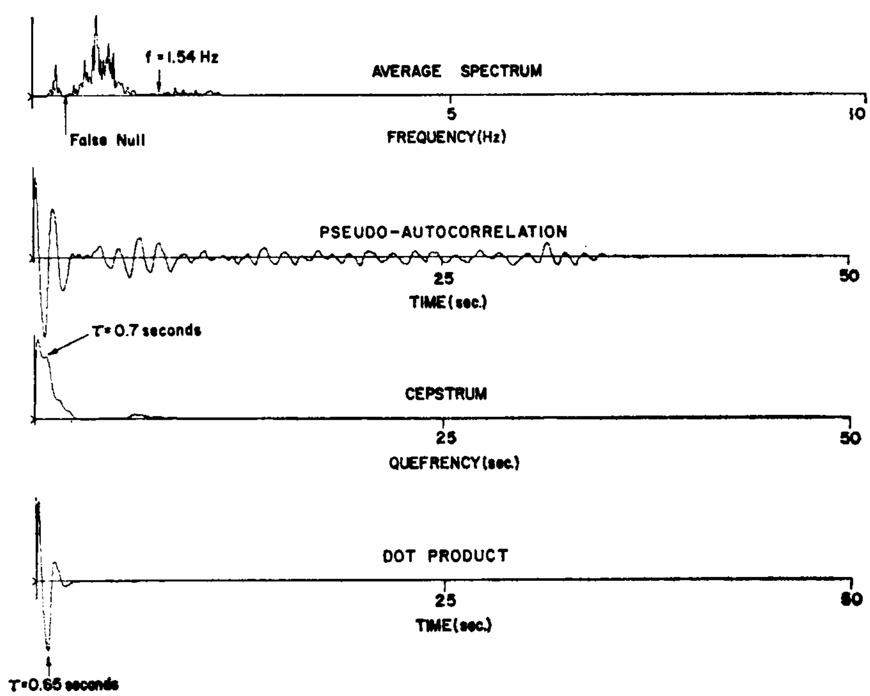

FIG. 4. Analysis results, Gasbuggy–*P*.

The first null frequency, $1/\tau = 1{\cdot}33$ Hz, is somewhat different, however, from what our analyst might judge to be the spectral null frequency. For assumed velocities of $V_1 = 3{\cdot}5 \text{ km s}^{-1}$ and $V_n = 8{\cdot}0 \text{ km s}^{-1}$, the time delay $\tau = 0{\cdot}75$ s yields a computed shot depth of 1·46 km, 0·15 km greater than the known shot depth.

Gasbuggy—P (Fig. 4)

Though only four seismograms at *P* distances were analysed, the results are considered acceptable. The positive peak in the cepstrum associated with the depth phase is seen at $\tau = 0{\cdot}7$ s, but the corresponding negative peak in the pseudo-autocorrelation is absent. If we pick the negative peak time of the dot-product function, $\tau = 0{\cdot}65$ s, the shot depth is 1·14 km, or 0·15 km shallower than the shot depth. The first null frequency, $f = 1/0{\cdot}65 = 1{\cdot}54$ Hz, is not clearly defined on the average spectrum.

The false null at $f = 0{\cdot}4$ Hz is caused by the presence of low frequency noise on several of the seismograms, and the subsequent superposition of signal and noise spectra.

Greeley—Pn (Fig. 5)

The results for seismograms taken at *Pn* distances are somewhat ambiguous. Presence of a depth phase is suggested by the character of the cepstrum around $\tau = 0{\cdot}75$ s, but there is no indication of this phase in the pseudo-autocorrelation function, and the dot-product function (heavily influenced by the large negative peak in the pseudo-autocorrelation function) has a large negative correlation at

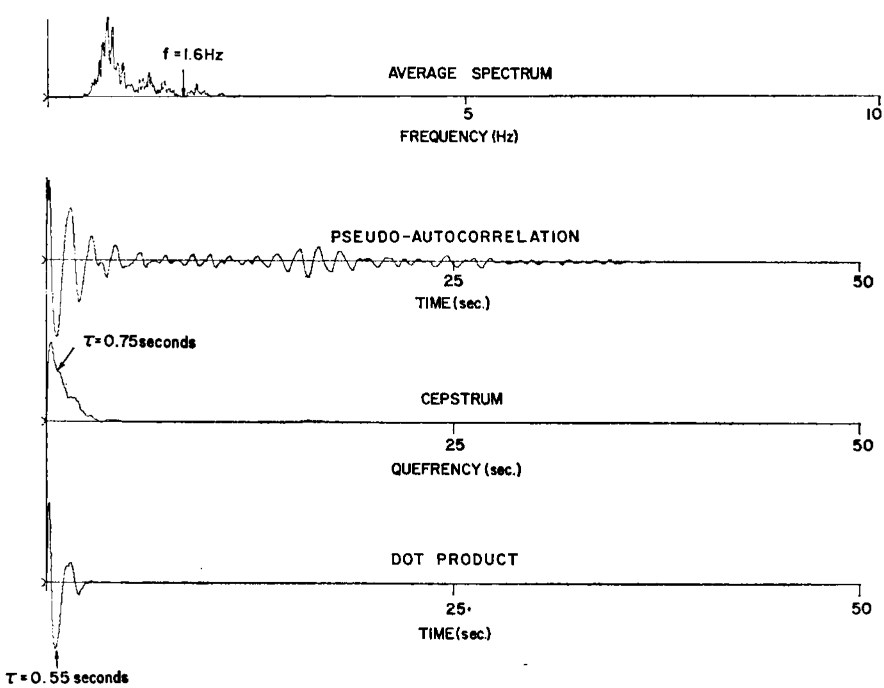

FIG 5. Analysis results, Greeley-*Pn*.

$\tau = 0{\cdot}55$ s. The average of these values—0·65 s—corresponds to a null frequency of 1·54 Hz. A null is observed at around 1·6 Hz, and we are inclined to adopt a delay-time value of 0·65 s. Using a previously assumed velocities of 3·5 and 8·0 km s^{-1}, the computed shot depth is 1·25 km, or 0·04 km deeper than the known shot depth.

Long Shot—P (*Fig.* 6)

Application of the analysis techniques to thirty-one Long Shot *P* records yielded a positive result. Though the pseudo-autocorrelation shows no multiple-arrival character for delay times less than one second, a positive peak in the cepstrum is clearly visible at $\tau = 0{\cdot}55$ s. The corresponding null frequency at $f = 1/0{\cdot}55 = 1{\cdot}82$ Hz is well defined in the average spectrum. For an average overburden velocity of 3·38 km s^{-1} (11100 ft s^{-1}; Day & Murrell 1967), the computed depth for $\tau = 0{\cdot}55$ s is 0·93 km, 0·23 km deeper than the actual shot depth.

It is somewhat disconcerting that a well-defined spectral null at 1·8 Hz is observed, while the depth determined from this null is deeper than the known depth by 0·24 km. One explanation for obtaining a deeper apparent depth for Long Shot is that non-linear effects may have distorted the overburden, lowering its velocity and causing a greater depth phase delay. Since such effects should be more pronounced for large shallow explosions, we might expect a greater error in the depth determination for such events, biased toward deeper depths.

5. Conclusions

We have shown that cepstral analysis of the type described here can successfully be used to determine depth of focus of seismic events. However, scalloping of spectra can also be caused by multi-path effects (including crustal reverberations) and multiple sources, as well as from depth-phase interference of the type *pP–P* (and

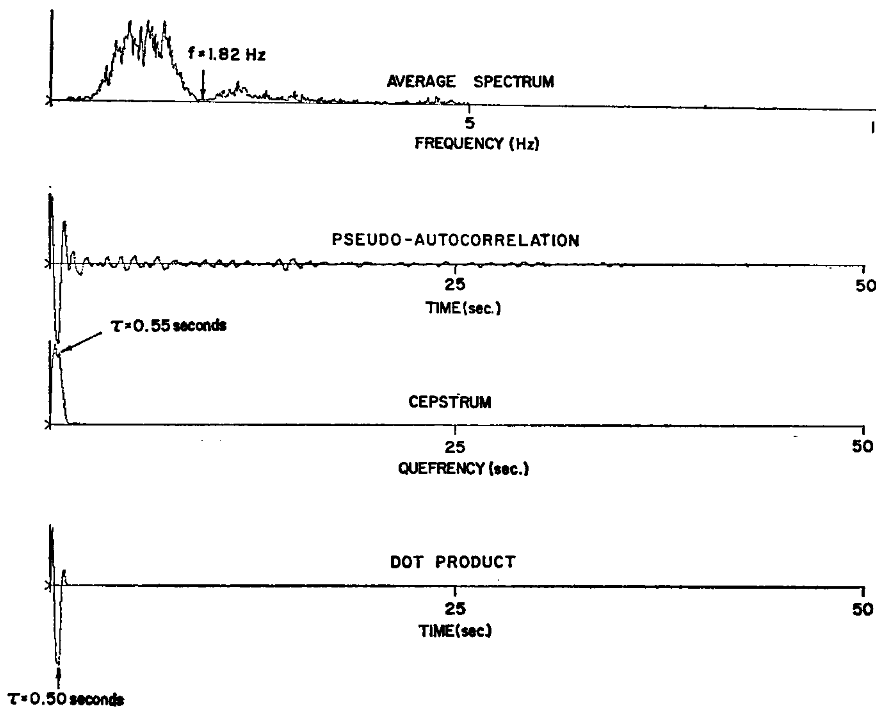

Fig 6. Analysis results, Long Shot–*P*.

pPn–Pn). Thus caution should be exercised in computing depths by the methods described here. To reduce reverberation noise introduced at the individual recording sites, analysis should be performed on a network-averaged spectrum. Further, the individual spectra from which the average spectrum is determined should have instrument response removed to enhance spectral components in the band 2–5 Hz.

An objective determination of depth-phase time delays can be computed using pseudo-autocorrelation and cepstral analysis with a network-averaged spectrum. Application of these described techniques to four data sets at *Pn* and *P* distances yielded the following depth determinations:

Event	*d* true (km)	*d* computed (km)	Error (km)
Gasbuggy (*Pn*)	1·29	1·46	+·15
Gasbuggy (*P*)	1·29	1·14	−·15
Greeley (*Pn*)	1·23	1·27	+·04
Long Shot (*P*)	0·70	0·93	+·23

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