Sovereign Default and the Choice of Maturity

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Abstract

This paper studies the term structure of interest rate spreads and the maturity composition of sovereign bonds. As observed in the literature, sovereign interest rate spreads increase during crises, with short term interest rate spreads rising more than long term spreads. The inversion of the yield curve is accompanied by lower debt issuance and a shortening of the maturity structure. In addition, sovereign debt restructurings may lead to a non-monotonic term structure of interest rate spreads, as evidenced during the recent sovereign debt crisis in Greece, when the yield curve developed a humped shape. To properly capture the observed variation of expected sovereign debt collection at different horizons and thus account for the dynamics in the maturity of debt issuances and its co-movement with the level of spreads across maturities found in the data, this paper introduces a new quantitative dynamic model of the term structure of interest rate spreads of government defaultable debt under incomplete markets.

1 Model

1.1 Preferences and Endowments

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. The country has endowment y_t each period. The endowment is stochastic and follows a finite-state Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition probability $\Pr\{y_{t+1} = y' \mid y_t = y\}$.

The country maximizes expected utility over consumption sequences. The discount factor is $\beta \in (0, 1)$. The momentary utility function is

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$

1.2 Market Structure

The country has the option to borrow in the international credit market. However, the country cannot commit to repay its obligations. If the country defaults, it is excluded from credit markets for a random length of time. When the country is excluded from credit market, it loses a fraction of its income $\phi(y)$.

There is only one type of bond that can be issued.[1] The bond pays a coupon $b \in B \subset \mathbb{R}_-$ every period for until maturity, N. Notice that at maturity the only payment is b, as in any other period; i.e. the face value that the bond pays at maturity is zero. Thus, the state variables will be (b, y, n), the current level of endowment, the size of the coupon payments promised, and the number of periods for which the coupon must be paid.

Lender as risk neutral.[2] The unit price of the bond is $q(b, y, n)$. Notice that (b, n) affect the price of the bond because they determine the characteristic of the bond, while y appears only because it helps predicting the probability of future default.

[1] This assumption is relaxed later, when bonds of every maturity from 1 to N can be issued.

[2] This assumption is relaxed later, when risk averse lenders are assumed.

1.3 Decision Problem

A country with asset b (debt if $b < 0$) has three actions to choose from: making the payment of b and nothing else (P), issuing new debt and keeping the maturity constant (R) and defaulting (D). Thus, the lifetime utility of a country is

$$V(b, y, n) = \max \left[V^P(b, y, n), V^R(b, y, n), V^D(y) \right]. \tag{2}$$

The policy function $I_P(b', y', n)$ is equal to one if the country chooses to make and payment and zero otherwise. Similarly, the policy function $I_R(b', y', n)$ is equal to one if the country chooses to issue new debt and zero otherwise. If the country chooses to default, lifetime utility can be represented by

$$V^D(y) = u(y - \phi(y)) + \beta E_{y'|y} \left[(1 - \lambda) V^D(y') + \lambda V(0, y', 0) \right] \tag{3}$$

where λ is the probability that the country gains access to credit markets in the next period.

In the case the country chooses to make the payment and doesn't issue new debt, lifetime utility can be represented by

$$V^P(b, y, n) = u(y + b) + \beta E_{y'|y} V(b, y', n - 1). \tag{4}$$

Notice that in this case the maturity of the debt decreases by exactly 1 year.

Finally, in the case the households issue new debt with maturity N, the lifetime utility can be represented by

$$V^R(b, y, n) = \max_{b' \in B} U(y + b + q(b', y', n - 1)b - q(b', y, N)b') + \beta E_{y'|y} V(b', y', n') \tag{5}$$

1.4 Equilibrium

Given the world interest rate r, the price of the country's debt must be consistent with zero expected discount profits. This implies that

$$q(b', y, n) = \frac{q^P(b', y, n) + q^R(b', y, n)}{1 + r} \tag{6}$$

where,

$$q^P(b', y, n) = E_{y'|y} I_P(b', y', n)(1 + q(b', y', n-1))$$

$$q^R(b', y, n) = E_{y'|y} I_R(b', y', n)(1 + q(B(b', y', n), y', n-1))$$

2 Calibration

The only shock in the model is the one of endowment. In particular, $y_t = Ae^{z_t}$, where A is a scaling parameter and z_t follows an AR(1) process:

$$log(z_t) = \rho log(z_{t-1}) + \epsilon_t,\ \epsilon_t \sim N(0, \sigma)$$

The parameters for the income process and also the other parameters are taken from Arellano and Ramanarayan (2012). They are summarized below.

Table 1. Parameters.

Risk aversion, γ	2
Interest rate, r	0.032
Discount factor, β	0.935
Probability of redemption, λ	0.17
Loss of output, ϕ	0.045
Output scale, A	1
Transitory shock std, σ	0.017
Transitory shock autocorrelation, ρ	0.9

3 Quantitative results

Table 2. Main statistics.

	Maximum maturity		
	1	3	5
Duration	1.00	1.90	2.68
$\sigma(\log(y))$	1.87	1.96	2.03
$\sigma(\log(c))$	3.14	3.32	3.25
$\sigma(TB/y)$	1.62	1.72	1.54
$\sigma(R_s)$	0.62	1.02	2.09
$E(R_s)$	0.68	0.69	1.42
Debt/GDP	0.09	0.09	0.09
$\rho(duration, \log(y))$		0.52	0.64
$\rho(\log(c), \log(y))$	0.91	0.91	0.93
$\rho(TB/y, \log(y))$	-0.47	-0.59	-0.63
$\rho(R_s, \log(y))$	-0.54	-0.58	-0.65
$\rho(TB/y, R_s)$	0.70	0.66	0.59
Default (per 100 years)	0.54	0.58	0.92

Table 3. Yields to maturities and debt maturity structure

	Maximum maturity		
	1	3	5
Bad times			
Share of debt that is 1-year debt	100.00	39.94	28.34
Median yield to maturity, zero coupon bonds			
1-year maturity	8.40	3.26	3.21
2-year maturity		6.38	28.34
3-year maturity		18.92	3.21
4-year maturity			4.64
5-year maturity			10.69
Good times			
Share of debt that is 1-year debt	100.00	37.65	25.31
Median yield to maturity, zero coupon bonds			
1-year maturity	3.43	3.20	3.20
2-year maturity		3.21	3.20
3-year maturity		3.69	3.22
4-year maturity			3.49
5-year maturity			4.35

Note: Bad times are the episodes between 1 to 2 periods before a default. Good times are the ones at least 20 periods before a default andwithout exclusion from the financial markets.

Appendix: Notes on variable definitions

Spread

For a bond with maturity n, the implied interest rate is r_q that satisfies:

$$q(b,y,n) = \sum_{s=1}^{n} \left(\frac{1}{1 + r_q(b,y,n)} \right)^s = \frac{1 - \left(\frac{1}{1+r_q(b,y,n)} \right)^{n+1}}{1 - \frac{1}{1+r_q(b,y,n)}}$$

Then the annual spread is given by:

$$R_s(b,y,n) \equiv r_q(b,y,n) - r.$$

Duration

For the duration of a bond, we use the Macaulay definition (as in Hatchondo and Martinez, 2008) which is a weighted sum of future coupon payments. Ignoring the indexes for states (b,y,n):

$$D \equiv \frac{1}{q} \sum_{s=1}^{n} s \left(\frac{1}{1 + r_q} \right)^s = \frac{1}{q} \frac{1}{1 + r_q} \frac{1 - (n+1) \left(\frac{1}{1+r_q} \right)^n + n \left(\frac{1}{1+r_q} \right)^{n+1}}{\left(1 - \frac{1}{1+r_q} \right)^2}$$

Yield to maturity

$YTM(b,y,s)$ solves:

$$q(b,y,n) = \sum_{s=1}^{n} \left(\frac{1}{1 + YTM(b,y,s)} \right)^s$$