

Space, and the propagation of Light.

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Stephen H. Jarvis.

<http://orcid.org/0000-0003-3869-7694> (ORCID)

EQUUS AEROSPACE PTY LTD

Web: www.equusspace.com

email: shj@equusspace.com

Abstract: In this paper, theoretical uncertainties fuelled by ad-hoc fixes of theory to explain observed astrophysical anomalies using models upholding the idea of space being a field of potential inertia for bodies in relative motion are discussed. A few of the basic discussion points include determining the difference between scientific space and mathematical space, the cosmic expansion of space, the ad-hoc theoretical localised effect of gravity in galaxies, and the need to use ideas such as dark matter and dark energy. This paper takes the current uncertainties and ad-hoc theoretical fixes in astrophysics theory and places those uncertainties and ad-hoc theoretical fixes together with the aim of determining the root cause of all the general anomalies, here in this paper identified as the idea of labelling space with “inertia”. Then, a general idea of space, more universal than contemporary theories hold, is presented, and demonstrated to uphold all observed astrophysical data. The general “fix” proposed here is a new theory for space, joining both the contemporary scientific and mathematical notions of space in the one construct without using the idea of “inertia”. To achieve this task, a general review of astrophysical data is presented, and each theoretical step used in history to explain that observed astrophysical data, while along the process here making mention both the uncertainty and the ad-hoc theoretical fix being used. all leading to “one” fundamental key oversight in physics theory, prompting the general fix to the standard model. Finally, a new landscape of the universe as a model is presented bearing reference to this new appraisal of space, further shedding light on the previous series of papers [1-10].

Keywords: astrophysics; Einstein; special relativity; general relativity; time; space; inertia; electromagnetism; gravity; non-inertia; golden ratio; quantum field theory; quantum electrodynamics; Minkowski space; Hilbert space; dark energy; dark matter; redshift effect; Euclidean space; cosmic inflation; Hubble’s law; big bang; doppler effect; Cantor; Dirac

1. INTRODUCTION

Einstein's Special and General relativity theory of gravitation, the general backdrop of astrophysical theory, takes root from the fundamental notion of space labelled with the feature of "inertia" [11]. The paper here shall present that although relativity theory works very well in explaining observed phenomena according to their basics of inertial frames of reference of particles relative to the idea of light, it still fails to deliver a number of key ideas, namely the linking of gravity with electromagnetism, and the explanation of the expansion of space locally in galaxies and less locally in other space considered to be expanding at a superluminal rate, requiring the addition of the ad-hoc ideas of dark matter and dark energy respectively. The problem though with replacing relativity theory is that it opens up so many new possibilities that would need to at least carry most if not all the basic ideas relativity theory has explored and successfully proved with observed data, together with all other off-shoot theories and models from relativity theory. Thus, to address the limitations of relativity theory, *it is necessary to look for a key flaw in relativity theory, and how that can be replaced, and what "other" replacements are required with that solution.* It would be fair therefore to propose that if a key flaw in relativity theory can be found, a broad-spectrum concept that is able to replace those theories that are associated to that model's key flaw with a new more correct version of observed data needs to be proposed. The suggestion previously in this series of papers [1-10] is that using "inertia" for the idea of space will not bring into effect a result of gravity linking with EM, for the reasons presented in the paper "The Physics Chimera" [9]. Although that paper addressed a key flaw in the theory of space as inertia, that paper did not explain "how" that problem came about in physics, how space became known to be considered as a feature of "inertia", how that idea itself seemed to merely be assumed. Here in this paper that assumption shall be fully addressed and rectified.

This paper will be divided into 8 key sections:

1. Introduction
2. Space (general ideas)
3. Space (mathematical theory)
4. Space (general observed phenomena)
5. Space (mathematically explaining general observed phenomena)
6. Space (inertia: The Physics Chimera)
7. The propagation of light in space
8. Conclusion.

The aim here is to provide a solution to contemporary problems in relativity theory, with the emphasis on providing a new definition for the idea of space in regard to time, not though in using the idea of "inertia" with space, yet space as a new creature of definition. All known data of the universe, observable data, will not change, only the theories trying to explain that data. Subsequently, the idea of relativistic mass sprung from inertial space will be in dispute, together with the idea of gravity being a curvature of "spacetime", as the idea of "spacetime" will undertake a specific revision giving genesis to the idea of gravity as a new associated feature of that new definition for space and time. Consequently, a new model for the universe will become self-evident, a model devoid of the need for dark energy and dark matter, devoid of the need thus also for those adjunct theories of general and special relativity respectively, and relativistic mass. As it shall be highlighted, what Einstein presented was perfectly natural and absolutely necessary for his time, it ticked all the then required boxes, yet a thorough review was required given the number of unsubstantiated fixes being used to uphold the theory of relativity, especially in light of quantum field theory and those ideas of light and energy.

2. SPACE (GENERAL IDEAS)

“Space” is generally defined as an infinite three-dimensional realm in which objects and events have relative position and direction of motion in time, considered as three linear dimensions with time as part of a boundless four-dimensional continuum known as spacetime. The idea of “space” goes back to the ancients, to the likes of Plato, Socrates, and Aristotle, who all took a philosophical view of space as a feature of the greater cosmos and that geometry and mathematics of the time. Their ideas were further developed in the Renaissance period most notably by Isaac Newton who presented space in being absolute with the mind; his general philosophy of space took root from Galilean and Cartesian theories, primarily from Rene Descartes who aligned the idea of space with the mind in establishing a metaphysical/mental foundation for a mechanical explanation for objects in space, space described as an infinite and uniform construct. Other natural philosophers, notably Gottfried Leibniz, considered that space, as opposed to being entire of itself, was a collection of relations between objects given by their distance and direction from one another. Immanuel Kant on the other hand considered that the concepts of space and time are elements of an already given systematic framework that humans execute to structure all experiences. Kant referred to the phenomenon of “space” in his *Critique of Pure Reason* as being a subjective experience, as a “pure a priori form of intuition” and not existing as an objective reality per-se. To achieve this notion, he considered space to be both an *a priori* concept and artificial/manufactured, suggesting a far deeper intuitive meaning to space itself. He concluded that space and time cannot be objective features of the world via human understanding, yet features imposed by us as part of a phenomenal experience of consciousness, thus bringing the subjective experience of reality to the forefront. Henri Poincaré in the late 19th century argued that geometry describing space was a matter of convention, preferring the “simplest” explanation for something, as simple as what space should be, and thus Euclidean geometry.

All in all, the idea of space had two key notions, the first being the idea of its geometry, primarily Euclidean, and the second being the idea itself of how it can be “observed” by an observer, and those theoretical notions of calculating space. Those two issues represent the very ark itself of physics theory regarding space and the mathematics thereof, namely on the one hand space would have a geometry, yet on the other space has a component relating to one’s observational reference. Einstein brought both such ideas together in his theories of special and general relativity; in achieving this task, he proposed that light travelled through space at a constant speed, and that space and time can be viewed as a single construct known as *spacetime* to accommodate for what he considered to be the required fluidity of space, a space being measured, and the observer of that space in relation to light and thus time (except that light travels at a fixed speed yet time itself can be observed to slow down). In his theory, he considered that the speed of light in a vacuum is the same for all observers, or in other words our ability of observation ticks at the same speed of light, which has the result that two events that appear simultaneous to one particular observer will not be simultaneous to another observer if the observers are moving with respect to one another through space, changing their spatial dimensions to one another. Further to this, Einstein’s theory holds that any observer will measure a moving clock to tick more slowly than one that is stationary with respect to their own reference of consciousness and assumedly time. Furthermore, Einstein’s theory holds that objects are measured to be shortened in the direction that they are moving with respect to the observer, and that objects moving close to the speed of light become more massive and thus follow a curved trajectory in assuming a more massive status in space.

Subsequently, Einstein worked on his general theory of relativity, detailing how gravity is proposed to interact with spacetime, inviting gravity to the space-time, space-observer, symposium with the assumption of inertia being related to space and this a curvature of that manifold being as gravity; Einstein, instead of viewing gravity as a force field acting *in* spacetime as though a separate entity to spacetime, suggested that gravity “alters” the geometric structure of spacetime itself in the manner of a “curvature”, as though being a part of the fabric of

spacetime itself, altering the idea of Euclidean geometry of space to “Non-Euclidean” (non-straight lines). In fact, Einstein considered that bodies in motion relative to each other conjure the idea of gravity as a curvature of space to meet their required “relativity” principles of motion. Subsequent to these theoretical musings, in Einstein’s second paper subsequent to his special theory of reality, in his general theory of relativity, time is considered to tick more slowly at places with lower gravitational potentials (or rather, time ticks more slowly in more massive regions of space and thus gravity), and that rays of light bend in the presence of a gravitational field more greatly in those lower gravitational potentials (more massive regions). Essentially, Einstein proposed that space along with time as spacetime is directly related to the idea of mass and thus gravity to the effect that spacetime would be curved in regions that are more massive, regions that are also related to the effect of time “slowing down”.

On the one hand, there was the issue of space being Euclidean as geometrical space, and on the other was the principle of mind as the reference of the observer as a concept, assumedly, of time itself. Einstein joined these two concepts as his idea of spacetime, yet to do this the idea of “relativity” between bodies in motion as one observer to the next in the context of light being the “fixed” standard of domain, and thus the consciousness reference of an observer also being fixed (despite the observer being able to view other events differently as a warning of spacetime and thus light) had to be the standard reference. To explain this mathematically, Einstein in taking a step ahead of Newton’s ideas of force considered mass-based frames of reference as “inertial” frames of reference in space, thus by proxy giving space a type of “inertial” hue itself, and thus space became the platform for “inertial” frames of reference for objects in space, all of which things needed a strong mathematical basis. It was almost saying “a person as a reference of time in a spatial reality equals “mass” as “gravity” which can be explained using equations of “inertia” in space, and the closer that person travels in space to the speed of light the more curved their path of trajectory and the more massive they become, and of course the more warped their reference becomes to an observer. Clearly this was not the impetus behind Einstein’s idea of relativity, to create such a portrait of reality, yet the result itself of a more fundamental issue of relativity he was addressing regarding the observed nature of the universe. Simply the theory of relativity in tagging space with inertia provided the notion “that for bodies in relative motion in a spacetime where “c” is a constant, any particular body that approaches “c” must bend with spacetime bending”, and since space is tagged to the idea of inertia, thus bending must be “gravity”, he considered, and thus any object approaching this speed must become more “massive” (which isn’t really that useful for our normal reality here on Earth). Why therefore with such a portrait of mechanics would anyone go out of their way to present such a theory with such a strange result for human bodies in motion at such ridiculous speeds for humans in mind?

The question then beckons, “what was *that more important underlying issue of the universe* Einstein aimed to solve with his ideas of relativity?” In addressing that question, the idea of space as a concept of “mathematics” needs to be reviewed, as on general inspection of Einstein’s theory, there are a number of key flaws in the context of what that theory wants to achieve and what it actually can or cannot achieve, the case in point being both features of any set of relative bodies in different relative motion would “appear” distorted, in that light would be distorted regarding those two objects, according to the theory, yet not just “perceived” from each reference would there be a distortion (and thus of light) yet also “at” each reference would there be a distortion if indeed their relative motions produce a warping in spacetime, and thus presumably a distortion regarding how light would curve with that curved spacetime at each of those objects in relative motion. And this is why the theory of relativity fails, because in assuming light is altered “with” the curvature of spacetime and thus gravity, then where is the mathematical theoretical link between light (EM) and gravity (G)? There is none. Why is there none?

3. SPACE (MATHEMATICAL THEORY)

In modern mathematics “spaces” are defined as “sets” with the idea of structure, often described as different types of manifolds as spaces that *locally* approximate to Euclidean space, their properties defined largely on local connectedness of points that lie on the manifold, like fuzzy Euclidean geometry joined here there and everywhere by the care of the mathematical regard between those points in space. Basically therefore, any nominated space consists of arbitrarily selected mathematical objects that are treated as points, and so also consist of selected relationships between those points. The nature of the points can vary widely: for example, points can be elements of a set, functions on another space, or subspaces of another space. It is the mathematical relationship between those points, how that is defined, that defines the nature of the space. It is quite arbitrary, if not ad-hoc, depending on the use required, if whether a given mathematical object should be considered as a geometric “space”, or an algebraic “structure”. The great paradox that Einstein presented regarding space is that although on the one hand space is considered as one of the few fundamental quantities in physics, such that it cannot be defined via other quantities because nothing more fundamental is known at the present, yet on the other hand Einstein related space (as inertia) directly to idea of time (and thus the reference of the observer in time), and thence by that union, joined spacetime to gravity *via labelling space as an “inertial” platform to judge “inertial frames of reference” of objects, and thence “non-inertial” frames of reference to explain accelerating expanding space.* Why was Einstein using inertia tagged with space and light tagged with consciousness and time in the first place?

Unavoidably, to put this theory of Einstein’s to the test, space had to be explored via measurement of observed astronomical data, via the *mathematics* of space. To achieve this task of measurement certain mathematical tools and associated theories of time and space had to be implemented; three-dimensional space became viewed as embedded in a four-dimensional spacetime known as Minkowski space [12] which is a combination of three-dimensional Euclidean space and time into a four-dimensional manifold where the spacetime interval between any two events is independent of the inertial frame of reference in which they are recorded, to simply allow those inertial frames of reference to be each recorded. Although initially developed by mathematician Hermann Minkowski for Maxwell’s equations of electromagnetism, the mathematical structure of Minkowski spacetime was shown to be an immediate consequence of the postulates of special relativity, and thus was adopted for special relativity theory. Consequently, experimental tests of general relativity have confirmed that *non-Euclidean* geometries provide a better model for the shape of space on a gander astrophysical scale, requiring Einstein to hatch his “general” theory of relativity.

Einstein’s theories aimed to show that due to the relativity of motion between objects, our space and time can be mathematically combined into the one object, as spacetime, such that distances in space or in time separately are not the same (invariant) with respect to one type of mathematical code of coordinates (Lorentz coordinate transformations), yet distances in another code of coordinates (Minkowski space-time) along space-time intervals are the same (invariant), despite time and space dimensions not being viewed as exactly equivalent in such Minkowski space-time. Subsequently, using these different mathematical approaches, time and space coordinates are treated differently both in special relativity (where time is sometimes considered an imaginary coordinate) and in general relativity (where different signs are assigned to time and space components of spacetime metric). The whole process here is to adopt a mathematical tool to explain the nature of objects in space moving relative to each other, relative motion that is different between objects in “special” relativity compared to “general” relativity, requiring a different set of mathematical tools. Essentially, general relativity aims to explain expanding space per-se without bodies in motion, and thus a “non-inertial”, non-mass, explanation for space. And so, is space different for special relativity compared to general relativity? As relativity theory considers, the varying

nature of those objects “in” space upon the nature of space itself is the issue, and in the case here in our universe the issue of space is its *expanding* trait owing to our observation of the “redshift” effect. Special relativity holds that galaxies would fling apart, yet they don’t, with this redshift effect, so general relativity is required. The problem with relativity theory therefore is that it appears to be addressing the “problem” of the redshift effect which has been primarily put down to the idea of expanding space, requiring theories to explain that expanding space, namely relativity theory and the associated primary idea of a great beginning to the expansion, the “big bang”.

In short, compared to special relativity which adopts Minkowski space modeling, general relativity adopts/adopted a different mathematical tool to explain the nature of its cosmic phenomena, as per “Hilbert space” [13], a mathematical theory for space theory that generalizes the notion of Euclidean space [14], typically as infinite-dimensional function spaces; Hilbert space aims to bring a technical type of order in the space of the potential solutions (set of functions or spin vectors) of quantum mechanical problems, together with the explanation for the idea of expanding infinite space. The issue exists though that If the “speed” of light is a constant value, as Einstein’s relativity holds, and this is assumed with time as with the observer reference, the idea of the redshift suggests that light is being “stretched” by expanding space, yet still must move at “c” according to the observer and time, despite it being “stretched”. This suggests that the speed of light should be immune to the expansion of space, yet logically if space is moving faster than light away from an observer, and there is a star in that space advancing away from an observer at superluminal speed, then an observer shouldn’t see those points of light from that superluminal space moving away from it, which is the proposal in current astrophysical theories regarding the accelerating expansion of the universe. However, subluminal expanding space has light that is observed to be stretched in wavelength in alliance with the expansion of space despite it still moving at “c” in that space according to the observer. To note therefore, the metric of light as distance per time, as velocity, is unchanged in expanding space; the wavelength change suggestion as per the “redshift” (increasing wavelength) is that light is a just as fundamental creature as space, yet more so as ‘c’, in that although space may expand at a certain rate, the light within it remains at a constant velocity “c” as we would perceive it, and thus in the case of expanding space we would perceive a stretching of the wavelength and thus a redshift effect.

All of this leads to the question of the type of shape of the universe, and what space is, and of course where it came from if it presumably is ever-expanding. The current theory holds that space was created in the Big Bang [15] 13.8 billion years ago and has been expanding ever since. With that model, the overall shape of space is not known, yet space nonetheless is considered to be expanding very rapidly due to the cosmic inflation indicated by the redshift effect [16] of stars and that rate of redshift observed regarding the apparent distances between stars. As may be now apparent, there was an elephant in the room in the world of physics inspiring theorists to move ahead with their theories in the way they did, and that idea was the redshift effect of light in space, a phenomena that cannot be under-estimated as that vapor of thought guiding all types of theories to have the universe of stars and associated redshift of light from the stars explained.

4. SPACE (GENERAL OBSERVED PHENOMENA)

In physics, the redshift effect is a phenomenon of light such that electromagnetic radiation (as light) from an object undergoes an increase in wavelength, entering classically into the “red” spectrum of light; the “redshift” phenomena most fundamentally is an increase in wavelength, equivalent to a decrease in wave frequency and photon energy, in accordance with the wave and quantum theories of light. Examples of redshift are a gamma ray perceived as an X-ray, or initially visible light perceived as radio waves. The history of the idea of the “redshift effect” began with the development in the 19th century of wave mechanics and the exploration of phenomena

associated with the Doppler effect [17]; Doppler correctly predicted that the phenomenon of sound waves regarding the modification of wavelength through different relative motions of bodies to one another should apply to all waves, and thus to light, in particular the varying colours of stars. Before this phenomenon was tested with starlight, stellar colours were considered due to a star's temperature, not motion, so the revelation was exactly that.

The first Doppler redshift was described by French physicist Hippolyte Fizeau in 1848, who pointed to the shift in spectral lines seen in stars as being due to the Doppler effect. In 1868, British astronomer William Huggins was the first to determine the velocity of a star moving away from the Earth by applying the Doppler method of redshift analysis. In 1871, optical redshift was confirmed when the Doppler redshift phenomenon was observed in Fraunhofer lines using solar rotation. Shortly after, in 1887, Vogel and Scheiner discovered the *annual Doppler effect*, the yearly change in the Doppler shift of stars located near the ecliptic due to the orbital velocity of the Earth. Finally, in 1901, Aristarkh Belopolsky verified optical redshift in the laboratory using a system of rotating mirrors, all such events preceding the theory of relativity, yet all such discoveries creating a canvass for a theory of time and space regarding light to properly explain.

Vesto Slipher in 1912 discovered that most spiral galaxies, then thought to be spiral nebulae, had considerable redshifts. In subsequent years, Slipher reported the velocities for 15 spiral nebulae spread across the entire celestial sphere, all but three having observable "positive" (that is recessional) velocities. Subsequently, Edwin Hubble discovered an approximate relationship between the redshifts of such "nebulae" and the distances to them with the formulation of his eponymous Hubble's law. These observations corroborated Alexander Friedmann's 1922 work, in which he derived the Friedmann-Lemaître equations. They are today considered strong evidence for an expanding universe and the Big Bang theory, if not "irrefutable" as some would consider.

In all, there are three main causes of redshifts in astronomy and cosmology:

1. Objects are moving away from each other in space, as a basic example of the Doppler effect.
2. Space itself is expanding, causing objects to become separated without changing their positions in space, as an example of the cosmological redshift; all sufficiently distant light sources (generally more than a few million light years away) show redshift that is considered to correspond to the rate of increase in their distance from Earth, known as Hubble's Law.
3. Gravitational redshift which is a relativistic effect observed due to strong gravitational fields, which distorts spacetime, curling it, thus exerting a force on light leading to a redshift effect.

Simply, the gold rush in science was to explain the nature of the redshift effect of space and to address the question "was space expanding"? To answer that, the issue lay with the inertia involved in stars flying apart and that possibility, attributed perhaps to a feature itself of space, and why.

5. SPACE (MATHEMATICALLY EXPLAINING GENERAL OBSERVED PHENOMENA)

In the early part of the twentieth century, Slipher, Hubble and others made the first measurements of the redshifts and blueshifts of galaxies beyond the Milky Way. They initially interpreted these redshifts and blueshifts as due to random motions, but later Hubble discovered a rough correlation between the increasing redshifts and the increasing distance of galaxies. Theorists almost immediately realized that these observations could be explained by a mechanism for producing redshifts seen in certain cosmological solutions to Einstein's equations of general relativity, which lends one to consider if Einstein himself had not pre-designed his theory to accommodate for those observations of the stars known decades previously.

What needed to be determined was the source of the redshift effect, to distinguish between a cosmological redshift as compared to that witnessed when nearby objects exhibit a local Doppler-effect redshift. Basically, Rather than cosmological redshifts being a consequence of the relative velocities that are subject to the laws of special relativity (and thus subject to the rule that no two locally separated objects can have relative velocities with respect to each other faster than the speed of light), the photons instead increase in wavelength and redshift because of a global feature of the spacetime metric through which they are traveling. To make such a distinction, it can only be considered that space itself is expanding. The problem here is that due to the expansion increasing as distances increase, the distance between two remote galaxies can increase at more than $3 \cdot 10^8 \text{ m.s}^{-1}$, which though cannot imply that the galaxies move faster than the speed of light at their present location, as is forbidden by the idea of Lorentz covariance which is essential to the mathematics of special relativity. Ultimately, the cosmological effect is the theory to explain the phenomena of expanding space per-se, yet the local effects in galaxies are considered to be determined by the effect of gravity bunching stars together creating its own localised redshift phenomena. Subsequently therefore it was determined that it is the measurements of the stars further out and the associated progression of the redshift that suggests that space is expanding at an accelerating rate, whereas gravitational redshift itself is associated to galaxies where stars bunch together, which thus requires a different set of mathematics, all of which has required an exhaustive use of mathematics to describe the relative positions of inertial (special) and non-inertial (general) locations in spaces to better explain the sought for properties of space and associated energy structures, especially regarding the proposed nature of gravity with spacetime in Einstein's general relativity theory. The overlap between special and general relativity is such that while special relativity prohibits objects from moving faster than light with respect to a local reference frame where spacetime can be treated as flat and unchanging, it does not apply to situations where spacetime curvature or evolution in time become prominent, as here the model of space is best described by general relativity, which allows the separation between two distant objects to increase faster than the speed of light owing to the curved nature of space-time in play, although the definition of "separation" is different from that used in an inertial frame in standard special relativity.

A number of concurrent theories thus have been introduced to bridge the gap between the space of special and general relativity and that different mathematics being employed for each. For instance, physicists have postulated the existence of dark energy [18], appearing as a cosmological constant to explain the acceleration of expanding space, as that source of energy to allow such a thing of space to occur, namely inertial and non-inertial space to expand so greatly with such energy requirements, an energy level $\sim 10^{120}$ times greater than what is currently observed. Another "fix" is the need to use "dark matter" [19], a form of matter thought to account for approximately 85% of the matter in the universe and about a quarter of its total energy density, generally the mass that is required to keep spiral galaxies together in the context of an accelerating expanding universe. In short, their absence, their "darkness", are two of the biggest problems in physics. Despite these two key problems the scientific consensus is that *space itself is expanding*, and that it expanded very rapidly within the first fraction of a second after the proposed big bang. Theory and observations suggest that sometime very early in the history of the universe there was an inflationary phase where the metric of space and associated inertial qualities changed very rapidly, and that the remaining time-dependence of this metric is what we observe as the so-called Hubble expansion [20], *the moving apart of all gravitationally unbound objects in the universe*. The expanding universe is therefore a proposed fundamental feature of the universe, a step beyond Einstein's initial work, Einstein who had not prepared his theory for *the accelerating superluminal space model*. The ideas of Einstein nonetheless work well in the context of the idea of "galaxies" in an every expanding spatial universe being held together by the idea of gravity (and of course need for "dark matter") and associated slowing down of time (time dilation, in a gravitational "well"), as much as the need to find an extra 85% of mass in those galaxies as a gravitational "well" to keep them together, an 85% that is unaccounted for and thus termed dark-matter. When compared with the accelerating

expanding space which seems to have a massive amount of energy unaccounted for as dark-energy, the two systems together required such broad-scale fixes it seems almost impossible to consider them seriously. Key to the problem here is space being labelled with inertia. Yet on top of this, other problems include assuming the passage of time with light as held as an observation reference, despite this observation reference bending at high speeds as light in presumably a gravity field, and despite that light field theory (EM) still to this day not linking with a gravity (G) field theory.

As the previous paper “The Conception of Time” [10] aimed to highlight, it is not sufficient to merely assume time and light and the observer reference are as one; in that paper, it was theorized that there is a complex mathematical algorithm involved in such an association. Despite that finding, it is quite amazing that the model of relativity is still being used the way it is being used, and space being investigated through such a lens of consideration, given the dependence on unseen/dark factors (dark energy and dark matter) that outweigh what exists to our perception. There is one thing though that should be primarily examined, and that is the idea of associating space to the idea of inertia, following which the assumption of light with time and the reference of the observer, as all such assumptions in relativity theory had led to the need to employ ideas such as dark energy and dark matter together with failing to explain the link between gravity (G), presumably a curvature of spacetime, and light (EM), light which presumably as a constant is associated to the fabric of spacetime, space, and thus inertial frames of reference, held as a “link” according to the theory of relativity, of spacetime, yet fail to deliver an actual mathematical link between gravity and light, and thus upholding gravity theory and quantum mechanics as an impossible association (as observed data and calculations would hold).

6. SPACE (INERTIA: THE PHYSICS CHIMERA)

The sole problem for the idea of the required amount of energy for the observed expansion of space care of the redshift effect is the associating of space to the idea of inertia, as an expansion of inertia regarding space is the big question, the “**huge**” problem, requiring an enormous amount of energy, a factor of $\sim 10^{120}$ compared to what exists. A previous paper in this series of papers highlighted the trouble with using inertia as a reference per-se in considering the fundamentals of time and space, and not just considering space as associated to inertia. Here in this paper the exposition and inquisition are focused on what happens when space is regarded as that manifold of inertia for *particles in relative motion to each other*. In combination with the redshift effect there are two problems that need massive fixes, the first being the accelerating expansion of space incurring a massive amount of energy (and from where, hence the idea of dark energy), and how indeed the galaxies can keep themselves together in the context of this cosmic expansion of space above and beyond the general relativity fix of special relativity (hence the need for the idea of dark matter to account for at least 85% of the mass of the universe that would ideally be required to keep galaxies together). Both issues take root in using the idea of “inertia” with space. So why was it done in such a fashion and why was space defined in such a way? The simple answer is, “there was no other choice”.

At the time when inertia was labelled to space, of space harboring the idea of inertia, the idea of Minkowski space and tensors for Euclidean space was adopted in special relativity, and the use of Hilbert space for non-inertial (stretching) space in general relativity, both still requiring the fixes of dark matter and dark energy respectively. The great problem here is using the backdrop of space as the idea of inertia, to then investigate bodies apparently in natural motion with one another, the only exception of course being gravity which would incur an acceleration between objects in this inertial space which Einstein neatly described as being a “curvature” of spacetime, or rather a “curvature” of the spacetime grid, of the inertial grid. In realizing the limitation of the “inertial grid” for space, especially with the idea of the redshift effect and how galaxies bunch together, Einstein proposed,

via the mathematics employed, in his general theory of relativity, that there can be no preferred inertial status to any class of coordinate systems, thus allowing for “expansive” space for instance that does not involve gravitational frames of reference (and thus mass). The fundamental problem with expanding space and thus expanding inertia is “where does the energy come from”, that ridiculously huge amount of energy, to keep the expansion warranted, and what stops galaxies from flying apart, where’s that invisible mass? Simply, inertia, as its employment highlights, suggests that space has a type of “viscosity” compared to the field forces, a type of “inertia” as its name suggests, compared to for instance “gravity” which is a curvature of spacetime, a “curvature” of inertia.

In defense though of Einstein, what Einstein presented was perfectly natural and necessary for his time, it ticked all the then required boxes of what was available in theory at that time (QFT was not then available for light). The suggestion in this paper is that there is only one way forward in not employing dark energy and dark matter, and that is by abandoning the idea of inertial frames of reference for particles in motion, and thus not associating inertia with space, as was proposed in the 9th paper “The Physics Chimera” [9] in this series of papers [1-10], yet here explained in the manner of the spatial manifold itself, and what must become of it as a mathematical construct in abandoning its previously held inertial notions. Yet in not employing inertial frames of reference for objects in space, all the equations of objects in relative motion, all those mathematical algorithms, are no longer needed, a new mathematics is needed, and furthermore if not ultimately, a new idea of “relativity” is required. This was presented in papers 9 “The Physics Chimera” [9] and 10 “The conception of time” [10]. Paper 9 “The Physics Chimera” [9] addressed the problem of inertia with space and all those follow-through mathematical and associated theoretical consequences for the idea of time and light, and paper 10 “The Conception of Time” [10] addressed the idea of the reference of the observer and how this can be related to the idea of “time”, and then applied to space. Why discuss perception and time? As the theories of Einstein would highlight and those associated assumptions of theory, there is only so much we “can” know, so much our thinking can handle, depending on the a-priori of our theories. Besides, the question of “what makes us think we have a thought-process complete enough to piece everything together?” cannot be ignored, as it needs to be addressed considering the theoretical inadequacies of relativity theory itself. To address this, an examination of our “real” ability to be conscious in the regard of “time” needed examining, hence paper 10 “The Conception of Time” [10]. The assumption was at the time of Einstein that physics theory was sufficiently provided with the required theoretical tools to explain time and space, which has turned out not to be the case. How can that assumption be exercised again today without examining the nature of the facility of human consciousness when related to the concept of time itself? Basically, the current “front” for theoretical physics is to determine the relativistic equations for light as it bends with gravitational relativistic effects, yet there is no solution of EM (light) and gravity (G) field forces being linked via the inertial spatial platform. Thus, quite simply, a new approach is required, otherwise theoretical physics is not seeing the forest for the trees, not admitting the inherent failure of using space as a platform of inertial reference.

7. THE PROPAGATION OF LIGHT IN SPACE

The task seems impossible, namely voiding the idea of inertia from space. What will become of relativity theory? What will become of any field theory making use of the idea of space as “inertia”? Needing to depend on finding an extraordinarily huge amount of dark energy to the order of $\sim 10^{120}$, and dark matter unaccounted for by a factor of 85% of the mass of the universe, should be motivation enough to seek a new basis for space theory. Given the nature of the task, it may make sense to start from the top down with theoretical physics’ greatest issues of theory, and here would be to firstly address the idea itself of infinitely expanding space, and what mathematics is being used there. Then secondly, to explain the nature of the redshift effect of light, what is really happening there and why in terms of light passing through space given contemporary ideas of quantum physics, and then thirdly, if

superluminal expanding-space is false, and the redshift effect can be explained in a new way regarding space, what is the “resultant” nature of space, and thus finally what is the observable stand-alone evidence for this phenomena of space which would have previously, as evidence, been taken into the care of a different theory prescribing inertia as space. Thus, four things to follow here:

- (1.) The mathematics of infinite expanding space.
- (2.) Quantum theory instalment: phi-quantum wave-function
- (3.) The resultant nature of space.
- (4.) Observable stand-alone evidence for this new definition of space.

(1.) THE MATHEMATICS OF INFINITE SPACE

The drive to explain space mathematically as an infinitely expanding inertial and non-inertial reference (depending on the presence of mass) as per the reasoning delivered to explain the redshift effect led to a pursuit of the understanding of the mathematics of infinity and how that can relate to space, as of course infinite expanding space, space as a mathematical construct of infinite dimensions, could only be examined mathematically by mathematical models that rationalise the idea of infinity. Here the ideas of Cantor [21] were of interest to Hilbert’s mathematical modelling for space, known as Hilbert space [13]. Yet an alternative approach is required given the ineffectiveness of using space tagged with “inertia”. Simply, an alternative approach for space must have space disavowed of inertia. Why not just say space is as “nothing”, no quality other than the three dimensions it is perceived to have, and thus devoid of complicated inertia-based mathematical descriptors and associated physical theories? Yet how does light relate to “naught” space, and indeed thus “time”? If space is as “naught”, what defines space? This question was raised in a previous paper, “The Golden Ratio Time Algorithm” [8]. In that paper ([8], p4-8), it was suggested that if time is an a-priori on its own, then the feature that should define a notion of bodies in relative motion in space, space being a type of “nothing” yet infinite, should be time. There, together with references from paper 2 “Golden Ratio Axioms of Time and Space” [2], the idea of space was presented as derived from a fundamental basis of time. From there the idea of light was born from a new wave-function equation arriving at a value of that wave-function of light in paper 3 “The Emergence of Consciousness from Chaos” ([3]: p3) as the equation $E = e_c \left(\frac{19.8}{c}\right)^2 f$, an analogue of the Planck equation $E = hf$. Thus, in short, a new examination for space was taken from the reference of time, producing a new basis for space and associated feature of time as a wave-function of energy as light, reaching an analogue of Planck’s equation in paper 3 [3]. The task now is to use this new regard of space and primarily time as per the previous papers [1-10] to explain the idea of the redshift effect and the appearance of the stars.

(2.) QUANTUM THEORY INSTALMENT: PHI-QUANTUM WAVE-FUNCTION

Paper 3 “The Emergence of Consciousness from Chaos” ([3]: p3) delivered an analogue to the Planck equation using a new basis for the understanding of time and space. Preceding that paper was the explanation of a new wave-function for light designed for space as the ideal proposed naught (3-d) manifold, paper 2 “Golden Ratio Axioms of Time and Space” [2]. What can now be investigated is how that wave-function equation of energy and frequency would interact with space. Such a notion should have been investigated with the Planck equation itself from the Standard Model in Einstein’s gravitational

inertial spacetime. If it was, it would have been found that if $E = hf$ then as the energy of light decreases per surface area wave-front of propagation, then so could its frequency, and thus with a lowering of frequency would incur a raising of wavelength, and thus a natural redshift effect. What does this mean? Quite simply, take a point source of light and have it travel through space; naturally, as that point source of light fans out in 3-dimensions its energy in a steady state system would need to remain uniform along that wave-front, and thus as the surface area of that uniform 3-d wave-front increases with the propagation of light, the energy of that increasing 3-d wave-front remains the same, yet the energy per surface-area value could in theory “decrease” along that wave-front, clearly suggesting that the behaviour of light in space, irrespective of relativity, irrespective of doppler redshift, presents the potential case of a decrease of frequency per surface-area, and thus that a wave-front surface area of light could inherently be redshifted with its propagation through space, leading to the case that space may not need to be expanding in the presence of a redshift effect of light the further one sees the passage of light through space. How much this redshift occurs will be the subject of a subsequent paper, as clearly the frequency and this wavelength of a wavefront of light would be preserved to a large degree of the propagation of light through space.

In continuing with this logic nonetheless of a spherical wave-front of light in 3-d space, ultimately in theory light could become so-stretched regarding its wavelength it could, in effect, as infinitely as it could be as a wave-front, become a singularity, a wavelength infinitely long, much like a straight line, and thus as a wave-function collapse into nothingness; ultimately, light in space could reach the point of becoming a line, a singularity as an appearance of no energy, no light. The interesting feature for that source of light shining out into space would be how it could travel through space and pass through and reflect from objects in space with that associated redshift effect. How would this light manifest as a reflection to the source of light on its way to a dark singularity?

(3.) THE RESULTANT NATURE OF SPACE

One thing to note is that with this spherical front of light, light could merely be less compact as energy the more it moves out at a constant rate, the more it propagates through space. In a way, via the propagation of light, space could have the effect of deconstructing light through this potential redshift effect, and this “deconstruction” could be considered as a type of “repulsion” of light itself, as a type of energy that opposes light, in its purest infinite sense, that which makes light “change” in regard to itself as time, constantly. This leads to the notion that space could be considered as a form of *negative energy*, an idea not unfamiliar in physics theory as per the work of Dirac [22], together with the account here of this theory for time and space in paper 7 [7] “Golden ratio entropic gravity: Gravitational singularity “field” testing”, as per page 3:

1. “NEGATIVE ENERGY” V “EMERGENT GRAVITY”

The general Φ and $(-\frac{1}{\Phi})^2$ entropic manifolds of emergent electromagnetic and gravitational energy respectively were outlined in paper 5 [5], with greater emphasis there in that paper on emergent electromagnetism and its relationship to the CMBR. Now we need to explain the other emergent feature of the atom, namely gravity, which as the entropic realm ([5]; p 8-9), as $(-\frac{1}{\Phi})^2$, would be a form of energy release. As per paper 5 [5], the formation of the phi-quantum wave-function for the atom on the elementary particle level is more or less enthalpic as it undergoes a “contractive” dynamic process; see ([2]; eq.8, p12), ([2];

fig.16, p16), and ([4]; p3-4). Beyond this is emergent entropic electromagnetic radiation. Similarly, the formation of matter itself on the elementary particle level would be “enthalpic”, yet its emergence as “gravity” would be considered to be “entropic”.

The contemporary explanation for “entropic” gravity as kinetic energy associated to gravitational energy is simple: as the strength of the gravitational attraction between two objects represents the amount of gravitational energy [7] in the field which attracts them towards each other, when two objects are infinitely far apart the gravitational attraction and hence energy is close to zero. Yet when the two objects move towards each other, their motion accelerates by their mutual effect of gravity which causes an increase in the positive kinetic energy of the system. Yet, at the same time, the gravitational attraction (and thus energy) also increases in magnitude. The problem here is that the law of energy conservation [8] requires that the net energy of the system cannot change. Therefore, the change in gravitational energy must be negative to cancel out the positive change in kinetic energy. Paradoxically though, as the gravitational energy is getting stronger, this decrease can only mean that it is negative. There are a few problems with this idea though in the absence of negative energy, namely that in a universe in which positive energy dominates everything will eventually collapse in a “big crunch”, while in an “endless” universe where negative energy dominates everything will either expand indefinitely or cause a “big rip”. In a zero-energy universe [9] model (“flat” or “Euclidean”, the model proposed here), the total amount of energy in the universe is exactly zero where the amount of positive energy in the form of matter is exactly cancelled out by its negative energy in the form of gravity.

In this new description of space along the same theoretical basis for time (golden ratio algorithm) and space (naught-3d), the idea of light becoming a singularity lends to the idea of a gravitational singularity, an emergence of mass and thus gravity. Simply as light approaches a “naught” singularity of its own, upon this reference is manifest mass and gravity, and space in being integral to this process would act as a type of “negative energy” manifold which also as that back-drop to mass would have itself, itself as space, effect a type of “anti-matter” effect on mass itself, as per paper 7 “Golden Ratio Entropic Gravity: Gravitational Singularity Field Testing” ([7]: p3):

When the idea of negative energy is discussed, the idea of anti-particles cannot be ignored, as it is embedded in the current idea of negative energy; in regard to anti-particles, more specifically the positron, Dirac associated his “Dirac sea” as full of negative energy with “anti-particles” as a theoretical model of the vacuum containing an infinite sea of particles with negative energy [10]. Negative energy was first postulated to explain the anomalous negative-energy quantum states predicted by the Dirac equation [11] for relativistic electrons. The positron [12], the antimatter [13] counterpart of the electron, was originally conceived of as a hole in the Dirac sea, well before its experimental discovery in 1932. This idea was revised; although quantum field theory replaced the idea of the Dirac sea owing to the notion of anti-particles representing “real” matter, the theory presented a new explanation for anti-matter in paper 4 ([4]; p8-10), the idea of the positron being as an electron that has undergone a magnetic field “flip”, the case in point regarding the relationship here between positron and electron as a new explanation for a relativistic electron that when becoming super-massive would undergo a magnetic flip according to the phi-quantum wave-function. The question though with this theory, as per the phi-quantum wave-function, is why would there be a magnetic flip in the electron? The thinking is that electron in reaching relativistic speeds would undergo a magnetic flip according to the phi-quantum wave-function where the wavefunction would track back on itself as though mirroring the magnetic moment of the proton in taking on the signature of a massive particle at such a relativistic speed. It’s not a remarkable concept in this phi-quantum wave-function golden ratio theory, just a derivative of a newly defined process for time as the golden-ratio, yet a theoretical and research-based explanation. Therefore, the “idea” of anti-particles will not be used here in the process of describing “negative energy gravity”. The idea used here in this paper is “entropic” emergent gravity, which by its very nature in being

entropic allows for this increase in kinetic energy of its associated mass (being enthalpic); to properly explain this process, a further step of equations and field modelling for time is required.

In considering the proposed theory for time and space, and in considering paper 8 “The Golden Ratio Time Algorithm” [8], the “way” space would effect time and thus light is calculated to be in the manner of a golden ratio algorithm as that wave-function, or in other words, in a Fibonacci sequence “spiral” manner. How would this light manifest as a reflection to the source of light on its way to a dark singularity? In a Fibonacci sequence spiral manner, together with the idea of space seeming to repel light, and thus a spiral pattern of points of light coalescing around a light singularity behaving as a “massive” gravitational source. Adding the feature of atomic decay of points of light all convening from an exterior infinite sphere to a central observation point as our own, light would still be redshifted, and its manifestation as that redshift what would appear to be a telescopically magnified account of basic atomic phenomena releasing light from a far-away spherical location toward our own, which could potentially happen on an infinite scale, as perhaps it does as our view of starlight, which shall be further expanded upon in a subsequent paper.

(4.) OBSERVABLE STAND-ALONE EVIDENCE FOR THIS NEW DEFINITION OF SPACE

If an equation is formed to explain starlight as a type of redshift and compositional make-up and corresponding distance, using special and general relativity, then that is a “model” of the universe based on those theories and associated equations leading to a particular conclusion about the dynamic of the stars and their behaviour. Ideally those theories aim to predict further dynamic activity of those star’s behaviour as the dynamic of a tapestry of “spacetime” in the context of expanding space. Any number of predictions can be made and confirmed using a process of observation as a theory to further confirm “pieces” of that theory that accommodates for a broad swathe of assumptions, such as dark energy and dark matter. Much of what astrophysics seeks to achieve is joining the large scale with the small scale, galaxies with atomic matter. Yet what if the stars are already the construct itself of atomic matter and light? What if the stars are exactly what they appear to be, points of light, shining through dust clouds, demonstrating atomic behaviour, redshifted by their passage through space? Are they suns or relatively dynamic points of light demonstrating atomic behaviour, light naturally redshifted through its passage through space? Could those points of light perceived as stars represent a display of light from particles? The evidence for this theory proposal exists, whether it would be the idea of Fibonacci spiral points of light surrounding a gravitational singularity we would otherwise know as a galaxy, or an associated explanation of the redshift effect, yet “minus” the need to fix the numbers with “dark” ingredients such as dark energy and dark matter. An experiment can be constructed to demonstrate this feature of the “light singularity” and associated surrounding spiral phenomena, as an effect itself of “gravity”, as an upgrade of what was initially presented in paper 7 “Golden Ratio Entropic Gravity: Gravitational Singularity Field Testing” [7], which should yield more exact results, research which shall become the subject of a subsequent paper.

8. CONCLUSION

This paper has aimed to highlight relativity theory being a victim of its own time, not focussing on how it regarded the idea of inertia with space, yet merely assuming that association, together with assuming the reference of the observer with the idea of time and thus light. The problem could also exist that, owing to our reference of

observation and organisation of conscious logic, we are only able to structure a theory local to our evolutionary process on this planet, especially if it can be demonstrated that consciousness represents a type of reference for time and thus ideas of space. Besides, any theory on the nature of the universe beyond this local solar system would in any event of theory prove hazardous in upholding the idea of starlight existing as other suns given the vast distances that exist between stars and the unknowability of the nature of the fabric of those spaces in between, which in current theories of the universe is assumed to be practically linear. Ultimately, any theory that is able to join the field forces in this local solar system reality and do it successfully with proof should be held in higher regard than theories about a universe impossible to reach in any close lifetime together with requiring absurd amounts of energy and matter to bring resolution to those theories concerned. What would science rather, joining all the field forces locally in this solar system with an associated theory providing that with experimental proof of gravity linking with electromagnetism yet the trade-off being a completely different understanding of the stars, or the currently held view of starlight and associated universal model while searching for the highly mythical concepts of dark energy and dark matter?

The stars in their great numbers are a wonderful display of cosmic light, their patterns of which can be whatever we want them to be, as a story; the further we examine them, the more intricate the story. Historically those stories were relevant to the nature of human and that history in a celestial context, our reference, and so on. Today the story of the stars tries to paint a picture of where humanity came from based on our examination of the stars compared to atomic phenomena more locally here within this solar system; essentially the story of the stars seems to always be about where we came from, who we are, and where we are headed, except today it is based on a theory of relativity which may indeed be incorrect. The stars nonetheless always seem to be a process of “story telling”, and yet also the pursuit of truth. “Elusive” is perhaps their best description, yet also powerfully beautiful. Perhaps what comes from the outer darkness could only be questions, always, in comparison to what we have here in this solar system, and forever a question, an unknown. In the meantime, theories abound regarding the stars, some theories depending on “darkness”, dark energy and dark matter, and thus extreme uncertainties until they are found. If an alternative theory is developed minus those uncertainties, a theory that can neatly explain the phenomena of light of the stars, together with joining the EM and G field forces, without any assumptions, it would be a profound change compared to what is upheld in theory today. Perhaps the fundamental question for any physics theory is, “Is reality convincing or is what we want to believe more convincing than plain sight?”. For physics theory today, how long will a theory be upheld that requires dark energy and dark energy of the order of magnitude that is required, an order of magnitude that outweighs what already exists in plain sight?

Conflicts of Interest

The author declares no conflicts of interest; this has been an entirely self-funded independent project.

REFERENCES.

1. Jarvis S. H. (2017), Gravity's Emergence from Electrodynamics, <http://vixra.org/abs/1704.0169>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
2. Jarvis S. H. (2017), Golden Ratio Axioms of Time and Space, <http://vixra.org/abs/1706.0488>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
3. Jarvis S. H. (2017), The Emergence of Consciousness from Chaos, <http://vixra.org/abs/1707.0044>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.

4. Jarvis S. H. (2017), Phi-Quantum Wave-Function Crystal Dynamics, <http://vixra.org/abs/1707.0352>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
5. Jarvis S. H. (2017), Time as Energy, <http://viXra.org/abs/1711.0419>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
6. Jarvis S. H. (2018), The Relativity of Time, <http://viXra.org/abs/1801.0083>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
7. Jarvis S. H. (2019), Golden Ratio Entropic Gravity: Gravitational Field Testing, <http://vixra.org/abs/1904.0485>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
8. Jarvis S. H. (2019), The Golden Ratio Time Algorithm, <http://vixra.org/abs/1905.0081>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
9. Jarvis S. H. (2019), The Physics Chimera, <http://viXra.org/abs/1906.0127>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
10. Jarvis S. H. (2019), The Conception of Time, <http://vixra.org/abs/1906.0441>, http://www.equusspace.com/index_2.htm, page accessed 19th August 2019.
11. Einstein A. (1916), Relativity: The Special and General Theory (Translation 1920), New York: H. Holt and Company
12. Minkowski 1907–1908, pp. 53–111 *Wikisource translation: The Fundamental Equations for Electromagnetic Processes in Moving Bodies.
13. Hazewinkel, Michiel, ed. (2001) [1994], "Hilbert space", Encyclopedia of Mathematics, Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4
14. Ball, W.W. Rouse (1960) [1908]. A Short Account of the History of Mathematics (4th ed.). Dover Publications. pp. 50–62. ISBN 0-486-20630-0.
15. <https://www.britannica.com/science/big-bang-model>, page accessed 19th August 2019.
16. <https://www.britannica.com/science/redshift>, page accessed 19th August 2019.
17. <https://www.britannica.com/science/Doppler-effect>, page accessed 19th August 2019.
18. <https://www.britannica.com/science/dark-energy>, page accessed 19th August 2019.
19. <https://www.britannica.com/science/dark-matter>, page accessed 19th August 2019.
20. <https://www.britannica.com/science/expanding-universe>, page accessed 19th August 2019.
21. Dauben, Joseph Warren (1979). Georg Cantor His Mathematics and Philosophy of the Infinite. Princeton University press. pp. introduction. ISBN 9780691024479.
22. Dirac, P.A.M. (1982) [1958]. Principles of Quantum Mechanics. International Series of Monographs on Physics (4th ed.). Oxford University Press. p. 255. ISBN 978-0-19-852011-5.