Space–Time Block Codes from Orthogonal Designs

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***Abstract*— We introduce *space–time block coding*, a new paradigm for communication over Rayleigh fading channels using multiple transmit antennas. Data is encoded using a space–time block code and the encoded data is split into n streams which are simultaneously transmitted using n transmit antennas. The received signal at each receive antenna is a linear superposition of the n transmitted signals perturbed by noise. Maximum-likelihood decoding is achieved in a simple way through decoupling of the signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space–time block code and gives a maximum-likelihood decoding algorithm which is based only on linear processing at the receiver. Space–time block codes are designed to achieve the maximum diversity order for a given number of transmit and receive antennas subject to the constraint of having a simple decoding algorithm.**

The classical mathematical framework of orthogonal designs is applied to construct space–time block codes. It is shown that space–time block codes constructed in this way only exist for few sporadic values of n. Subsequently, a generalization of orthogonal designs is shown to provide space–time block codes for both real and complex constellations for any number of transmit antennas. These codes achieve the maximum possible transmission rate for any number of transmit antennas using any arbitrary real constellation such as PAM. For an arbitrary complex constellation such as PSK and QAM, space–time block codes are designed that achieve $1/2$ of the maximum possible transmission rate for any number of transmit antennas. For the specific cases of two, three, and four transmit antennas, space–time block codes are designed that achieve, respectively, all, $3/4$, and $3/4$ of maximum possible transmission rate using arbitrary complex constellations. The best tradeoff between the decoding delay and the number of transmit antennas is also computed and it is shown that many of the codes presented here are optimal in this sense as well.

***Index Terms*— Codes, diversity, multipath channels, multiple antennas, wireless communication.**

I. Introduction

SEVERE attenuation in a multipath wireless environment makes it extremely difficult for the receiver to determine the transmitted signal unless the receiver is provided with some form of *diversity*, i.e., some less-attenuated replica of the transmitted signal is provided to the receiver.

In some applications, the only practical means of achieving diversity is deployment of antenna arrays at the transmitter and/or the receiver. However, considering the fact that receivers are typically required to be small, it may not be practical to deploy multiple receive antennas at the remote station. This motivates us to consider transmit diversity.

Transmit diversity has been studied extensively as a method of combatting impairments in wireless fading channels [2]–[4], [6], [9]–[15]. It is particularly appealing because of its relative simplicity of implementation and the feasibility of multiple antennas at the base station. Moreover, in terms of economics, the cost of multiple transmit chains at the base can be amortized over numerous users.

Space–time trellis coding [10] is a recent proposal that combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas. Specific space–time trellis codes designed for 2–4 transmit antennas perform extremely well in slow-fading environments (typical of indoor transmission) and come close to the outage capacity computed by Telatar [12] and independently by Foschini and Gans [4]. However, when the number of transmit antennas is fixed, the decoding complexity of space–time trellis codes (measured by the number of trellis states in the decoder) increases exponentially with transmission rate.

In addressing the issue of decoding complexity, Alamouti [1] recently discovered a remarkable scheme for transmission using two transmit antennas. This scheme is much less complex than space–time trellis coding for two transmit antennas but there is a loss in performance compared to space–time trellis codes. Despite this performance penalty, Alamouti's scheme [1] is still appealing in terms of simplicity and performance and it motivates a search for similar schemes using more than two transmit antennas. It is a starting point for the studies in this paper, where we apply the *theory of orthogonal designs* to create analogs of Alamouti's scheme, namely, *space–time block codes*, for more than two transmit antennas.

The theory of orthogonal designs is an arcane branch of mathematics which was studied by several great number theorists including Radon and Hurwitz. The encyclopedic work of Geramita and Seberry [5] is an excellent reference. A classical result in this area is due to Radon who determined the set of dimensions for which an orthogonal design exists [8]. Radon's results are only concerned with real square orthogonal designs. In this work, we extend the results of Radon to both nonsquare and complex orthogonal designs and introduce a *theory of generalized orthogonal designs*. Using this theory, we construct space–time block codes for any number of transmit antennas. Since we approach the theory of orthogonal designs from a communications perspective, we also study designs which correspond to combined coding and linear processing at the transmitter.

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The outline of the paper is as follows. In Section II, we describe a mathematical model for multiple-antenna transmission over a wireless channel. We review the diversity criterion for code design in this model as established in [10]. In Section III, we review orthogonal designs and describe their application to wireless communication systems employing multiple transmit antennas. It will be proved that the scheme provides maximum possible spatial diversity order and allows a remarkably simple decoding strategy based only on linear processing. In Section IV, we generalize the concept of the orthogonal designs and develop a theory of *generalized orthogonal designs*. Using this mathematical theory, we construct coding schemes for any arbitrary number of transmit antennas. These schemes achieve the full diversity order that can be provided by the transmit and receive antennas. Moreover, they have very simple maximum-likelihood decoding algorithms based only on linear processing at the receiver. They provide the maximum possible transmission rate using totally real constellations as established in the theory of space–time coding [10]. In Section V, we define complex orthogonal designs and study their properties. We will recover the scheme proposed by Alamouti [1] as a special case, though it will be proved that generalization to more than two transmit antennas is not possible. We then develop a theory of *complex generalized orthogonal designs*. These designs exist for any number of transmit antennas and again have remarkably simple maximum-likelihood decoding algorithms based only on linear processing at the receiver. They provide full spatial diversity and $1/2$ of the maximum possible rate (as established previously in the theory of space–time coding) using complex constellations. For complex constellations and for the specific cases of two, three, and four transmit antennas, these diversity schemes are improved to provide, respectively all, $3/4$, and $3/4$ of maximum possible transmission rate. Section VI presents our conclusions and final remarks.

For the reader who is interested only in the code construction but is not concerned with the details, we provide a summary of the material at the beginning of each subsection.

II. The Channel Model and the Diversity Criterion

In this section, we model a multiple-antenna wireless communication system under the assumption that fading is quasi-static and flat. We review the diversity criterion for code design assuming this model. This diversity criterion is crucial for our studies of space–time block codes.

We consider a wireless communication system where the base station is equipped with n and the remote is equipped with m antennas. At each time slot t, signals $c_t^i, i = 1, 2, \cdots, n$ are transmitted simultaneously from the n transmit antennas. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to receive antenna j. The path gains are modeled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension. The wireless channel is assumed to be quasi-static so that the path gains are constant over a frame of length l and vary from one frame to another.

At time t the signal r_t^j received at antenna j is given by

$$r_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^i + \eta_t^j \tag{1}$$

where η_t^j are independent samples of a zero-mean complex Gaussian random variable with variance $1/(2\mathrm{SNR})$ per complex dimension. The average energy of the symbols transmitted from each antenna is normalized to be $1/n$.

Assuming perfect channel state information is available, the receiver computes the decision metric

$$\sum_{t=1}^{l} \sum_{j=1}^{m} \left| r_t^j - \sum_{i=1}^{n} \alpha_{i,j} c_t^i \right|^2 \tag{2}$$

over all codewords

$$c_1^1 c_1^2 \cdots c_1^n c_2^1 c_2^2 \cdots c_2^n \cdots\cdots c_l^1 c_l^2 \cdots c_l^n$$

and decides in favor of the codeword that minimizes this sum.

Given perfect channel state information at the receiver, we may approximate the probability that the receiver decides erroneously in favor of a signal

$$\boldsymbol{e} = e_1^1 e_1^2 \cdots e_1^n e_2^1 e_2^2 \cdots e_2^n \cdots e_l^1 e_l^2 \cdots e_l^n$$

assuming that

$$\boldsymbol{c} = c_1^1 c_1^2 \cdots c_1^n c_2^1 c_2^2 \cdots c_2^n \cdots c_l^1 c_l^2 \cdots c_l^n$$

was transmitted. (For details see [6], [10].) This analysis leads to the following diversity criterion.

- Diversity Criterion For Rayleigh Space–Time Code: In order to achieve the maximum diversity mn, the matrix

$$B(\boldsymbol{c}, \boldsymbol{e}) = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & \cdots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \ddots & \vdots & e_l^3 - c_l^3 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & \cdots & e_l^n - c_l^n \end{pmatrix}$$

  has to be full rank for any pair of distinct codewords $\boldsymbol{c}$ and $\boldsymbol{e}$. If $B(\boldsymbol{c}, \boldsymbol{e})$ has minimum rank r over the set of pairs of distinct codewords, then a diversity of rm is achieved.

Subsequent analysis and simulations have shown that codes designed using the above criterion continue to perform well in Rician environments in the absence of perfect channel state information and under a variety of mobility conditions and environmental effects [11].

III. Orthogonal Designs as Codes for Wireless Channels

In this section, we consider the application of *real orthogonal designs* (Section III-A) to coding for multiple-antenna wireless communication systems. Unfortunately, these designs only exist in a small number of dimensions. Encoding using orthogonal designs is shown to be trivial in Section III-B. Maximum-likelihood decoding is shown to be achieved by decoupling of the signals transmitted from different antennas and is proved to be based only on linear processing at the receiver (Section III-C). The possibility of linear processing at the transmitter, leads to the concept of *linear processing orthogonal designs* developed in Section III-D. We then prove a normalization result (Theorem 3.4.1) which allows us to

focus on a specific class of linear processing orthogonal designs. To study the set of dimensions for which linear processing orthogonal designs exist, we need a brief review of the Hurwitz–Radon theory which is provided in Section III-E. Using this theory, we prove that allowing linear processing at the transmitter only increases the hardware complexity at the transmitter and does not expand the set of dimensions for which a real orthogonal design exists.

A reader who is only interested in code construction and applications of space–time block codes may choose to focus attention on Sections III-A, III-B, and III-C as well as Theorem 3.5.1, Definition 3.5.2, and Lemma 3.5.1.

A. Real Orthogonal Designs

A real orthogonal design of size n is an $n \times n$ orthogonal matrix with entries the indeterminates $\pm x_1, \pm x_2, \cdots, \pm x_n$. The existence problem for orthogonal designs is known as the Hurwitz–Radon problem in the mathematics literature [5], and was completely settled by Radon in another context at the beginning of this century. In fact, an orthogonal design exists if and only if $n = 2, 4$, or 8.

Given an orthogonal design $\mathcal{O}$, one can negate certain columns of $\mathcal{O}$ to arrive at another orthogonal design where all the entries of the first row have positive signs. By permuting the columns, we can make sure that the first row of $\mathcal{O}$ is $x_1, x_2, \cdots, x_n$. Thus we may assume without loss of generality that $\mathcal{O}$ has this property.

Examples of orthogonal designs are the 2×2 design

$$\begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix} \tag{3}$$

the 4×4 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix} \tag{4}$$

and the 8×8 design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}. \tag{5}$$

The matrices (3) and (4) can be identified, respectively, with complex number $x_1 + x_2 \boldsymbol{i}$ and the quaternionic number $x_1 + x_2\boldsymbol{i} + x_3\boldsymbol{j} + x_4\boldsymbol{k}$.

B. The Coding Scheme

In this section, we apply orthogonal designs to construct *space–time block codes* that achieve diversity. We assume that transmission at the baseband employs a real signal constellation $\mathcal{A}$ with 2^b elements. We focus on providing a diversity order of nm. Corollary 3.3.1 of [10] implies that the maximum transmission rate is b bits per second per hertz (bits/s/Hz). We provide this transmission rate using an $n \times n$ orthogonal design. At time slot 1, nb bits arrive at the encoder and select constellation signals $s_1, \cdots, s_n$. Setting $x_i = s_i$ for $i = 1, 2, \cdots, n$, we arrive at a matrix $\mathcal{C} = \mathcal{O}(s_1, \cdots, s_n)$ with entries $\pm s_1, \pm s_2, \cdots, \pm s_n$. At each time slot $t = 1, 2, \cdots, n$, the entries $\mathcal{C}_{ti}$, $i = 1, 2, \cdots, n$ are transmitted simultaneously from transmit antennas $1, 2, \cdots, n$.

Clearly, the rate of transmission is b bits/s/Hz. We now demonstrate that the diversity order of such a space–time block code is nm.

Theorem 3.2.1: The diversity order of the above coding scheme is nm.

Proof: The rank criterion requires that the matrix $\mathcal{O}(\tilde{s}_1, \cdots, \tilde{s}_n) - \mathcal{O}(s_1, \cdots, s_n)$ be nonsingular for any two distinct code sequences $(\tilde{s}_1, \cdots, \tilde{s}_n) \neq (s_1, \cdots, s_n)$. Clearly,

$$\mathcal{O}(\tilde{s}_1 - s_1, \cdots, \tilde{s}_n - s_n) = \mathcal{O}(\tilde{s}_1, \cdots, \tilde{s}_n) - \mathcal{O}(s_1, \cdots, s_n)$$

where $\mathcal{O}(\tilde{s}_1 - s_1, \cdots, \tilde{s}_n - s_n)$ is the matrix constructed from $\mathcal{O}$ by replacing x_i with $\tilde{s}_i - s_i$ for all $i = 1, 2, \cdots, n$. The determinant of the orthogonal matrix $\mathcal{O}$ is easily seen to be

$$\det(\mathcal{O}\mathcal{O}^T)^{1/2} = \left[\sum_i x_i^2\right]^{n/2}$$

where $\mathcal{O}^T$ is the transpose of $\mathcal{O}$. Hence

$$\det[\mathcal{O}(\tilde{s}_1 - s_1, \cdots, \tilde{s}_n - s_n)] = \left[\sum_i |\tilde{s}_i - s_i|^2\right]^{n/2}$$

which is nonzero. It follows that $\mathcal{O}(\tilde{s}_1, \cdots, \tilde{s}_n) - \mathcal{O}(s_1, \cdots, s_n)$ is nonsingular and the maximum diversity order nm is achieved. □

C. The Decoding Algorithm

Next, we consider the decoding algorithm. Clearly, the rows of $\mathcal{O}$ are all permutations of the first row of $\mathcal{O}$ with possibly different signs. Let $\epsilon_1, \cdots, \epsilon_n$ denote the permutations corresponding to these rows and let $\delta_k(i)$ denote the sign of x_i in the kth row of $\mathcal{O}$. Then $\epsilon_k(p) = q$ means that x_p is up to a sign change the (k, q)th element of $\mathcal{O}$. Since the columns of $\mathcal{O}$ are pairwise-orthogonal, it turns out that minimizing the metric of (2) amounts to minimizing

$$\sum_{i=1}^{n} S_i, \tag{6}$$

where

$$S_i = \left(\left| \left[\sum_{t=1}^{n} \sum_{j=1}^{m} r_t^j \alpha^*_{\epsilon_t(i),j} \delta_t(i) \right] - s_i \right|^2 + \left(-1 + \sum_{k,l} |\alpha_{k,l}|^2 \right) |s_i|^2 \right) \tag{7}$$

and where $\alpha^*_{\epsilon_t(i),j}$ denotes the complex conjugate of $\alpha_{\epsilon_t(i),j}$. The value of S_i only depends on the code symbol s_i, the received symbols $\{r_t^j\}$, the path coefficients $\{\alpha_{i,j}\}$, and the structure of the orthogonal design $\mathcal{O}$. It follows that minimizing the sum given in (6) amounts to minimizing (7) for all $1 \le i \le n$. Thus the maximum-likelihood detection rule is to form the decision variables

$$R_i = \sum_{t=1}^{n}\sum_{j=1}^{m} r_t^j \alpha^*_{\epsilon_t(i),j}\delta_t(i)$$

for all $i = 1, 2, \cdots, n$ and decide in favor of s_i among all the constellation symbols s if

$$s_i = \arg\min_{s\in\mathcal{A}} |R_i - s|^2 + \left(-1 + \sum_{k,l} |\alpha_{k,l}|^2\right)|s|^2. \quad (8)$$

This is a very simple decoding strategy that provides diversity.

D. Linear Processing Orthogonal Designs

There are two attractions in providing transmit diversity via orthogonal designs.

- There is no loss in bandwidth, in the sense that orthogonal designs provide the maximum possible transmission rate at full diversity.
- There is an extremely simple maximum-likelihood decoding algorithm which only uses linear combining at the receiver. The simplicity of the algorithm comes from the orthogonality of the columns of the orthogonal design.

The above properties are preserved even if we allow linear processing at the transmitter. Therefore, we relax the definition of orthogonal designs to allow linear processing at the transmitter. Signals transmitted from different antennas will now be linear combinations of constellation symbols.

Definition 3.4.1: A linear processing orthogonal design in variables $x_1, x_2, \cdots, x_n$ is an $n \times n$ matrix $\mathcal{E}$ such that:

- The entries of $\mathcal{E}$ are real linear combinations of variables $x_1, x_2, \cdots, x_n$.
- $\mathcal{E}^T\mathcal{E} = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix with (i,i)th diagonal element of the form $(l_1^i x_1^2 + l_2^i x_2^2 + \cdots + l_n^i x_n^2)$, with the coefficients $l_1^i, l_2^i, \cdots, l_n^i$ all strictly positive numbers.

It is easy to show that transmission using a linear processing orthogonal design provides full diversity and a simplified decoding algorithm as above. The next theorem shows that we may, with no loss of generality, constrain the matrix $\mathcal{D}$ in Definition 3.4.1 to be a scaled identity matrix.

Theorem 3.4.1: A linear processing orthogonal design $\mathcal{E}$ in variables $x_1, x_2, \cdots, x_n$ exists if and only if there exists a linear processing orthogonal design $\mathcal{L}$ such that

$$\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + x_2^2 + \cdots + x_n^2)I.$$

Proof: Let $\mathcal{E} = x_1A_1 + \cdots + x_nA_n$ be a linear processing orthogonal design, and let

$$\mathcal{E}^T\mathcal{E} = x_1^2\mathcal{D}_1 + \cdots + x_n^2\mathcal{D}_n$$

where the matrices $\mathcal{D}_i$ are diagonal and full-rank (since the coefficients $l_1^i, l_2^i, \cdots, l_n^i,\ i = 1, 2, \cdots, n$ are strictly positive). Then it follows that

$$A_i^TA_i = \mathcal{D}_i, \qquad i = 1, \cdots, n \quad (9)$$
$$A_i^TA_j = -A_j^TA_i, \qquad 1 \le i < j \le n \quad (10)$$

and $\mathcal{D}_i$ is a full-rank diagonal matrix with positive diagonal entries. Let $\mathcal{D}_i^{1/2}$ denote the diagonal matrix having the property that $\mathcal{D}_i^{1/2}\mathcal{D}_i^{1/2} = \mathcal{D}_i$. We define $U_i = A_i\mathcal{D}_i^{-1/2}$. Then the matrices U_i satisfy the following properties:

$$U_i^TU_i = I, \qquad i = 1, \cdots, n \quad (11)$$
$$U_i^TU_j = -U_j^TU_i, \qquad 1 \le i < j \le n. \quad (12)$$

It follows that $\mathcal{L} = x_1U_1 + \cdots + x_nU_n$ is a linear processing orthogonal array having the property

$$\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + x_2^2 + \cdots + x_n^2)I. \qquad \square$$

In view of the above theorem, we may, without any loss of generality, assume that a linear processing orthogonal design $\mathcal{L}$ satisfies

$$\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + x_2^2 + \cdots + x_n^2)I.$$

E. The Hurwitz–Radon Theory

In this section, we define a Hurwitz–Radon family of matrices. These matrices encode the interactions between variables in an orthogonal design.

Definition 3.5.1: A set of $n \times n$ real matrices $\{B_1, B_2, \cdots, B_k\}$ is called a size k Hurwitz–Radon family of matrices if

$$B_i^TB_i = I$$
$$B_i^T = -B_i, \qquad i = 1, \cdots, k$$

and

$$B_iB_j = -B_jB_i, \qquad 1 \le i < j \le k.$$

We next recall the following theorem of Radon [8].

Theorem 3.5.1: Let $n = 2^ab$, where b is odd and $a = 4c + d$ with $0 \le d < 4$ and $0 \le c$. Any Hurwitz–Radon family of $n \times n$ matrices contains strictly less than $\rho(n) = 8c + 2^d$ matrices. Furthermore $\rho(n) \le n$. A Hurwitz–Radon family containing $n - 1$ matrices exists if and only if $n = 2$, 4, or 8.

Definition 3.5.2: Let $A = [a_{ij}]$ be a $p \times q$ matrix and let B be any arbitrary matrix. The tensor product $A \otimes B$ is the matrix given by

$$\begin{pmatrix} a_{11}B & a_{12}B & \dots & \dots & a_{1q}B \\ a_{21}B & a_{22}B & \dots & \dots & a_{2q}B \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a_{p1}B & a_{p2}B & \dots & \dots & a_{pq}B \end{pmatrix}. \quad (13)$$

Definition 3.5.3: A matrix is called an integer matrix if all of its entries are in the set $\{-1, 0, 1\}$.

The proof of the next Lemma is directly taken from [5] and we include it for completeness.

Lemma 3.5.1: For any n there exists a Hurwitz–Radon family of matrices of size $\rho(n)-1$ whose members are integer matrices.

Proof: The proof is by explicit construction. Let I_b denote the identity matrix of size b. We first notice that if $n = 2^a b$ with b odd, then $\rho(n) = \rho(2^a)$. Moreover, given a family of $2^a \times 2^a$ Hurwitz–Radon integer matrices $\{A_1, A_2, \cdots, A_s\}$ of size $s = \rho(2^a) - 1$, the set $\{A_1 \otimes I_b, A_2 \otimes I_b, \cdots, A_s \otimes I_b\}$ is a Hurwitz–Radon family of $n \times n$ integer matrices of size $\rho(n) - 1$. In light of this observation, it suffices to prove the lemma for $n = 2^a$. To this end

$$R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{14}$$

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{15}$$

and

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{16}$$

Let

$$\begin{aligned} n_1 &= 2^{4s+3} \\ n_2 &= 2^{4(s+1)} \\ n_3 &= 2^{4(s+1)+1} \\ n_4 &= 2^{4(s+1)+2} \end{aligned}$$

and

$$n_5 = 2^{4(s+1)+3}.$$

Then

$$\begin{aligned} \rho(n_2) &= \rho(n_1) + 1 \\ \rho(n_3) &= \rho(n_1) + 2 \\ \rho(n_4) &= \rho(n_1) + 4 \\ \rho(n_5) &= \rho(n_1) + 8. \end{aligned}$$

We observe that R is a Hurwitz–Radon integer family of size $\rho(2) - 1$, $\{R \otimes I_2, P \otimes R, Q \otimes R\}$ is a Hurwitz–Radon integer family of size $\rho(2^2) - 1$, and

$$\{I_2 \otimes R \otimes I_2,\ I_2 \otimes P \otimes R,\ Q \otimes Q \otimes R,\ P \otimes Q \otimes R,\ R \otimes P \otimes Q,\ R \otimes P \otimes P,\ R \otimes Q \otimes I_2\}$$

is an integer Hurwitz–Radon family of size $\rho(2^3) - 1$.

The reader may easily verify that if $\{A_1, A_2, \cdots, A_s\}$ is an integer Hurwitz–Radon family of $n \times n$ matrices, then

$$\{R \otimes I_n\} \cup \{Q \otimes A_i, i = 1, 2, \cdots, s\} \tag{17}$$

is an integer Hurwitz–Radon family of $s + 1$ integer matrices $(2n \times 2n)$.

If, in addition, $\{L_1, L_2, \cdots, L_m\}$ is an integer Hurwitz–Radon family of $k \times k$ matrices, then

$$\{P \otimes I_k \otimes A_i, i = 1, 2, \cdots, s\} \cup \{Q \otimes L_j \otimes I_n, j = 1, 2, \cdots, m\} \cup \{R \otimes I_{nk}\} \tag{18}$$

is an integer Hurwitz–Radon family of $s + m + 1$ integer matrices $(2nk \times 2nk)$.

We proceed by induction. For $n = 2^3$, we already constructed an integer Hurwitz–Radon family of size $\rho(2^3) - 1$ with entries in the set $\{-1, 0, 1\}$. Now (17) gives the transition from n_1 to n_2. By using (18) and letting $k = n_1$, $n = 2$, we get the transition from n_1 to n_3. Similarly, with $k = n_1$, $n = 4$ and $k = n_1$, $n = 8$, we get the transition from n_1 to n_4 and to n_5. □

The next theorem shows that relaxing the definition of orthogonal designs to allow linear processing at the transmitter does not expand the set of dimensions n for which there exists an orthogonal design of size n.

Theorem 3.5.2: A linear processing orthogonal design of size $n \geq 2$ exists if and only if $n = 2$, 4, and 8.

Proof: Let $\mathcal{L}$ denote a linear processing orthogonal design. Since the entries of $\mathcal{L}$ are linear combinations of variables $x_1, x_2, \cdots, x_n$, we can write row i of $\mathcal{L}$ as XA_i, where A_i is an appropriate real-valued $n \times n$ matrix and $X = (x_1, x_2, \cdots, x_n)$. Orthogonality of $\mathcal{L}$ translates into the following set of matrix equalities:

$$A_i A_i^T = A_i^T A_i = I, \qquad i = 1, 2, \cdots, n \tag{19}$$

$$A_i A_j^T = -A_j A_i^T, \qquad 1 \leq i < j \leq n \tag{20}$$

where I is the identity matrix. We now construct a Hurwitz–Radon set of matrices from the original design. Let $B_i = A_1^T A_i$ for $i = 1, 2, \cdots, n$. Then $B_1 = I$ and we have

$$B_i^T B_i = I, \qquad i = 2, \cdots, n \tag{21}$$

$$B_i^T = -B_i, \qquad i = 2, \cdots, n \tag{22}$$

$$B_i B_j = -B_j B_i, \qquad 2 \leq i < j \leq n. \tag{23}$$

These equations imply that $\{B_2, B_3, \cdots, B_n\}$ is a *Hurwitz–Radon* family of matrices. By the Hurwitz–Radon Theorem (3.5.1), we can conclude that $\rho(n) = n - 1$ and $n = 2$, 4, or 8. □

In particular, we have the following special case.

Corollary 3.5.1: An orthogonal design of size n exists if and only if $n = 2$, 4, and 8.

Proof: Immediate from Theorem 3.5.2. □

To summarize, relaxing the definition of orthogonal designs, by allowing linear processing at the transmitter, fails to provide new transmission schemes and only adds to the hardware complexity at the transmitter.

IV. Generalized Real Orthogonal Designs

The previous results show the limitations of providing transmit diversity through linear processing orthogonal designs based on square matrices. Since the simple maximum-likelihood decoding algorithm described above is achieved because of orthogonality of columns of the design matrix, we may generalize the definition of linear processing orthogonal designs. Not only does this create new and simple transmission schemes for any number of transmit antennas, but also generalizes the Hurwitz–Radon theory to nonsquare matrices.

In this section, we introduce *generalized real orthogonal designs* and pose the *fundamental question of generalized orthogonal design theory*. The answer to this fundamental question provides us with transmission schemes that are in some sense optimal in terms of the decoding delay. We then settle the fundamental question of generalized orthogonal design theory for full-rate orthogonal designs (in a sense to be defined in the sequel) and construct full-rate transmission schemes for any number of transmit antennas.

A reader who is interested only in code construction and applications of space–time block codes is advised to go through the results of this section.

A. Construction and Basic Properties

Definition 4.1.1: A generalized orthogonal design $\mathcal{G}$ of size n is a $p \times n$ matrix with entries $0, \pm x_1, \pm x_2, \cdots, \pm x_k$ such that $\mathcal{G}^T\mathcal{G} = \mathcal{D}$ where $\mathcal{D}$ is a diagonal matrix with diagonal $\mathcal{D}_{ii}, i = 1, 2, \cdots, n$ of the form $(l_1^i x_1^2 + l_2^i x_2^2 + \cdots + l_k^i x_k^2)$ and coefficients $l_1^i, l_2^i, \cdots, l_k^i$ are strictly positive integers. The rate of $\mathcal{G}$ is $R = k/p$.

The following theorem is analogous to Theorem 3.4.1

Theorem 4.1.1: A $p \times n$ generalized orthogonal design $\mathcal{E}$ in variables $x_1, x_2, \cdots, x_k$ exists if and only if there exists a generalized orthogonal design $\mathcal{G}$ in the same variables and of the same size such that

$$\mathcal{G}^T\mathcal{G} = \left(x_1^2 + x_2^2 + \cdots + x_k^2\right)I.$$

In view of the above theorem, without any loss of generality, we assume that any $p \times n$ generalized orthogonal design $\mathcal{G}$ in variables $x_1, x_2, \cdots, x_k$ satisfies

$$\mathcal{G}^T\mathcal{G} = \left(x_1^2 + x_2^2 + \cdots + x_k^2\right)I.$$

Transmission using a generalized orthogonal design is discussed next. We consider a real constellation $\mathcal{A}$ of size 2^b. Throughput of kb/p can be achieved as described in Section III-A. At time slot 1, kb bits arrive at the encoder and select constellation symbols $s_1, s_2, \cdots, s_n$. The encoder populates the matrix by setting $x_i = s_i$, and at time $t = 1, 2, \cdots, p$ the signals $\mathcal{G}_{t1}, \cdots, \mathcal{G}_{tn}$ are transmitted simultaneously from antennas $1, 2, \cdots, n$. Thus kb bits are sent during each p transmissions. It can be proved, as in Theorem 3.1, that the diversity order is nm. It should be mentioned that the rate of a generalized orthogonal design is different from the throughput of the associated code. To motivate the definition of the rate, we note that the theory of space–time coding proves that for a diversity order of nm, it is possible to transmit b bits per time slot and this is best possible (see [10, Corollary 3.3.1]). Therefore, the rate R of this coding scheme is defined to be kb/pb which is equal to k/p.

The goal of this section is to construct high-rate linear processing orthogonal designs with low decoding complexity and full diversity order. We must, however, take the memory requirements into account. This means that given R and n, we must attempt to minimize p.

Definition 4.1.2: For a given R, n, we define $A(R, n)$ to be the minimum number p such that there exists a $p \times n$ generalized orthogonal design with rate at least R. If no such orthogonal design exists, we define $A(R, n) = \infty$. A generalized orthogonal design attaining the value $A(R, n)$ is called delay-optimal.

The value of $A(R, n)$ is the *fundamental question of generalized orthogonal design theory*. The most interesting part of this question is the computation of $A(1, n)$ since the generalized orthogonal designs of full rate are bandwidth-efficient. To address this question, we will need the following construction.

Construction I: Let $X = (x_1, x_2, \cdots, x_p)$ and $n \leq \rho(p)$. In Lemma 3.5.1, we explicitly constructed a family of integer $p \times p$ matrices with $\rho(p) - 1$ members $\{A_1, A_2, \cdots, A_{\rho(p)-1}\}$. Let $A_0 = I$ and consider the $p \times n$ matrix $\mathcal{G}$ whose jth column is $A_{j-1}X^T$ for $j = 1, 2, \cdots, n$. The Hurwitz–Radon conditions imply that $\mathcal{G}$ is a generalized orthogonal design of full rate.

Theorem 4.1.2: The value $A(1, n)$ is the smallest number p such that $n \leq \rho(p)$.

Proof: Let p be a number such that $n \leq \rho(p)$. Let $X = (x_1, x_2, \cdots, x_p)$ and apply Construction I to arrive at $\mathcal{G}$, a $p \times n$ generalized orthogonal design of full rate. By definition, $A(1, n) \leq p$, and hence

$$A(1, n) \leq \min_{n \leq \rho(p)} (p) < \infty. \tag{24}$$

Next, we consider any generalized orthogonal design $\mathcal{G}$ of size $p \times n$ in p variables (rate one) where $p = A(1, n)$. The columns of $\mathcal{G}$ are linear combinations of the variables $x_1, x_2, \cdots, x_p$. The ith column can be written as $B_i X^T$ for some real-valued $p \times p$ matrix B_i. Since the columns of $\mathcal{G}$ are orthogonal we have

$$B_i^T B_i = I, \qquad i = 1, 2, \cdots, n \tag{25}$$

$$B_i^T B_j = -B_j^T B_i, \qquad 1 \leq i < j \leq n. \tag{26}$$

This means that the matrices $A_j = B_1^T B_j, j = 2, \cdots, n$ are a Hurwitz–Radon family of size $n - 1$. Thus $n - 1 \leq \rho(p) - 1$ and $n \leq \rho(p)$, and $A(1, n) = p \geq \min_{n \leq \rho(p)}(p)$. Combining this result with inequality (24) concludes the proof. □

Corollary 4.1.1: For any R, $A(R, n) < \infty$.

Proof: The proof follows immediately from Theorem 4.1.2. □

Corollary 4.1.2: The value $A(1, n) = \min(2^{4c+d})$, where the minimization is taken over the set

$$\{c, d \mid 0 \leq c, 0 \leq d < 4 \text{ and } 8c + 2^d \geq n\}.$$

In particular, $A(1, 2) = 2$, $A(1, 3) = A(1, 4) = 4$, and $A(1, n) = 8$ for $5 \leq n \leq 8$.

Proof: Let $p = A(1, n)$. We first claim that p is a power of two. To this end, suppose that $p = 2^a b$ where $b > 1$ is an odd number. Then $\rho(2^a) = \rho(p) \geq n$. But $2^a < p$. This contradicts the fact that $p = \min_{n \leq \rho(p)}(p)$ and proves the claim. Thus $p = 2^a$ for some a. An application of the explicit formula for $\rho(2^a)$ given in Theorem 3.5.1 completes the proof. □

It follows that orthogonal designs are delay optimal for $n = 2, 4$, and 8.

We have explicitly constructed a Hurwitz–Radon family of matrices of size p with $\rho(p)$ members such that all the matrices in the family have entries in the set $\{-1, 0, 1\}$. Given such a family of Hurwitz–Radon matrices of size $p = A(1, n)$, we can apply Construction I to provide a $p \times n$ generalized orthogonal design with full rate. This full-rate generalized orthogonal design has entries of the form $\pm x_1, \cdots, \pm x_p$. This is the method used to prove the following theorem which completes the construction of delay-optimal generalized orthogonal designs of rate one for $n \leq 8$ transmit antennas.

Theorem 4.1.3: The orthogonal designs

$$\mathcal{G}_3 = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{pmatrix} \tag{27}$$

$$\mathcal{G}_5 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 \end{pmatrix} \tag{28}$$

$$\mathcal{G}_6 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 \end{pmatrix} \tag{29}$$

and

$$\mathcal{G}_7 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 \end{pmatrix} \tag{30}$$

are delay-optimal designs with rate one.

Proof: The orthogonal designs constructed above achieve the value $A(1, n)$ for $n = 3, 5, 6, 7$. □

V. Generalized Complex Orthogonal Designs as Space–Time Block Codes

The simple transmit diversity schemes described above assume a real signal constellation. It is natural to ask for extensions of these schemes to complex signal constellations. Hence the notion of *complex orthogonal designs* is introduced in Section V-A. We recover the Alamouti scheme as a 2×2 complex orthogonal designs in Section V-B. Motivated by the possibility of linear processing at the transmitter, we define *complex linear processing orthogonal designs* in Section V-C, but we shall prove that complex linear processing orthogonal designs only exist in two dimensions. This means that the Alamouti Scheme is in some sense unique. However, we would like to have coding schemes for more than two transmit antennas that employ complex constellations. Hence the notion of *generalized complex orthogonal designs* is introduced in Section V-E. We then prove by explicit construction that rate $1/2$ generalized complex orthogonal designs exist in any dimension. In Section V-F, it is shown that this is not the best rate that can be achieved. Specifically, examples of rate $3/4$ generalized complex linear processing orthogonal designs in dimensions three and four are provided.

A reader who is only interested in code construction and the application of space–time block codes may choose to read Section V-B, Definition 5.4.1, Definition 5.5.2, the proof of Theorem 5.5.2, Corollary 5.5.1, the remark after Corollary 5.5.1, and Section V-F.

A. Complex Orthogonal Designs

We define a *complex orthogonal design* $\mathcal{O}_c$ of size n as an orthogonal matrix with entries the indeterminates $\pm x_1, \pm x_2, \cdots, \pm x_n$, their conjugates $\pm x_1^*, \pm x_2^*, \cdots, \pm x_n^*$, or multiples of these indeterminates by $\pm \boldsymbol{i}$ where $\boldsymbol{i} = \sqrt{-1}$. Without loss of generality, we may assume that the first row of $\mathcal{O}_c$ is $x_1, x_2, \cdots, x_n$.

The method of encoding presented in Section III-A can be applied to obtain a transmit diversity scheme that achieves the full diversity nm. The decoding metric again separates into decoding metrics for the individual symbols $x_1, x_2, \cdots, x_n$. An example of a 2×2 complex orthogonal design is given by

$$\begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}. \tag{31}$$

B. The Alamouti Scheme

The space–time block code proposed by Alamouti [1] uses the complex orthogonal design

$$\begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}. \tag{32}$$

Suppose that there are 2^b signals in the constellation. At the first time slot, $2b$ bits arrive at the encoder and select two complex symbols s_1 and s_2. These symbols are transmitted simultaneously from antennas one and two, respectively. At the second time slot, signals $-s_2^*$ and s_1^* are transmitted simultaneously from antennas one and two, respectively.

Maximum-likelihood detection amounts to minimizing the decision statistic

$$\sum_{j=1}^{m} \left(\left| r_1^j - \alpha_{1,j} s_1 - \alpha_{2,j} s_2 \right|^2 + \left| r_2^j + \alpha_{1,j} s_2^* - \alpha_{2,j} s_1^* \right|^2 \right) \tag{33}$$

over all possible values of s_1 and s_2. The minimizing values are the receiver estimates of s_1 and s_2, respectively. As in the

previous section, this is equivalent to minimizing the decision statistic

$$\left|\left[\sum_{j=1}^{m}\left(r_1^j\alpha_{1,j}^* + \left(r_2^j\right)^*\alpha_{2,j}\right)\right] - s_1\right|^2 + \left(-1 + \sum_{j=1}^{m}\sum_{i=1}^{2}|\alpha_{i,j}|^2\right)|s_1|^2$$

for detecting s_1 and the decision statistic

$$\left|\left[\sum_{j=1}^{m}\left(r_1^j\alpha_{2,j}^* - \left(r_2^j\right)^*\alpha_{1,j}\right)\right] - s_2\right|^2 + \left(-1 + \sum_{j=1}^{m}\sum_{i=1}^{2}|\alpha_{i,j}|^2\right)|s_2|^2$$

for decoding s_2. This is the simple decoding scheme described in [1], and it should be clear that a result analogous to Theorem 3.2.1 can be established here. Thus Alamouti's scheme provides full diversity $2m$ using m receive antennas. This is also established by Alamouti [1], who proved that this scheme provides the same performance as $2m$ level maximum ratio combining.

C. On the Existence of Complex Orthogonal Designs

In this section, we consider the existence problem for complex orthogonal designs. First, we show that a complex orthogonal design of size n determines a real orthogonal design of size $2n$.

Construction II: Given a complex orthogonal design $\mathcal{O}_c$ of size n, we replace each complex variable $x_i = x_i^1 + x_i^2\boldsymbol{i}$, $1 \le i \le n$ by the 2×2 real matrix

$$\begin{pmatrix} x_i^1 & x_i^2 \\ -x_i^2 & x_i^1 \end{pmatrix}. \tag{34}$$

In this way x_i^* is represented by

$$\begin{pmatrix} x_i^1 & -x_i^2 \\ x_i^2 & x_i^1 \end{pmatrix} \tag{35}$$

$\boldsymbol{i}x_i$ is represented by

$$\begin{pmatrix} -x_i^2 & x_i^1 \\ -x_i^1 & -x_i^2 \end{pmatrix} \tag{36}$$

and so forth. It is easy to see that the $2n \times 2n$ matrix formed in this way is a real orthogonal design of size $2n$.

We can now prove the following theorem:

Theorem 5.3.1: A complex orthogonal design $\mathcal{O}_c$ of size n exists only if $n = 2$ or $n = 4$.

Proof: Given a complex orthogonal design of size n, apply Construction II to provide a real orthogonal design of size $2n$. Since real orthogonal designs can only exist for $n = 2$, 4, and 8, it follows that complex orthogonal designs of size n cannot exist unless $n = 2$ or $n = 4$. □

For $n = 2$, Alamouti's scheme gives a complex orthogonal design. We will prove later that complex orthogonal designs do not exist even for four transmit antennas.

D. Complex Linear Processing Orthogonal Designs

Definition 5.4.1: A complex linear processing orthogonal design in variables $x_1, x_2, \cdots, x_n$ is an $n \times n$ matrix $\mathcal{E}_c$ such that

- the entries of $\mathcal{E}_c$ are complex linear combinations of variables $x_1, x_2, \cdots, x_n$ and their conjugates;
- $\mathcal{E}_c^*\mathcal{E}_c = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix where all diagonal entries are linear combinations of $|x_1|^2$, $|x_2|^2, \cdots, |x_n|^2$ with all strictly positive real coefficients.

The proof of the following theorem is similar to that of Theorem 3.4.1.

Theorem 5.4.1: A complex linear processing orthogonal design $\mathcal{E}_c$ in variables $x_1, x_2, \cdots, x_n$ exists if and only if there exists a complex linear processing orthogonal design $\mathcal{L}_c$ such that

$$\mathcal{L}_c\mathcal{L}_c^* = \mathcal{L}_c^*\mathcal{L}_c = (|x_1|^2 + |x_2|^2 + \cdots + |x_n|^2)I.$$

In view of the above theorem, without any loss of generality, we assume that any complex linear processing orthogonal design $\mathcal{L}_c$ satisfies

$$\mathcal{L}_c\mathcal{L}_c^* = \mathcal{L}_c^*\mathcal{L}_c = (|x_1|^2 + |x_2|^2 + \cdots + |x_n|^2)I.$$

We can now prove the following theorem:

Theorem 5.4.2: A complex linear processing orthogonal design of size n exists if and only if $n = 2$.

Proof: We apply Construction II to the complex linear processing orthogonal design of size n to arrive at a linear processing orthogonal design of size $2n$. Thus $2n = 4$ or $2n = 8$ which implies that $n = 2$ or $n = 4$. For $n = 2$, Alamouti's matrix is a complex linear processing orthogonal design. Therefore, it suffices to prove that for $n = 4$ complex linear processing orthogonal designs do not exist. The proof is given in the Appendix.

We can now immediately recover the following result.

Corollary 5.4.1: A complex orthogonal design of size n exists if and only if $n = 2$.

Proof: Immediate from Theorem 5.4.2. □

We conclude that relaxing the definition of complex orthogonal designs to allow linear processing will only add to hardware complexity at the transmitter and fails to provide transmission schemes in new dimensions.

E. Generalized Complex Orthogonal Designs

We next define generalized complex orthogonal designs.

Definition 5.5.1: Let $\mathcal{G}_c$ be a $p \times n$ matrix whose entries are

$$0, \pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, \cdots, \pm x_k, \pm x_k^*$$

or their product with $\boldsymbol{i}$. If $\mathcal{G}_c^*\mathcal{G}_c = \mathcal{D}_c$ where $\mathcal{D}_c$ is a diagonal

matrix with (i,i)th diagonal element of the form

$$(l_1^i|x_1|^2 + l_2^i|x_2|^2 + \cdots + l_k^i|x_k|^2)$$

and the coefficients $l_1^i, l_2^i, \cdots, l_k^i$ all strictly positive numbers, then $\mathcal{G}_c$ is referred to as a generalized orthogonal design of size n and rate $R = k/p$.

The following Theorem is analogous to Theorem 3.4.1.

Theorem 5.5.1: A $p \times n$ complex generalized linear processing orthogonal design $\mathcal{E}_c$ in variables

$$0, \pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, \cdots, \pm x_k, \pm x_k^*$$

exists if and only if there exists a complex generalized linear processing orthogonal design $\mathcal{G}_c$ in the same variables and of the same size such that

$$\mathcal{G}_c^* \mathcal{G}_c = (|x_1|^2 + |x_2|^2 + \cdots + |x_k|^2)I.$$

In view of the above theorem, without any loss of generality, we assume that any $p \times n$ generalized orthogonal design $\mathcal{G}_c$ in variables

$$0, \pm x_1, \pm x_1^*, \pm x_2, \pm x_2^*, \cdots, \pm x_k, \pm x_k^*$$

satisfies the equality

$$\mathcal{G}_c^* \mathcal{G}_c = (|x_1|^2 + |x_2|^2 + \cdots + |x_k|^2)I$$

after the appropriate normalization.

Transmission using a complex generalized orthogonal design is similar to that of a generalized orthogonal design. Maximum-likelihood decoding is analogous to that of Alamouti's scheme and can be done using linear processing at the receiver. The goal of this section is to construct high-rate complex generalized linear processing orthogonal designs with low decoding complexity that achieve full diversity. We must, however, take the memory requirements into account. This means that given R and n, we must attempt to minimize p.

Definition 5.5.2: For a given R and n, we define $A_c(R,n)$ the minimum number p for which there exists a complex generalized linear processing orthogonal design of size $p \times n$ and rate at least R. If no such orthogonal design exists, we define $A_c(R,n) = \infty$.

The question of the computation of the value of $A_c(R,n)$ is the *fundamental question of generalized complex orthogonal design theory*. To address this question to some extent, we will establish the following Theorem.

Theorem 5.5.2: The following inequalities hold.

- i) For any R, we have $A(R,2n) \le 2A_c(R,n)$.
- ii) For $R \le 0.5$, we have $A_c(R,n) \le 2A(2R,n)$.

Proof: We first prove Part i). If $A_c(R,n) = \infty$, then there is nothing to be proved. Thus we assume that $p = A_c(R,n) < \infty$ and consider a complex generalized linear processing orthogonal design $\mathcal{G}_c$ of rate at least equal to R and size $p \times n$. By applying Construction II, we arrive at a $2p \times 2n$ real generalized linear processing orthogonal design of rate at least equal to R. Thus $2A_c(R,n) = 2p \ge A(R,2n)$.

To prove Part ii), we consider a real orthogonal design $\mathcal{G}$ of size $p \times n$ and rate at least equal to $2R$ in variables $x_1, x_2, \cdots, x_k$, where $p = A(2R,n)$. We construct a complex array $\mathcal{G}_c$ of size $2p \times n$. We replace the symbols $x_1, x_2, \cdots, x_k$ everywhere in $\mathcal{G}$ by their symbolic conjugates $x_1^*, x_2^*, \cdots, x_k^*$ to arrive at a new array $\mathcal{G}^*$. Then we define $\mathcal{G}_c$ to be the $2p \times n$ array with the row $i \le p$ the ith row of $\mathcal{G}$ and the row $p < i \le 2p$ the $(i-p)$th row of $\mathcal{G}^*$. It is easy to see that $\mathcal{G}_c$ is a complex generalized orthogonal design of rate at least equal to R. Thus $A_c(R,n) \le 2p = 2A(2R,n)$. □

Corollary 5.5.1: For $R \le 0.5$, we have $A_c(R,n) < \infty$.

Proof: It follows immediately from Part ii) of Theorem 5.5.2 and Corollary 4.1.1. □

Remark: Corollary 5.5.1 proves there exists rate $1/2$ complex generalized orthogonal designs, and the proof of Part ii) of Theorem 5.5.2 gives an explicit construction for these designs. For instance, rate $1/2$ codes for transmission using three and four transmit antennas are given by

$$\mathcal{G}_c^3 = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{pmatrix} \tag{37}$$

and

$$\mathcal{G}_c^4 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{pmatrix}. \tag{38}$$

These transmission schemes and their analogs for higher n give full diversity but lose half of the theoretical bandwidth efficiency.

F. Few Sporadic Codes

It is natural to ask for higher rates than $1/2$ when designing generalized complex linear processing orthogonal designs for transmission with n multiple antennas. For $n = 2$, Alamouti's scheme gives a rate one design. For $n = 3$ and 4, we construct rate $3/4$ generalized complex linear processing orthogonal designs given by

$$\begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{(-x_1 - x_1^* + x_2 - x_2^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{(x_2 + x_2^* + x_1 - x_1^*)}{2} \end{pmatrix} \tag{39}$$

for $n = 3$ and

$$\begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(-x_2-x_2^*+x_1-x_1^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} & -\frac{(x_1+x_1^*+x_2-x_2^*)}{2} \end{pmatrix} \tag{40}$$

for $n = 4$. These codes are designed using the theory of amicable designs [5].

Apart from these two designs, we do not know of any other generalized designs in higher dimensions with rate greater than 0.5. We believe that the construction of complex generalized designs with rate greater than 0.5 is difficult and we hope that these two examples stimulate further work.

VI. Conclusion

We have developed the theory of space–time block coding, a simple and elegant method for transmission using multiple transmit antennas in a wireless Rayleigh/Rician environment. These codes have a very simple maximum-likelihood decoding algorithm which is only based on linear processing. Moreover, they exploit the full diversity given by transmit and receive antennas. For arbitrary real constellations such as PAM, we have constructed space–time block codes that achieve the maximum possible transmission rate for any number n of transmit antennas. For any complex constellation, we have constructed space–time block codes that achieve half of the maximum possible transmission rate for any number n of transmit antennas. For arbitrary complex constellations and for the specific cases $n = 2$, 3, and 4, we have provided space-time block codes that achieve, respectively, all, 3/4, and 3/4 of the maximum possible transmission rate. We believe that these discoveries only represent the tip of the iceberg.

Appendix

Theorem: A complex orthogonal design of size 4 does not exist.

Proof: The proof is divided into six steps.

Step I: In this step, we provide necessary and sufficient conditions for a 4×4 matrix of indeterminates to be a complex linear processing generalized orthogonal design. To this end, let $\mathcal{L}_c$ be a complex linear processing generalized orthogonal design of size $n = 4$. Each entry of $\mathcal{L}_c$ is a linear combination of $x_1, x_1^*, x_2, x_2^*, \cdots, x_4, x_4^*$. It follows that

$$\mathcal{L}_c = x_1A_1 + x_1^*B_1 + x_2A_2 + x_2^*B_2 + \cdots + x_4A_4 + x_4^*B_4 \tag{41}$$

where $A_1, B_1, A_2, B_2, \cdots, A_4, B_4$ are complex 4×4 matrices. Since

$$\mathcal{L}_c\mathcal{L}_c^* = \mathcal{L}_c^*\mathcal{L}_c = (|x_1|^2 + |x_2|^2 + \cdots + |x_4|^2)I$$

we can conclude from the above that

$$\begin{aligned} A_iA_i^* + B_iB_i^* &= A_i^*A_i + B_i^*B_i = I, && i = 1, \cdots, 4 \\ A_iA_j^* + B_jB_i^* &= A_i^*A_j + B_j^*B_i = 0, && 1 \le i \ne j \le 4 \\ B_iA_j^* + B_jA_i^* &= A_iB_j^* + A_jB_i^* = 0, && 1 \le i \ne j \le 4, \\ A_iB_i^* &= A_i^*B_i = 0, && 1 \le i \le 4. \end{aligned} \tag{42}$$

Conversely, any set of 4×4 complex matrices $A_1, B_1, A_2, B_2, \cdots, A_4, B_4$ satisfying the above equations defines a linear processing complex orthogonal design.

Step II: In this step, we will prove that given a complex linear processing generalized orthogonal design $\mathcal{L}_c$, we could construct another complex linear processing generalized orthogonal design $\mathcal{E}$ such that for any row, one of x_j and x_j^* does not occur in the entries of that row of $\mathcal{E}$. In other words, for any $i = 1, 2, 3, 4$

$$\mathcal{E}_{ik} = \sum_{j=1}^{4} a_{i,j,k}x_j + \sum_{j=1}^{4} b_{i,j,k}x_j^*$$

where for any fixed i either $b_{i,j,k} = 0$ for all $k = 1, 2, 3, 4$ or $a_{i,j,k} = 0$ for all $k = 1, 2, 3, 4$. In the former (respectively, latter) case we say x_j^* (respectively, x_j) does not occur in the ith row of $\mathcal{E}$.

Using (42), we first observe that

$$A_i = A_i(I) = A_i(A_i^*A_i + B_i^*B_i) = A_iA_i^*A_i.$$

Hence

$$A_iA_i^* = A_iA_i^*A_iA_i^* = (A_iA_i^*)^2.$$

Similarly, $B_iB_i^* = (B_iB_i^*)^2$. This means that the matrices $A_iA_i^*$ and $B_iB_i^*$ are idempotent for $i = 1, 2, 3, 4$. Since $A_iA_i^* + B_iB_i^* = I$, the matrices $A_iA_i^*$ and $B_iB_i^*$ represent projections onto perpendicular vector spaces W_i and $W_i^\perp$ and thus are diagonalizable with all eigenvalues in the set $\{0, 1\}$. If $p_i = \mathrm{rank}\,(W_i)$ and $q_i = \mathrm{rank}\,(W_i^\perp) = 4 - p_i$, then exactly p_i (respectively, q_i) of eigenvalues of $A_iA_i^*$ (respectively, $B_iB_i^*$) are 1.

Next, using (42), we observe that for $i \ne j$

$$\begin{aligned} A_iA_i^*A_jA_j^* &= -A_iB_j^*B_iA_j^* = A_jB_i^*B_iA_j^* \\ &= -A_jB_i^*B_jA_i^* = A_jA_j^*A_iA_i^*. \end{aligned}$$

Thus the matrices $A_iA_i^*$ and $A_jA_j^*$ commute. Similarly, it follows that $\{A_iA_i^*, B_iB_i^*, i = 1, 2, 3, 4\}$ is a commuting family of diagonalizable matrices. Hence, these matrices are simultaneously diagonalizable. Since the eigenvalues of $A_iA_i^*$ are in the set $\{0, 1\}$, we conclude that there exists a unitary transformation U such that

$$\begin{aligned} UA_iA_i^*U^* &= D_i^1 \\ UB_iB_i^*U^* &= D_i^2 \end{aligned}$$

where $D_i^1, D_i^2, i = 1, 2, 3, 4$ are diagonal matrices with diagonal entries in the set $\{0, 1\}$. Moreover, because

$$D_i^1 + D_i^2 = U(A_iA_i^* + B_iB_i^*)U^* = I$$

the (j, j)th entry of D_i^1 is zero (respectively, one) if and only if the (j, j)th entry of D_i^2 is one (respectively, zero). Since

$$D_i^1UB_iU^* = UA_iA_i^*U^*UB_iU^* = 0$$

the nonzero entries of UB_iU^* appear in those rows j where the (j, j)th element of D_i^1 is zero. Similarly,

$$D_i^2UA_iU^* = UB_iB_i^*U^*UA_iU^* = 0$$

implies that the nonzero entries of UA_iU^* appear in those rows, where the corresponding diagonal element of D_i^2 is zero. Thus the nonzero entries of UA_iU^* and UB_iU^* occur in different rows.

Let

$$\mathcal{E} = \sum_{i=1}^{4} (UA_iU^*x_i + UB_iU^*x_i^*)$$

then it follows from the matrix equations given in Step I that $\mathcal{E}$ is a complex linear processing generalized orthogonal design with the desired property.

Step III: We can now assume without any loss of generality that $\mathcal{L}_c$ is a complex linear processing generalized orthogonal design with the properties described in Step II.

In this step, we apply Construction II to $\mathcal{L}_c$ and study the properties of the associated real linear processing generalized orthogonal design.

By interchanging x_i with x_i^* everywhere in the design if necessary, we can further assume that only x_1, x_2, x_3, x_4 occur in the first row of $\mathcal{L}_c$. We next apply Construction II to $\mathcal{L}_c$ and construct a real orthogonal design $\mathcal{O}$ of size 8 in variables $x_1^1, x_1^2, x_2^1, x_2^2, \cdots, x_4^1, x_4^2$. The matrix $\mathcal{O}$ can be written as

$$\mathcal{O} = C_1x_1^1 + C_2x_1^2 + C_3x_2^1 + C_4x_2^2 + \cdots + C_7x_4^1 + C_8x_4^2 \tag{43}$$

where $C_1, C_2, \cdots, C_7, C_8$ are real 8×8 matrices. Furthermore, assuming the property established in Step II, we can easily observe by direct computation that

$$\begin{aligned} C_2 &= J_1C_1 \\ C_4 &= J_2C_3 \\ C_6 &= J_3C_5 \\ C_8 &= J_4C_7 \end{aligned} \tag{44}$$

where

$$J_i = F_i \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

where F_i is a diagonal matrix of size 4 whose diagonal entries belong to the set $\{-1, 1\}$. Moreover, the $(1,1)$th entry of $F_i, i = 1,2,3,4$ equals 1. We let $\epsilon_i = (\epsilon_{i,1}, \cdots, \epsilon_{i,4})$ denote the vector whose jth component is the (j,j)th element of F_i. The jth element of ϵ_i is equal to 1 (respectively, -1) if x_i (respectively, x_i^*) occurs in row j.

Using (43) and

$$\mathcal{O}\mathcal{O}^T = \mathcal{O}^T\mathcal{O} = \left(\sum_{i=1}^{4} \left[(x_i^1)^2 + (x_i^2)^2\right]\right) I$$

we arrive at the following set of equations:

$$\begin{aligned} C_iC_i^T &= I_8, && i = 1, \cdots, 8 \\ C_iC_j^T &= -C_jC_i^T, && 1 \le i < j \le 8. \end{aligned} \tag{45}$$

Let $E_i = C_iC_1^T$, then using (44) and (45) we have

$$E_1 = I_8 \tag{46}$$

$$E_2 = J_1 \tag{47}$$

$$E_4 = J_2E_3 \tag{48}$$

$$E_6 = J_3E_5 \tag{49}$$

$$E_8 = J_4E_7 \tag{50}$$

$$E_i^T = -E_i, \qquad i = 2, \cdots, 8 \tag{51}$$

$$E_i^TE_i = E_iE_i^T = I_8, \qquad i = 1, \cdots, 8 \tag{52}$$

$$E_iE_j^T = -E_jE_i^T, \qquad 1 \le i \ne j \le 8. \tag{53}$$

Step IV: We next prove that the matrices E_{2i-1}, E_{2i}, $i = 2,3,4$ anticommute with J_1 and J_i but commute with J_j, $j \ne 1$, and $j \ne i$. First, we observe that by (51) and (53)

$$\begin{aligned} E_{2i-1}J_1^T + J_1E_{2i-1}^T &= E_{2i-1}E_2^T + E_2E_{2i-1}^T = 0 \\ J_iE_{2i-1} + E_{2i-1}^TJ_i^T &= E_{2i} + E_{2i}^T = 0. \end{aligned}$$

Since the matrices J_1, J_i, and E_{2i-1} are antisymmetric, the above equations prove that E_{2i-1} anticommutes with J_1 and J_i. Furthermore, since $(J_jE_{2j-1})^T = E_{2j-1}^TJ_j^T$, we conclude from (46)–(53) that when $j \ne 1$ and $j \ne i$

$$\begin{aligned} J_jE_{2j-1}E_{2i-1}^T + E_{2i-1}E_{2j-1}^TJ_j^T &= E_{2j}E_{2i-1}^T + E_{2i-1}E_{2j}^T = 0 \\ E_{2j-1}E_{2i-1}^T + E_{2i-1}E_{2j-1}^T &= 0 \end{aligned}$$

which implies that

$$\begin{aligned} -J_jE_{2j-1}E_{2i-1} &- E_{2j-1}E_{2i-1}J_j \\ &= J_jE_{2j-1}E_{2i-1}^T - E_{2j-1}E_{2i-1}^TJ_j^T \\ &= J_jE_{2j-1}E_{2i-1}^T + E_{2i-1}E_{2j-1}^TJ_j^T = 0. \end{aligned}$$

Since J_j anticommutes with E_{2j-1}, we arrive at

$$E_{2j-1}J_jE_{2i-1} = E_{2j-1}E_{2i-1}J_j.$$

Because E_{2j-1} is orthogonal, it is invertible and thus whenever $j \ne 1$ and $j \ne i$, we have

$$J_jE_{2i-1} = E_{2i-1}J_j.$$

The assertion for E_{2i} now easily follows since $E_{2i} = J_iE_{2i-1}$.

Step V: Recall that $\epsilon_i = (\epsilon_{i,1}, \cdots, \epsilon_{i,4})$ is the vector whose jth component is the (j,j)th element of F_i. In this step, we prove that any two vectors ϵ_i and ϵ_j have Hamming distance exactly equal to two.

To this end, since E_{2i-1} commutes with J_j for $j \ne 1, j \ne i$ and anticommutes with J_1 and J_i, we can easily conclude from the nonsingularity of E_{2i-1} that $J_j \ne J_i$, for $1 \le i \ne j \le 4$ and $J_j \ne -J_i$ for $1 \le i \ne j \le 4$. Thus the Hamming distance of any two distinct vectors ϵ_i and ϵ_j is neither zero nor four. We first prove that the Hamming distance of any two distinct vectors ϵ_i and ϵ_j cannot be one. To this end, let us suppose that two distinct vectors ϵ_i and ϵ_j have Hamming distance one and differ only in the kth position. Then in the kth row of $\mathcal{L}_c$, we have either occurrences of x_i and x_j^* or occurrences of x_i^* and x_j but not both. In any other row of $\mathcal{L}_c$, we have either occurrences of x_i and x_j or occurrences of x_i^* and x_j^* but not both. It is easy to see that the columns of $\mathcal{L}_c$ cannot be orthogonal to each other.

We next prove that the Hamming distance of any two distinct vectors ϵ_i and ϵ_j cannot be three. To this end, let us suppose that two distinct vectors ϵ_i and ϵ_j have Hamming

distance three. Since $\epsilon_{k,1} = 1$ for all $k = 1,2,3,4$, we conclude that $\epsilon_{i,k} = -\epsilon_{j,k}$ for all $k = 2,3,4$. We can now choose $l \neq i,j$ and observe that the vector ϵ_l is distinct with both ϵ_i and ϵ_j. Moreover, it coincides with both ϵ_i and ϵ_j in the first position. It follows using a simple counting argument that ϵ_l has Hamming distance one with either ϵ_i or ϵ_j. But we just proved that this is not possible.

We conclude that any two distinct vectors ϵ_i and ϵ_j have Hamming distance exactly equal to two.

Step VI: In this step, we will arrive at a contradiction that concludes the proof.

Because any two distinct vectors ϵ_i and ϵ_j have Hamming distance exactly equal to two, the matrix H whose ith row is ϵ_i is a Hadamard matrix. It follows that any two distinct columns of H also have Hamming distance 2. Thus we can now assume without loss of generality that (after possible renaming of the variables and by exchanging the role of some variables with their conjugates) x_1, x_2, x_3, x_4 occur in row one and x_1^*, x_2^*, x_3, x_4 occur in row two of $\mathcal{L}_c$. The first row of $\mathcal{L}_c$ is thus expressible as $x_1v_1 + x_2v_2 + x_3v_3 + x_4v_4$ and the second row of $\mathcal{L}_c$ is of the form $x_1^*w_1 + x_2^*w_2 + x_3w_3 + x_4w_4$ for appropriate vectors $v_i, w_i, i = 1,2,3,4$. Because

$$\mathcal{L}_c\mathcal{L}_c^* = (|x_1|^2 + \cdots + |x_4|^2)I$$

we observe that v_i, w_i are vectors of unit length. Moreover, if $i \neq j$ the vectors v_i and v_j are orthogonal to each other. Since the first and second rows of $\mathcal{L}_c$ are orthogonal, we observe that w_3 is orthogonal to $v_i, i = 1,2,3,4$. This means that $\{w_3, v_1, v_2, v_3, v_4\}$ contains a set of five orthonormal vectors in complex space of dimension 4. This contradiction proves the result. □

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