

Space-Time Coding and Space-Time Channel Modelling for Wireless Communications

Tharaka Anuradha Lamahewa

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Department of Information Engineering
Research School of Information Sciences and Engineering
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Declaration

The contents of this thesis are the results of original research and have not been submitted for a higher degree to any other university or institution.

Much of the work in this thesis has been published or has been submitted for publication as journal papers or conference proceedings. These papers are:

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Tharaka A. Lamahewa
Research School of Information Sciences and Engineering,
The Australian National University,
Canberra,
ACT 0200,
Australia.

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Abstract

In this thesis we investigate the effects of the physical constraints such as antenna aperture size, antenna geometry and non-isotropic scattering distribution parameters (angle of arrival/departure and angular spread) on the performance of coherent and non-coherent space-time coded wireless communication systems. First, we derive analytical expressions for the exact pairwise error probability (PEP) and PEP upper-bound of coherent and non-coherent space-time coded systems operating over spatially correlated fading channels using a moment-generating function-based approach. These analytical expressions account for antenna spacing, antenna geometries and scattering distribution models. Using these new PEP expressions, the degree of the effect of antenna spacing, antenna geometry and angular spread is quantified on the diversity advantage (robustness) given by a space-time code. It is shown that the number of antennas that can be employed in a fixed antenna aperture without diminishing the diversity advantage of a space-time code is determined by the size of the antenna aperture, antenna geometry and the richness of the scattering environment.

In realistic channel environments the performance of space-time coded multiple-input multiple output (MIMO) systems is significantly reduced due to non-ideal antenna placement and non-isotropic scattering. In this thesis, by exploiting the spatial dimension of a MIMO channel we introduce the novel use of linear spatial precoding (or power-loading) based on fixed and known parameters of MIMO channels to ameliorate the effects of non-ideal antenna placement on the performance of coherent and non-coherent space-time codes. The spatial precoder virtually arranges the antennas into an optimal configuration so that the spatial correlation between all antenna elements is minimum. With this design, the precoder is fixed for fixed antenna placement and the transmitter does not require any feedback of channel state information (partial or full) from the receiver. We also derive precoding schemes to exploit non-isotropic scattering distribution parameters of the scattering channel to improve the performance of space-time codes applied on MIMO systems in non-isotropic scattering environments. However, these schemes

require the receiver to estimate the non-isotropic parameters and feed them back to the transmitter.

The idea of precoding based on fixed parameters of MIMO channels is extended to maximize the capacity of spatially constrained dense antenna arrays. It is shown that the theoretical maximum capacity available from a fixed region of space can be achieved by power loading based on previously unutilized channel state information contained in the antenna locations. We analyzed the correlation between different modal orders generated at the transmitter region due to spatially constrained antenna arrays in non-isotropic scattering environments, and showed that adjacent modes contribute to higher correlation at the transmitter region. Based on this result, a power loading scheme is proposed which reduces the effects of correlation between adjacent modes at the transmitter region by nulling power onto adjacent transmit modes.

Furthermore, in this thesis a general space-time channel model for down-link transmission in a mobile multiple antenna communication system is developed. The model incorporates deterministic quantities such as physical antenna positions and the motion of the mobile unit (velocity and the direction), and random quantities to capture random scattering environment modeled using a bi-angular power distribution and, in the simplest case, the covariance between transmit and receive angles which captures statistical interdependency. The Kronecker model is shown to be a special case when the power distribution is separable and is shown to overestimate MIMO system performance whenever there is more than one scattering cluster. Expressions for space-time cross correlations and space-frequency cross spectra are given for a number of scattering distributions using Gaussian and Morgenstern's family of multivariate distributions. These new expressions extend the classical Jake's and Clarke's correlation models to general non-isotropic scattering environments.

List of Acronyms

AOD	angle of departure
AOA	angle of arrival
AWGN	additive white Gaussian noise
BER	bit-error rate
BPSK	binary phase shift keying
CSI	channel state information
MGF	moment generating function
MISO	multiple-input single-output
MIMO	multiple-input multiple-output
OFDM	orthogonal frequency-division multiplexing
PEP	pair-wise error probability
PSD	power spectral density
QPSK	quadrature phase shift keying
SIMO	single-input multiple-output
SISO	single-input single-output
SNR	signal to noise ratio
STBC	space-time block code
STTC	space-time trellis code
UCA	uniform circular array
ULA	uniform linear array
UGA	uniform grid array

Notations and Symbols

\mathbf{A}^\dagger	complex conjugate transpose of matrix \mathbf{A}
\mathbf{a}^\dagger	complex conjugate transpose of vector \mathbf{a}
\mathbf{A}^T	transpose of matrix \mathbf{A}
\mathbf{a}^T	transpose of vector \mathbf{a}
\mathbf{A}^*	complex conjugate of matrix \mathbf{A}
\mathbf{a}^*	complex conjugate of vector \mathbf{a}
$\overline{f(\cdot)}$	complex conjugate of scalar or function $f(\cdot)$
$\ \mathbf{a}\ $	euclidian norm of vector \mathbf{a}
$\ \mathbf{A}\ ^2$	squared norm of matrix \mathbf{A}
$ \mathbf{A} $	determinant of matrix \mathbf{A}
$\text{tr}\{\mathbf{A}\}$	trace of matrix \mathbf{A}
$\text{vec}(\mathbf{A})$	matrix vectorization operator: stacks the columns of \mathbf{A}
\otimes	matrix Kronecker product
$\delta(\cdot)$	Dirac delta function
$\lceil \cdot \rceil$	ceiling operator
$\mathcal{E}\{\cdot\}$	mathematical expectation
\mathbf{I}_n	$n \times n$ identity matrix
$\mathbf{1}$	vector of all ones
\mathbb{S}^1	unit circle
\mathbb{S}^2	unit sphere
$Q(x)$	Gaussian Q -function: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$

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