

section increases with Q^2 , while the two- (or three-) prong fraction decreases.

The s dependence (averaged over Q^2) for the charged multiplicity from both neutron and proton targets is shown in Fig. 3. Also presented are the results from photoproduction,¹⁰ πp scattering,¹¹ the recent experiment on associated multiplicity in $pp \rightarrow pX$,¹² e^+e^- colliding beam results,¹³ and $\bar{p}p$ data.¹⁴ The impressive thing about this comparison is that, except at low s , where one might reasonably expect individual differences, the value of the average multiplicity and its s dependence are remarkably similar, independent of what the colliding particles are, and, in fact, independent of how far off the mass shell they are. This suggests that although the scattered electron may transfer energy and momentum to a single pointlike constituent of the nucleon, the excitation is rapidly thermalized and the final-state multiplicity depends only on center-of-mass energy, in the same way as in any other high-energy collision.

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Space-Time Structure of Hadronic Collisions and Nuclear Multiple Production*

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Nuclear interactions at high energy are sensitive to the space-time structure of basic hadronic collisions. A model of this structure is proposed; it provides a parameter-free account of National Accelerator Laboratory and cosmic-ray emulsion data, which shows that the multiplicity of mesons produced in nuclei exceeds that in hydrogen by an energy-independent ratio remarkably close to unity.

Let τ be a time that characterizes a pp collision in its c.m. frame. In the target rest frame τ is dilated to¹ $\tau' = \tau(E/2m)^{1/2}$. Should this collision occur in a nucleus, and E be sufficiently large, τ' will exceed² the nuclear mean free path λ . At the point the nuclear process becomes sensitive to the short-time behavior of hadronic in-

teractions, and yields information that cannot be inferred directly from the S-matrix elements observed in hydrogen experiments.

One might have expected nuclear multiple production to be a messy phenomenon, but it is enigmatically simple: The mean multiplicity and angular distributions of relativistic secondaries

TABLE I. Multiplicity ratios.

E (GeV)	Target	R	Ref.
67 ^a	Em	1.65 ± 0.04	5
200 ^a	Em	1.73 ± 0.04	5
200 ^a	Em	1.68 ± 0.06	6
~ 1000 ^b	Em	1.71 ± 0.31	7
~ 3000 ^c	Em	1.81 ± 0.17 ^c	8
~ 8000 ^c	Em	1.63 ± 0.12 ^c	9
110 ^d	C	1.18 ± 0.10	10
200 ^d	C	1.10 ± 0.08	10
290 ^d	C	1.15 ± 0.11	10
410 ^d	C	1.16 ± 0.21	10
670 ^d	C	1.33 ± 0.19	10
~ 3000 ^c	C, N, O	1.38 ± 0.19 ^c	8

^a Accelerator proton beam.

^b E determined from heavy primary breakup.

^c E determined by Castagnoli method.

^d E determined by Echo Lake calorimeter.

^e R found by extrapolating the logarithmic fit to \bar{n}_{ch} by M. Antinucci *et al.*, Lett. Nuovo Cimento **6**, 13 (1973). If σ_{in} rises with s , these values of R increase, but so does $\bar{\nu}$, and the success of Eq. (7) is fully maintained. Both n_s and \bar{n}_{ch} are multiplicities for inelastic collisions.

produced in emulsion (Em) nuclei ($\langle A \rangle = 69$) do not differ markedly from what emerges in pp collisions.^{3,4} To be precise, let $R \equiv n_s / \bar{n}_{ch}$, where n_s is the mean number of charged secondaries with $\beta > 0.7$ (95% mesons) in a nuclear collision irrespective of the number of knock-out and evaporation protons, and \bar{n}_{ch} is the mean pp charge multiplicity at the same energy. Values⁵⁻¹⁰ of R measured in C and Em for $E \geq 67$ GeV are listed in Table I. Note the remarkable constancy of R with E . Possibly more astonishing are the small values of R , for the mean number $\bar{\nu}$ of collisions of a proton within one nucleus is¹ 3.2 for Em, and 2.2 for C. The angular distributions of relativistic tracks also reveal little dependence on target size. The mean transverse momentum measured in cosmic-ray emulsions has been confirmed at National Accelerator Laboratory and the CERN intersecting storage rings, while the 200-GeV rapidity distribution^{5,6} in Em agrees with that from pp collisions throughout the projectile hemisphere.¹¹

These facts indicate that the hadronic state traversing the nucleus bears little resemblance to what is finally observed in a pp collision. For if the degrees of freedom of the produced particles were already active in the nucleus, a cas-

cade would ensue and lead to a catastrophic growth of R with both E and A .

As early as 1951 Pomeranchuk had argued¹² that the particle degrees of freedom only become relevant when the volume of the evolving state has grown large enough to enclose the \bar{n} produced particles. Before that time, collective variables must be used, and it was this observation that led to Landau's hydrodynamical model. While we are unwilling to accept that model *in toto*,¹³ we do adapt one essential concept from it¹⁴: *The energy flux of hadronic matter is the essential variable that governs the early evolution of the system, and it is a cascade of this flux, and not of conventional hadrons, that occurs in a nuclear collision.*

To compute the development of the energy-flux cascade we must know how the stress tensor evolves and scatters from nucleons. In lieu of a detailed dynamical theory, we postulate a very naive though not implausible recipe:

(I) Subsequent to a pp collision, hadronic matter is contained in a cylinder expanding¹⁵ uniformly from a disk¹⁶ at $t = z = 0$; the stress tensor in the cylinder is determined by projecting the observed asymptotic flux backward in t via the classical trajectories of free particles.

(II) This flux scatters from nucleons as if it were a set $\mathcal{K}(t)$ of conventional hadrons, $\mathcal{K}(t)$ being determined by dividing the flux into slices each of which has the spatial thickness appropriate to a hadron moving with the mean rapidity of that slice.

Some comments about this recipe are called for. Concerning rule (I), we do *not* assume that the observed \bar{n} particles exist as $t \rightarrow 0$ —we merely assert that their trajectories serve to estimate the energy flux as $t \rightarrow 0$. Indeed, we shall see that all the observed particles only materialize when $t \sim E$. We resort to classical free-particle motion to avoid unfathomable complications, but can offer the following alibis: (i) Classical motion may be a valid approximation because we deal with states having sizable occupation numbers and very short wavelengths. (ii) The observed dominance of short-ranged rapidity correlations, and the quasifree behavior revealed by deep inelastic scattering, may justify the neglect of interactions over the short time intervals needed here. Rule (II) is a universality hypothesis: Whenever any hadronic state occupies the same volume as a nucleon or pion moving with the same rapidity, its behavior in a collision is close to that of an ordinary hadron.¹⁷

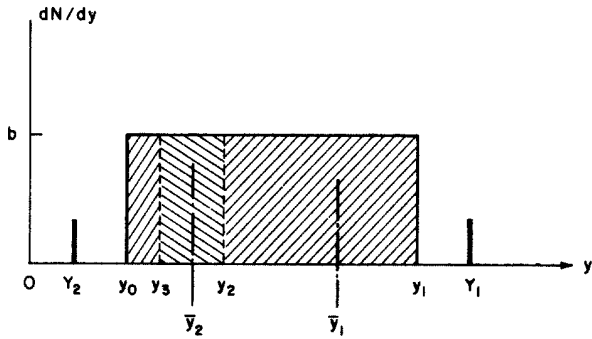


FIG. 1. Rapidity distribution in target frame. Three slices H_i are shown; $Y_1 = \ln(xs/m^2)$ and $Y_2 = -\ln(x)$ are the mean rapidities of the nucleons in the final state.

Our recipe is conveniently formulated with the light-cone coordinates $x_{\pm} = t \pm z$; x_- is small throughout, and $x_+ \approx 2t \approx 2z$. A free particle moving with rapidity y has the trajectory $x_- = e^{-2y}x_+$. The energy flux subsequent to a pp collision can be described by the component T_{+-} of the stress tensor. From rule (I) it is

$$T_{+-} = \sum_k \mu_k e^y (dN_k/dy) \partial y / \partial x_-, \quad (1)$$

where dN_k/dy is the inclusive distribution of secondaries of type k . For simplicity, assume the final state always contains two nucleons with momentum fractions $\pm x$, and pions in a rectangular distribution (see Fig. 1) of height b , and bounded by $y_0 = \ln[b\mu/m(1-x)]$ and $y_1 = \ln[s(1-x)/mb\mu]$. The pionic contribution to (1) is then $-\frac{1}{2}b\mu(x_+/x_-)^{1/2}$ provided $x_+ \exp(-2y_0) \geq x_- \geq x_+ \times \exp(-2y_1)$, and zero otherwise. This flux is highly compressed towards the light cone ($x_- \approx 0$) even after traveling one mean free path ($x_+ \approx 2\lambda$). Therefore a second nucleon is struck by an object that is not readily distinguishable from the incident particle, and if we were to ignore this distinction completely we would expect the multiplicity ratio $R = 1$.

Although the preceding sentence captures the essential point, the spatial expansion of the flux is not entirely negligible. In particular, its thickness at a finite x_+ is s independent, whereas a single hadron has a thickness $\sim s^{-1}$. The flux is therefore equivalent to some set $\mathcal{K}(t) = (H_1, H_2, \dots)$ of hadrons that we now determine from rule (II). Designate the i th slice by H_i , and define the mean rapidity \bar{y}_i of H_i as $\bar{y}_i = \frac{1}{2} \ln(P_{i+}/P_{i-})$, where $P_{i\pm} = E_i \pm P_{iz}$ are the indicated momentum components of the matter in H_i . For the pionic slices ($i > 1$) this just gives $\bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$,

while for H_1 , which has baryon number 1, $\bar{y}_1 = \frac{1}{2}(y_1 + y_2) - \frac{1}{2} \ln(1-x)$ provided $\exp(y_1) \gg \exp(y_2)$. Rule (II) then reads $T_0/\bar{y}_i = (v_i - v_{i+1})t$, where T_0 is the rest-frame thickness of a typical hadron [$T_0 = \frac{4}{3}(\bar{r}^2)^{1/2}$], $\bar{y}_i = \frac{1}{2} \exp(\bar{y}_i)$, and $v_i = 1 - 2 \exp(2y_i)$. For the pionic slices we therefore obtain the recursion formula

$$y_{i+1} = \frac{1}{3}y_i + \frac{2}{3} \ln\{\xi[1 - \exp(-2\Delta_i)]\}, \quad (2)$$

where $\xi = z/T_0 = t/T_0$, and $\Delta_i = y_i - y_{i+1}$ is the thickness of H_i in y ; for H_1

$$y_2 = \frac{1}{3}y_1 + \frac{2}{3} \ln[\xi(1-x)^{-1/2}], \quad (3)$$

if $\exp(y_1) \gg \exp(y_2)$.

It is instructive to ask when and where the secondaries actually materialize. This question can only be answered if one asserts that particles form when the rapidity thickness Δ_i of H_i reaches a minimum value—otherwise our continuum description gives an infinite number of secondaries. The only sensible choice for this value is $1/b$, for then each pion condenses out of an equal portion of y space. It then follows from (2) that H_i becomes a physical pion at $t_i \propto s(e^{-1/b})^{i-1}$. Thus the softest pions materialize when $t \sim s^0$, the hardest when $t \sim s$. Furthermore, the materialization points z_i lie on the invariant surface¹⁸ $t_i^2 - z_i^2 = \text{const}$. In particular, the first pion to emerge does so at a distance

$$T_0 \frac{b\mu}{m(1-x)} \frac{e^{-3/2b}}{1 - e^{-2/b}} \approx 3T_0.$$

But $3T_0 > \lambda$, and therefore real pions have not yet been formed when the pulse strikes the next nucleon.

The constitution of the set $\mathcal{K}(t)$ when $t \approx \lambda$ ($\approx z$) is found from (2) and (3); if (as is the case) $\exp(-2\Delta_i) \ll 1$, and $\alpha_i \equiv 1/3^i$, $\Lambda = \lambda/T_0$,

$$y_{i+1} = \alpha_i y_1 + (1 - \alpha_i) \ln \Lambda - \frac{1}{2} \alpha_i \ln(1-x). \quad (4)$$

This gives the y interval contained in H_i . The total number K of slices is

$$K = (\ln 3)^{-1} \ln \left(\frac{y_1 - \ln \Lambda (1-x)}{y_0 - \ln \Lambda} \right), \quad (5)$$

which grows like $\ln \ln s$; thus,¹⁹ $K = 2$ when $E \approx 100$ GeV, and reaches 3 when $E \approx 10^6$ GeV. Let E_i be the energy of H_i ; then

$$E_1 = E(1 - Cs^{-2/3}), \quad (6)$$

$$E_2 = Cs^{1/3}(1 - \text{const} \times s^{-2/9})/2m,$$

etc., where $C = (b\mu m \Lambda)^{2/3} \approx 1.5$ of s is in GeV^2 . Equation (6) is important, for it shows that H_1

has virtually the full incident energy, whereas H_2 is in a low-energy regime (if $E \approx 10^4$ GeV, $E_2 \approx 20$ GeV). This is also clear from (3), for H_1 contains all $y > \frac{1}{3}y_1 + 0.7$ when $\xi = \Lambda$.

Now imagine a linear array of ν nucleons with spacings of order λ , and let E be well below the value required for $K=3$. When the pulse reaches nucleon 2 it consists of H_1 and H_2 . By rule (II), the H_1 - N and N - N collisions are alike, and therefore another H_1 - H_1 + H_2 transformation occurs by the time the pulse reaches nucleon 3. The H_2 - N collision, on the other hand, does not induce $H_2 \rightarrow H_2 + H_3$ unless E is sufficient for $K=3$. The upshot¹⁹ of all this is that the last nucleon is struck by one H_1 having energy $E[1 - (\nu - 1)Cs^{-2/3}]$, and $\nu - 1$ hadrons H_2 , each of energy E_2 . In this event the multiplicity is therefore $[1 + \frac{1}{3}(\nu - 1)]b \times \ln s + C'$. On averaging over nucleon positions (and noting that only C' depends on their location) we obtain

$$R = 1 + \frac{1}{3}(\bar{\nu} - 1) + O(\ln^{-1}s). \quad (7)$$

With the quoted values of $\bar{\nu}$ one has $R_{EM} = 1.7$ and $R_C = 1.4$, in astonishing agreement with the data in Table I. [Please note that (7) contains no adjustable parameters.] Equation (7) is not truly asymptotic—eventually the third slice H_3 cannot be ignored.^{19,20} But it is adequate for the data at hand.

For the rapidity distribution we expect no difference between nuclei and hydrogen for $y \gtrsim y_2 = 0.7 + \frac{1}{3}y_1$, and an excess by a factor $\sim (\bar{\nu} - 1)$ for $y \lesssim y_2$. This is consistent with the only reliable data presently available.^{5,6,11}

Unfortunately the model makes no detailed statement concerning the leading particle effect. It is clear that there should be only one highly energetic nucleon, and that it must be in the leading slice with $y > y_2$. But this does not tell us whether it has a finite x as $E \rightarrow \infty$.

I am indebted to P. K. F. Grieder, A. J. Herz, and K. Rybicki for bringing this problem to my attention; to L. W. Jones and P. K. Malhotra for unpublished data; and to E. Amaldi, S. J. Chang, E. L. Feinberg, E. M. Friedlander, A. S. Goldhaber, O. Kofoed-Hansen, A. Marin, L. Stodolsky, L. Van Hove, and D. R. Yennie for helpful discussions.

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¹ E is the incident lab energy; m the proton mass; μ_k the mean longitudinal mass of particle k , with $\mu_\pi \equiv \mu \equiv 0.38$ GeV; z the incident direction; $s = 2mE$; b (≈ 2.9) the coefficient of $\ln s$ in the total pp multiplicity; and x the nucleon's mean longitudinal c.m. momentum fraction (≈ 0.6). For a nucleus of radius r , $\bar{\nu} = \frac{1}{3}\rho^3[\frac{1}{2}\rho^2 + e^{-\rho}(1+\rho) - 1]^{-1}$, with $\rho \equiv 2r/\lambda$.

²For protons with $\sigma_{in} = 32$ mb, $\lambda = 1.8$ fm. Thus if $\tau = m^{-1}$ (a rather short time scale), $\tau' \approx \lambda$ when $E \geq 160$ GeV.

³This fact had been surmised long ago from cosmic rays. For a summary see K. Gottfried, in Proceedings of the Fifth International Conference on High Energy Physics and Nuclear Structure, Uppsala, Sweden, June 1973 (to be published), and CERN Report No. TH-1735 (to be published). This report also contains a review of earlier theoretical work on nuclear multiple production.

⁴Gottfried, Ref. 3.

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¹⁰K. N. Erickson, University of Michigan Report No. 03028-4-T, 1970 (unpublished).

¹¹Thus the nuclear excess lies wholly in the target hemisphere at 200 GeV.

¹²Cf. E. L. Feinberg, Phys. Rep. **5C**, No. 5 (1972).

¹³Landau postulated an initial condition which, in modern jargon, assumes that all partons stop momentarily when the projectiles meet. This cannot be reconciled with the data on deep-inelastic electron scattering.

¹⁴A preliminary account of these considerations appeared in Ref. 3. Related ideas have also been studied independently by A. S. Goldhaber, private communication.

¹⁵Transverse motion is ignored throughout.

¹⁶A cautionary remark: A. H. Mueller [Phys. Rev. D **2**, 2241 (1970)] has shown that a hadron observed electromagnetically does *not* contract as $E \rightarrow \infty$.

¹⁷This accords with the discovery that coherently produced multiboson states apparently have the same cross section on nucleons as do pions; cf. W. Beusch, Acta Phys. Pol. B **3**, 679 (1972). For other evidence concerning this hypothesis, see C. Quigg, State University of New York at Stony Brook Report No. ITP-SB-73-47 (to be published).

¹⁸This is also the space-time structure of the multiperipheral model; cf. J. Kogut and L. Susskind, Phys. Rep. **8C**, No. 2 (1973).

¹⁹These K thresholds are very sensitive to x : If $x = 0.5$ instead of 0.6, they change to 15 and 3300 GeV, respectively. This is only important for secondaries

with $y \lesssim y_3$.

²⁰Equation (7) gives a reliable value at all large E for the ratio of secondaries with $y \gtrsim y_3$. The evolution of

the cascade for $y \lesssim y_3$ is somewhat ambiguous when K is not an integer; fortunately this is a very small portion of phase space.

Improved Limit on Photon Rest Mass

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Recent observations of Alfvén waves in the interplanetary medium provide an improved upper limit on the photon rest mass. We find a reliable upper limit $\mu \leq 3.6 \times 10^{-11} \text{ cm}^{-1}$, $m_{\text{ph}} \leq 1.3 \times 10^{-48} \text{ g}$, and a stronger, but less certain upper limit $\mu < 3.1 \times 10^{-12} \text{ cm}^{-1}$, $m_{\text{ph}} < 1.1 \times 10^{-49} \text{ g}$. These represent improvements on the heretofore best reliable estimate by 0.5 and 1.5 orders of magnitude, respectively.

One of the various methods for experimentally determining if the photon has a nonzero rest mass involves the propagation of low-frequency waves in an electrically conducting medium. It is well known¹ that a highly conducting medium can support a variety of propagating waves at frequencies below the proton gyrofrequency. The mode of particular interest here is the intermediate, or Alfvén, wave, with a dispersion relation given by

$$k^2 \cos^2 \theta = \omega^2 / v_A^2, \quad (1)$$

where k is the wave number, ω the (circular) frequency, θ the angle between the direction of propagation and the unperturbed magnetic field, and v_A the Alfvén speed in the medium which supports the wave. It is easy to show^{2,3} that if the photon rest mass m_{ph} is different from zero, then Eq. (1) is modified to read

$$k^2 \cos^2 \theta = \omega^2 / v_A^2 - \mu^2, \quad (2)$$

where $\mu = m_{\text{ph}} c / \hbar$ is the inverse reduced Compton wavelength associated with m_{ph} . Thus if $|\omega| < \mu v_A$, the intermediate mode does not propagate, but becomes evanescent. Conversely, if propagating waves of frequency ω are observed, then one must have

$$\mu < \omega / v_A. \quad (3)$$

This fact has already been used^{3,4} in connection with early spacecraft data to establish that $\mu \leq 10^{-9} \text{ cm}^{-1}$ (i.e., $m_{\text{ph}} \leq 4 \times 10^{-47} \text{ g}$).

Our purpose here is to use recent observations of propagating intermediate waves in the interplanetary medium to set an improved upper limit on μ . It is now well established⁵ that propagating

intermediate waves represent a major contribution to the fluctuations of the interplanetary plasma detected by spacecraft at 1 astronomical unit (A.U.). This identification is based on observations of the correlation between fluctuations in the magnetic field and plasma velocity, which are observed^{5,6} to have a phase shift of 0 or 180°. Such a phase shift is consistent with propagating Alfvén waves, but not consistent with evanescent waves, for which the phase shift would be $\pm 90^\circ$. These waves are of extremely low frequency,⁶ with period $T' \leq 1 \text{ day} = 8.6 \times 10^4 \text{ sec}$ in the spacecraft frame. To get the frequency ω in the local rest frame of the plasma, we must use the Doppler formula

$$2\pi/T' = \omega + \vec{k} \cdot \vec{v}, \quad (4)$$

where \vec{v} is the solar-wind velocity. There is evidence^{7,8} that the waves with periods of several hours or less are propagating nearly parallel to the average interplanetary magnetic field, so that for these waves $\theta \cong 0^\circ$. There is no evidence concerning the direction of propagation of waves with period near 1 day, however. But geometrical "optics" provides an explanation⁹ for the observation that $\theta \cong 0^\circ$ which is independent of the wave period, and so we can assert that $\theta \cong 0^\circ$ also for the longer period waves which have $T' = 1 \text{ day}$. Thus, since the average magnetic field at 1 A.U. is nearly at 45° to \vec{v} , we have $\vec{k} \cdot \vec{v} = 0.707kv$. To obtain k we can make either of two extreme assumptions. First we can assume that it is possible to neglect any effects of μ at even the lowest frequency observed [this assumption is consistent with all of the data, if we assume that the absence of waves with periods longer than 1 day is